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One Dimensional Acoustic Phonon Linearly Coupled to Electron Density

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ABSTRACT We have studied one dimensional acoustic phonon linearly coupled to the electron density. The phonon spectrum is linear with cutoff at the Debye frequency, it is straightforward modification of three dimensional optical phonons. Phonon field results in substituting the dynamical coupling for the screened coulomb interaction. We have shown that the electron-phonon coupling quantitatively changed the phase diagram of Luttinger liquid with a single impurity. The existence of slow and fast polaron modes with different weights in different regimes resulted in different renormalization group flow for a weak scatterer and a weak tunneling link. The resulting phase diagram depending on the parameters of the problem, regimes corresponding to purely metallic or purely insulating behavior and an intermediate regime with two stable fixed points and one unstable, finite conductance fixed point. we have found that the direction of renormalization group flow change upon varying either the relative strength of the electron-electron and electron-phonon coupling or the ratio of Fermi to sound velocities. The obtained results were found in good agreement with previously obtained results.

KEYWORDS Acoustic phonon, Linearly Coupled, Electron Density, Debye Frequency, Coulomb Interaction, Impurity, Renormalization Group, Scattering, Conductance.

INTRODUCTION

Loss et al.[1] and Mathey et al.[2] presented that the electron-phonon coupling in addition to the coulomb electron-electron repulsion in the Luttiger liquid is known to result in the formation of two plaron branches with different propagation velocities. Kane and Fisher[3] studied that the renormalization group flows always go in the direction of stronger scattering for weaker transmission resulted at $T=0$ in the insulting phase for any scatterer. San-Jose etal[4] studied the electron-phonon coupling leaded to $\tilde{\gamma}_{\pm} \to \gamma_{\pm}$, with each of the exponents $\gamma_{\pm}$ changing sign at different values of bulk parameters. The change of sign of γ_{-} was reversed the renormalization group flow, indicating that the weak scatterer became irrelevant for sufficiently strong electron-phonon interaction and the Luttiger liquid thus remained in metallic state. Tomonaga et al.[5] showed that the interacting electron in one dimension are known to form a Luttiger liquid characterized by powerlaw correlation functions. This characterizatic feature of the Luttiger liquid has been established via conductance measurements and scanning tunneling microscopy in both carbon nanotubes[6] and semiconductor quantum wires[7]. Matveev et al.[8] presented that the embedding a potential impurity into the Luttiger liquid leads to a universal impurity independent power law suppression of the transmission amplitude through the Luttiger liquid and suppression of the tunneling density of states near the impurity, with the latter fading away with increasing distance[9-10]. Pandey[11] studied transmission through an impurity in quantumwire is strongly affected by the applied magnetic fields and the shape of the cross section. He found that when the magnetic field entered along the small axis of cross-section, electron transmission was strongly enhanced since the overlap between incident and reflected wave functions became smaller and back scattering was decreased. Vikas et al.[12] studied one dimensional extended Hubbard model coupled to optical bond phonon. They found that the Mott-Peierls transitions occurred at a finite value of electron-phonon coupling. They also found that in the dimerized peierls phase the bond correlation function approached a constant at long distances. Sharma et al.[13] studied that the non interacting systems with linear dispersion, the origin of the spatial oscillations were easily seen. These noise oscillations were affected by band curvature and coulomb interaction. A finite band curvature calculations showed that the spatial oscillations decayed away from the impurity site and showed a beating behaviour. Guang-ZhenKang et al.[14] studied the electron-phonon interaction to the electronic properties of graphene. They presented that the electron phonon interaction has a slight effect on the band structure renormalization at lower doping level. They calculated the renormalization conduction band energy spectrum at higher doping $n=4.5$, $12\times10^{13}cm^{-2}$ and found that the electron-phonon interaction effect on the band structure renormalization became larger with increasing doping. Shi et al.[15] studied the influence of the structural disorder on the spatial distribution of the plasmonic field and its propagation in one dimensional arrays of coupled metallic nanowires. They found that random variation of the radius of coupled plasmonic nanowires are sufficient to induced the Anderson localization of surface plasmon polaritons, the size of these trapped modes being significantly smaller than the optical wavelength. Phan et al.[16] studied and showed that coupling to vibrational degree of freedom can derive a semimetal excitonic-insulator quantum phase transition in a one dimensional two band $f-c$ electron system at zero temperature. The insulating state typifies an excitonic conductance accompanied by a finite lattice distortion. They found that the phonon spectral function indicated that the phonon mode involved in the transition softens or hardness in the adiabatic non adiabatic and extreme adiabatic phonon frequency regime.

METHOD

We have considered on dimensional model acoustic phonons. Integrating out the phonon field in the standard way resulting in substiting the dynamical coupling $V(\varepsilon)=V_0+D_0(\varepsilon)$ for the screened coulomb interaction V_0 in the Luttiger liquid action.

$$S_{LL}=\sum_{\eta=\pm1}\int d\xi\bar{\psi}_{\eta}(\xi)i\partial_{\eta}\psi_{\eta}(\xi)-\frac{1}{2}\int d\xi V(\xi)n^2(\xi) \qquad \text{... (A)}$$

where $\partial_{\eta}=\partial_t+\eta v_F\partial_x$ the spin less electron field is decoupled into sum of left - $(\eta=-1)$ and right $(\eta=1)$ moving terms, $\psi(\xi)=\psi_R(\xi)e^{ip_Fx}+\psi_L(\xi)e^{-ip_Fx}$ with $\xi=(x,t)$ and $n\equiv\bar{\psi}_R\psi_R+\bar{\psi}_L\psi_L$. We have used the Keldysh formalism employing time integration along the Keldysh contour. The free phonon propagator $D_0(\xi)$ in $V(\xi)=V_0+D_0(\xi)$ is obtained by the Fourier transform of its related component.

$$D_0^r(\omega,q)=\frac{v_0^{-1}\alpha_{ph}\omega_q^2}{\omega_{\pm}^2-\omega_q^2},\ \omega_q=cq,\ \ \omega_+\equiv\omega+i0 \qquad \text{... (B)}$$

where c is the sound velocity, α_{ph} is the dimensionless electron-phonon coupling constant and $v_0 = (\pi v_F)^{-1}$ is the free spinless electron desnsity of state. The Hubbard-Stratonovich transformation decouples the n^2 terms in equation (A) and resulted in the mixed fermionic bosonic action in terms of auxiliary bosonic field ψ minimally coupled to ψ.

$$S_{eff} = -\frac{1}{2}\int d\xi \varphi V^{-1}\varphi + i\int d\xi \overline{\psi}_\eta \left(\partial_\eta - \varphi\right)\psi_\eta \qquad \text{... (C)}$$

The coupling term by the transformation

$$\psi_\eta(\xi) \to \psi_\eta(\xi) e^{i\theta_\eta(\xi)}, \quad i\partial_\eta \theta(\xi) = \varphi(\xi) \qquad \text{... (D)}$$

The Fourier transform of its retarded component is given as

$$g_\eta^r(\omega, q) = \left[\omega_+ - \eta v_F q\right]^{-1}$$

The phase θ_η is related to the auxiliary field φ

$$\theta_\eta(\xi) = \int d\xi' g_\eta^B(\xi, \xi')\varphi(\xi')$$

Where g_η^B is the bosonic Green function. Its retarded components coincides with the free fermionic g_η^r. The Green function of the interacting polarons is not gange invariant with respect to the transformation and depends on the correlation function $iU_{\eta\eta'} = \left\langle \theta_\eta \theta_{\eta'} \right\rangle$

The retarded Fourier component of $U_{\eta,\eta'}$ is found as

$$U_{\eta\eta'}^r(\omega, q) = \frac{\omega_+ + \eta' v_F q \; V_0\left(\omega_+^2 - \omega_q^2\right) + v_0^{-1}\alpha_{ph}\omega_q^2}{\omega_+ - \eta\, v_F q \; \left(\omega_+^2 - v_+^2 q^2\right)\left(\omega_+^2 - v_-^2 q^2\right)} \qquad \text{... (E)}$$

Where $v_\pm$ are the velocities of the composite bosonic modes.

$$v_\pm^2 = \frac{1}{2}\left[v^2 + c^2 \pm \sqrt{\left(v^2 - c^2\right)^2 + 4\alpha v^2 c^2}\right] \qquad \text{... (F)}$$

Where v is the speed of plasmonic excitations in the phonon lessLuttiger liquid and $v = v_F\left(1 + v_0 V_0\right)^{\frac{1}{2}} \equiv v_F K^{-1}$, where K is the standard Luttiger parameter. Without phonons when $c = 0$, we have $v = 0$ and $v_+ = v$, so that in this case as well as for $\omega > \omega_0$ or $\alpha_{ph} = 0$, $\psi_{\eta\eta'}$ reduces to the usual Luttiger liquid plasmonic propagator. We have assumed the parameter $\alpha \equiv \alpha_{ph} k^2$ obeying the inequality $\alpha < 1$ to avoid the Wentzel-Bardeen instability corresponding to $v_-^2 < 0$. Then the velocities $v_\pm$ of the slow and fast composite bosonic modes, obey the inequalities $v_- < c$, $v < v_+$. These modes mean that the Luttiger liquid in the presence of the electron-phonon coupling becomes two component. It is the existence of these two modes that leads to rich phase diagram when scatterer is embedded. Renormalization group equation for the backscattering amplitude λ acquires a different prefactor γ_- and is given by

$$\gamma_\sigma = \gamma_{LL} + \sigma\gamma_{LR} = \kappa_\sigma K^{-\sigma} - 1, \; \sigma = \pm 1,$$

$$\kappa_\sigma = \left\{\left[1 - \alpha\delta_{\sigma,-1}\right]\left[1 + \frac{\alpha}{\left(\beta^\sigma + \sqrt{1-\alpha}\right)^2}\right]\right\}^{-\frac{1}{2}}$$

Where $\gamma_{L,L} = \gamma_{R,R}$, $\gamma_{L,R} = \gamma_{R,L}$ and $\beta \equiv \frac{v}{c}$

$$\delta_l \lambda(E) = \begin{cases} (1-K)\lambda(E), & E > \omega_D \\ -\gamma_- \lambda(E), & E < \omega_D \end{cases}$$

Where E is a running cut off and $l \equiv In\frac{E_0}{E}$ with $E_0 \sim \varepsilon_F$ is the band width.

RESULTS AND DISCUSSION

Figure (1) shows that renormalization group flows changed the directions depending on the values of k', σ_{ph} and $\beta_F \equiv \frac{v_F}{c}$. When $k_+(k) \le 1$ while $k_-(k) \ge 1$ resulted in $\gamma_\pm$ changind sign as shown in Figure (1) when the interaction strength k equals $k_\pm$ with $\gamma_\pm(k_\pm) = 0,\ \gamma_-(k_-) = 0$. We have found that $\alpha_{ph} < 1$, when $k_- < k < 1$ there exists three regimes specified by sign of the exponents $\gamma_\pm$ as shown in Figure (1) (a). In the first regime $k_+ < k < 1$ then both $\gamma_\pm$ have a sign opposite to that in the phononless case so that the weak link and weak scatterer amplitudes decreased so that a strong resulted in insulating behavior, while a weak scatterer leaves the Luttiger liquid in the metallic phase. Thus a line of unstable fixed points with a finit $g(k)$ exist, separating the metallic and insulating regimes. In the second regime an intermediate electron-electron interaction, $k_- < k < k_+$ only γ_- has a sign opposite to that in the phononless case. In the third regime a strong electron-electron interaction $k < k_-$,the renormalization group flows remain qualitatively the same as in the phononless case for both weak link and weak scatterer amplitudes. For β_F the purely metallic regime is no longer accessible as shown in graph (a) (b). thus for corresponding values of α_{ph} ie. $\alpha_{ph} > \alpha^*_{ph} \equiv \frac{1}{2} + \frac{1}{2}\sqrt{1 + \frac{4}{\beta_F^2}}$, only two regimes exist on the phase diagram as shown in graph (2). The other properties of the Luttiger liquid are strongly affected by the electron-phonon coupling. The obtained results were compared with previously obtained results and were found in good agreement.

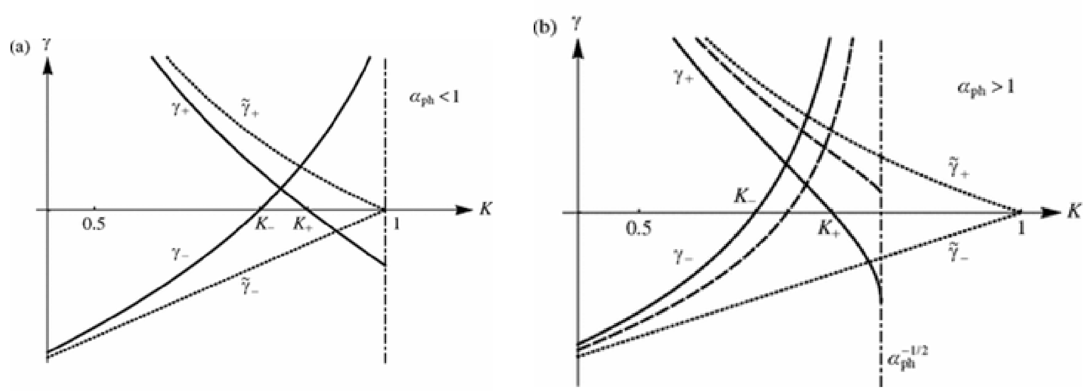

Figure 1: Weak tunneling, γ_+, and weak scattering, γ_-, exponents as function of the Luttinger parameter $K < 1$ weak $\sigma_{ph} < 1$.

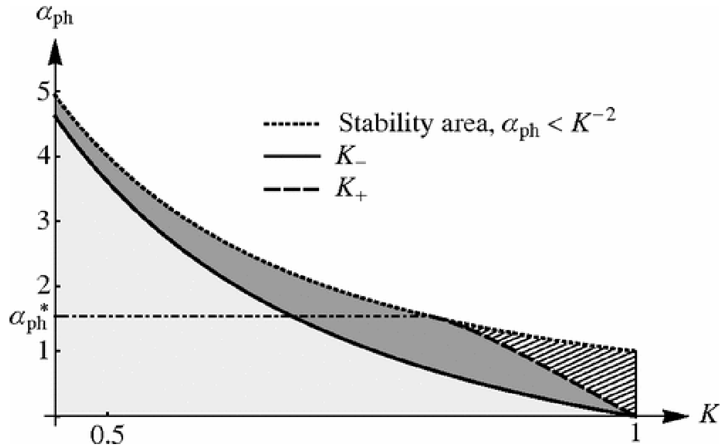

Figure 2: A line of unstable fixed points separating the metallic state for a weak scatterer from the insulating phase for a strong one.

CONCLUSION

We have studied one dimensional acoustic phonons linearly coupled to the electron density. We have also studied the influence of electron-phonon coupling electron transport through a Luttiger liquid with an embedded weak scatterer. We have derived the renormalization group equations, which indicated the direction of renormalization group flows and changed the varying relative strength of the electron-electron and electron-phonon coupling. To obtain result we have employed the functional bosoriztion formalism which allowed including the electron-phonon interaction and leaded to electrons and dressed with phonon. The formation of polarons was made and this was justified in the presence of the electron-phonon coupling for low temperatures. We have found that the electron-phonon coupling qualitatively changed the phase diagram of Luttiger liquid with a single impurity. The obtained results were compared with previously obtained results of theoretical and experimental research works and were found in good agreement.

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