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An Ambiguity in the Relativistic Dynamics and Electromagnetism Regarding Force Transformation

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ABSTRACT Einstein's relativity has always been the subject of considerable controversy among physicists. So far, a few scientists have tried to demonstrate an inconsistency in the theory, whereas the majority of the academics believe that finding a real paradox via, e.g., designing a *thought experiment* is something absurd and impossible. Although many of the paradoxes introduced in the framework of relativity have truly been resolved using precise mathematical expressions, the paradox introduced in this article can be considered as an exception. A simple thought experiment is indeed carried out through which it is shown that the accepted transformation for the electromagnetic (EM) fields, as well as the same transformation for force, culminates in possible inconsistencies in the framework of the special theory of relativity. A tiny charge in a uniform electric field is set in motion over a charged lid located in a compartment shielded against electromagnetic fields within which there is a similar electric field. It is shown that the observer at rest with respect to the tiny charge claims that the lid is easily pushed downward as the charge passes over it, whereas the observer in the compartment's rest frame claims otherwise.

KEYWORDS Special relativity, Reference frames, Charge, Force, Electric field, Magnetic field

INTRODUCTION

When dealing with the relativistic transformations for force and acceleration, it is important to remember that force and acceleration vectors, contrary to the Galilean transformation, are not necessarily superimposed on each other. [1] This means that, according to special relativity, it is possible to have a force vector on a particle in a specific direction, while the mentioned force can neither participate in producing any pressure or do any *work* on the particle in a clear manner, nor can it change or affect the particle's situation, e.g., by accelerating it. This odd phenomenon results in complexities for explaining some relativistic problems. However, in some special cases, it has been proved that force is parallel to acceleration, i.e., when the force vector is exactly parallel *or* perpendicular to the direction of motion. [1] p. 125

In this article, to avoid any complication, we investigate the electromagnetic field transformations for the special case where forces and accelerations as well as the EM fields are all considered to be perpendicular to the relative velocity of the observers. Then, we use the relativistic force transformation to validate the obtained results.

Two charged objects, having a relative velocity v, are subjected to independent, yet similar electric fields in their rest frames. Although each of the observers in the objects' rest frame solely detects an electric field, each one attributes a magnetic field, besides a different electric one, to the other moving object by applying the Lorentz transformation for fields. The related net force measured by one observer, however, is directed oppositely to that measured by the other one though having the same magnitude. We believe this thought experiment shows a possible inconsistency in the theory of special relativity.

MOVING CHARGED OBJECTS IN EM FIELDS

Assume that there is a uniform electric field $E'_y = E$ inside a vehicle shielded with a material that covers the inner space of the compartment from any electric or magnetic field to be penetrated in or out. (This means that the inner fields can neither affect any outer fields, nor vice versa.) A one-way nonconductive valve (lid) is electrically charged ($+q$) on its lower surface and is forcefully pushed upward due to the mentioned electric field. [See Figure 1.] Besides, there is a cylinder attached to the ceiling in which there is a tiny charge $+q$, acting as a frictionless piston, as well as a uniform electric field of $E_y = -E$. The tiny charge is hence pushed down onto the upper frictionless surface of the vehicle. (Remember that these fields cannot act onto each other due to the shielding effect of the vehicle's cover.)

If the vehicle moves at v, from the perspective of the lab observer (N), the electric field inside the cabin, applying the Lorentz transformation for fields, would increase to $E_y = \gamma E$, and a magnetic field of $B_z = \gamma E\, v/c^2$ would grow as well. [1] p. 166 [See Figure 2-a.] By the time the tiny charge, sliding onto the upper surface of the vehicle, reaches the lid, it can easily push the lid into the compartment because the net force exerted on the lid is negative (downward). To prove, we can write:

$$F_1 = -Eq\,, \tag{1}$$

where F_1 is the force of the tiny charge exerted from above on the lid. We also have:

$$F_2 = \gamma Eq - qvB_z\,, \tag{2}$$

where F_2 is the force of the charges exerted from below under the lid. The total force exerted on the lid is thus calculated to be:

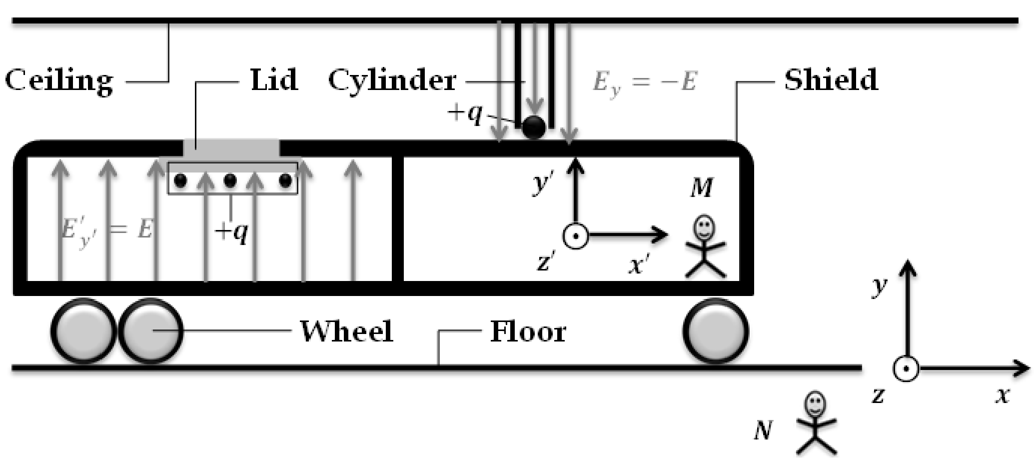

Figure 1: A uniform electric field pushes a charged lid upward in a vehicle at rest with respect to both M & N so that the upper surface of the lid is at the level of the outer surface of the shield. Moreover, a similar electric field pushes a tiny charge (as piston) down onto the frictionless surface of the shield.

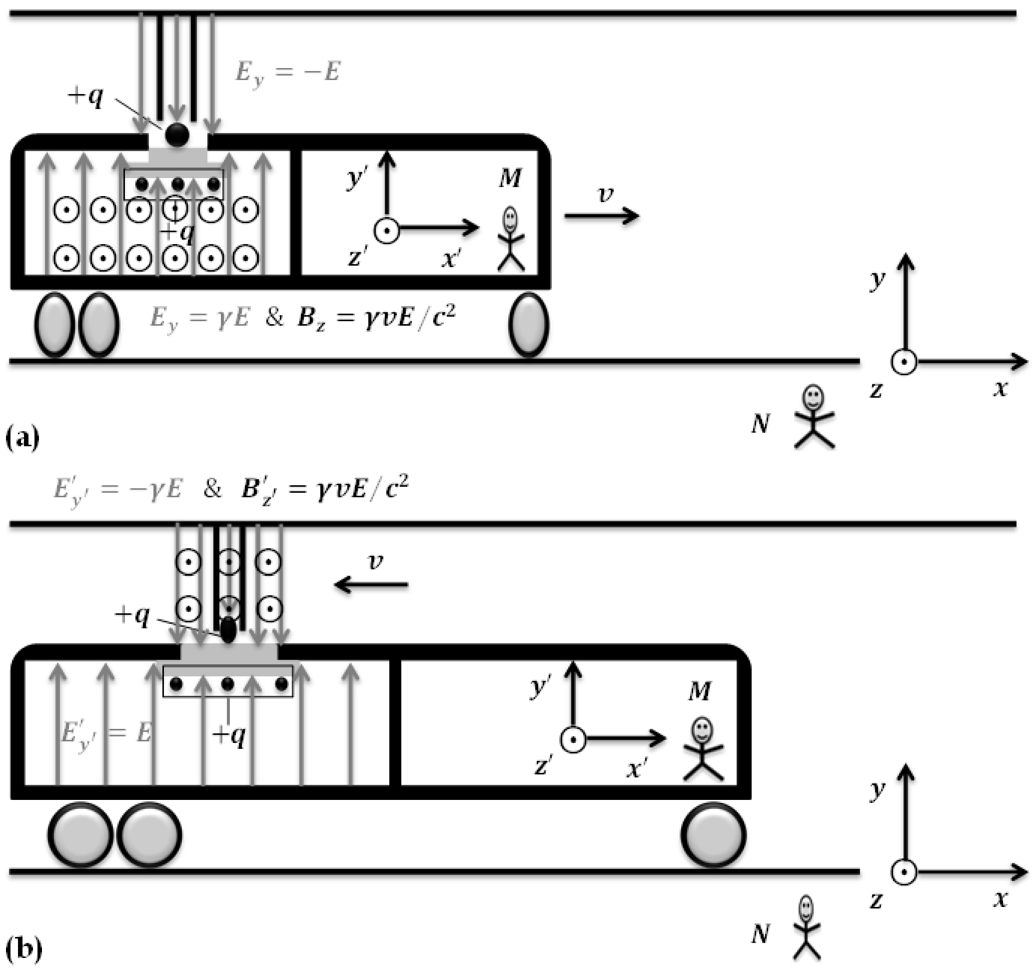

Figure 2: The vehicle is set in motion at v. (a) From the standpoint of N, the vehicle has a velocity v along $+x$ and is thus Lorentz contracted. Using the Lorentz transformation for the EM fields, N asserts that the fields inside the vehicle are $E_y = \gamma E$ & $B_z = \gamma v E/c^2$. (b) From the standpoint of M, the cylinder has a velocity v along $-x$ and is thus Lorentz contracted. Using the Lorentz transformation for the EM fields, M asserts that the fields inside the cylinder are $E'_{y'} = -\gamma E$ & $B'_{z'} = \gamma v E/c^2$.

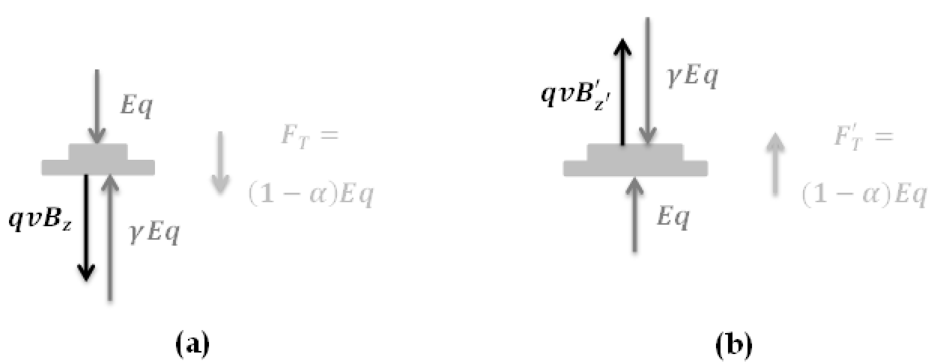

Figure 3: Force diagrams. (a) From the standpoint of N, the total force of F_T is considered to be downward. (b) From the standpoint of M, the total force F'_T is considered to be upward.

$$F_T = F_1 + F_2 = -(1-\alpha)Eq\,, \tag{3}$$

where $\alpha = 1/\gamma = \sqrt{1-v^2/c^2}$. Recall that the minus sign indicates the downward direction. [See Figure 3-a.] Now, from the standpoint of the observer (M) inside the vehicle, the electric field near the cabin would remain unchanged, while the electric field inside the moving cylinder increases to $E'_{y'} = -\gamma E$, and a magnetic field of $B'_{z'} = \gamma E\, v/c^2$ is produced. [1] p. 166 [See Figure 2-b.] When the tiny charge, sliding onto the upper surface of the vehicle, reaches the lid, it *cannot* push the lid into the compartment because the net force exerted on the lid is *positive* (upward). To prove, we can write:

$$F'_1 = -\gamma Eq + qvB'_{z'}\,, \tag{4}$$

where F'_1 is the force of the tiny charge exerted from above on the lid. We also have:

$$F'_2 = Eq\,, \tag{5}$$

where F'_2 is the force of the charges exerted from below under the lid. The total force exerted on the lid is thus calculated to be:

$$F'_T = F'_1 + F'_2 = +(1-\alpha)Eq\,. \tag{6}$$

Recall that the plus sign indicates the upward direction. [See Figure 3-b.] Indeed, N asserts that the tiny charge would fall into the compartment, while M says that the charge can easily pass over the lid and survives falling into the compartment. (Paradox)

The obtained results can easily be verified using the relativistic force transformation without involving with any EM fields straightforwardly. If we denote $F_1 = -Eq$ the force exerted by the tiny charge on the lid in the lab frame of reference (N), and $F'_2 = Eq$ the force exerted on the lid from below in M's rest frame of reference, the relativistic force transformation, as applied by N, implies: [1] p. 148, [2]

$$F_2 = \alpha F'_2 = \alpha Eq\,. \tag{7}$$

The total force is calculated as follows:

$$F_T = F_1 + F_2 = -Eq + \alpha Eq = -(1-\alpha)Eq\,, \tag{8}$$

which is compatible with Eq. (3). Moreover, when M applies the relativistic force transformation, he obtains:

$$F'_1 = \alpha F_1 = -\alpha Eq\,. \tag{9}$$

(Recall that each rest observer measures a reduction for the force of the other moving frame by factor α.) The total force is calculated as follows:

$$F'_T = F'_1 + F'_2 = -\alpha Eq + Eq = +(1-\alpha)Eq\,, \tag{10}$$

which is compatible with Eq. (6). Using force transformation, it is indeed demonstrated that regardless of whatever devices or apparatuses produce the forces, either electrical or mechanical, this paradox is always valid. The calculations presented here verify the result of the author's previous article in which two parallel springs are used instead of the EM fields. [3]

IMPORTANT NOTES REGARDING THIS PARADOX

1. The thought experiment is carried out away from any gravitational field.
2. The lid, as such, is a shield. Therefore, the effect of the Coulomb force between the charges, as the tiny charge meets the lid, is neglected.
3. The lower surface of the lid can be charged either with a point particle $+q$, or with a uniform linear charge density $\lambda = +q/L$, where L is the length of the lid. Remember that the increase of the linear charge density due to length contraction as measured by N does not affect the magnitude of the force exerted from below on the lid because this force only depends on the uniform electric field in the compartment and lid's total charge of $+q$
4. When the lid moves downward, in terms of its velocity v_y along $-y$, N asserts that an additional Lorentz force of $F_x = -qv_yB_z$ acts on the charged lid along $-x$ due to its motion at v_y in B_z, while there is no such a horizontal force exerting on the tiny charge. Furthermore, a similar force of $F'_{x'} = -qv_yB'_{z'}$ is exerted on the tiny charge along $-x$ due to its motion at v_y in $B'_{z'}$ from the viewpoint of M, while there is no such a horizontal force exerting on the lid! Although this odd phenomenon needs additional explanation to show whether or not any horizontal acceleration is produced along $-x$ due to F_x & $F'_{x'}$, we can completely neglect it by supposing that the velocity v_y of the lid / tiny charge is very small compared to c.
5. We did not involve the mass of the tiny charge and that of the lid because, as mentioned previously, the acceleration vectors are parallel to and in the direction of the force vectors. Therefore, whatever the magnitudes of the masses are, the net force direction determines the motion direction of the lid and tiny charge.
6. This paradox somehow reminds us of Supplee's submarine paradox in which two opposite forces of buoyancy and gravitation act on a submarine. [4] However, since there is no gravitational field in our paradox, no resolutions based on general relativity are applicable to it contrary to those traditionally applied to Supplee's. [5,6]

CONCLUSION

Although the relativistic force transformation is compatible with the transformation of EM fields, it is demonstrated through a thought experiment that these transformations can bring about anomalies in the realm of the special theory of relativity.

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