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The Quantization of a Theory of Charged Scalar Fields

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ABSTRACT The quantum theory of the charged scalar field will be developed for an action consistent with a real component of the equal-time commutator in the near classical limit. A conserved current is derived, and the charge $(5/3)^3$ is times larger than the conventional value. A connection between the mass hierarchies of the weak and strong nuclear interactions are found as a result of the theoretical predictions of a mixed theory of vector and pseudoscalar particles.

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1. INTRODUCTION

The charged scalar field is one of the fields that arises in the description of pion interactions [1][2] and phenomena with an electromagnetic field [3][4]. Given that a classical limit of the quantum theory requires a modification of the field equations, the perturbative expansion of the scattering elements shall be considered.

The $(\varphi^\dagger\varphi)^2$ theory is very similar to the φ^4 theory, except that antiparticles are im- mediately introduced. Furthermore, there is symmetry of the Lagrangian for complex scalar fields which is not present for φ^4 theory. The consequences of this symmetry will be investigated in Section 2. It is found that the definition of the current through the variation of the Lagrangian under the transformation $\phi \to e^{i\alpha}\phi,\ \phi^\dagger \to e^{-i\alpha}\phi^\dagger$ yields two cancelling terms to leading order in α resulting from the opposite signs of the contributions of the scalar field and its conjugate. Selecting one of the terms yields a nonvanishing expression. The variational method produces a current density that is a contravariant vector only in three and not four dimensions. Nevertheless, it may be established that there is a constant charge with an increased magnitude at nearly classical scales by Stokes' theorem in three dimensions.

The renormalizability of the theory will be examined by an evaluation of diagrams that arise with the new set of momentum space rules. It is found that the theory remains renormalizable.

2. CONSERVED CURRENTS

The charged scalar field action requires a scalar field and it is written customarily in the form

$$\mathcal{L} = \frac{1}{2}(\partial^\mu\phi)^\dagger(\partial_\mu\phi) - \frac{1}{2}m^2\phi^\dagger\phi \tag{2.1}$$

Consistency of the quantum commutators of the scalar field and the conjugate momentum operators at classical scales engenders the introduction of a real component and matrix term to preserve hermiticity [5]. The differential form of the momentum and energy then would be

$$\begin{aligned}\tilde{p}_i &= \kappa_0 \gamma^i \frac{\partial}{\partial x_i} - i\hbar \frac{\partial}{\partial x_i} \\ \tilde{E} &= \kappa_0 \gamma_5 \frac{\partial}{\partial t} - i\hbar \frac{\partial}{\partial t}\end{aligned} \tag{2.2}$$

From the relativistic formula

$$E = (\vec{p_i})^2 c^2 + m^2 c^4,$$

the trace of the matrix equation [6] gives

$$-3\frac{\partial^2 \phi}{\partial t^2} - 5\sum_i \frac{\partial^2 \phi}{\partial x_i^2} + m^2 \phi = 0 \tag{2.3}$$

with $\kappa_0 = \hbar$ in units with $\hbar = c = 1$.

With the generalized momentum and energy operators, the Lagrangian be

$$\frac{1}{2}\left(\frac{\partial \phi^\dagger}{\partial t}\frac{\partial \phi}{\partial t}\right) + \frac{5}{6}\sum_i \frac{\partial \phi^\dagger}{\partial x_i}\frac{\partial \phi}{\partial x_i} + \frac{1}{6}m^2 \phi^\dagger \phi \tag{2.4}$$

An interaction Lagrangian may be introduced through

$$\mathcal{L}_I = \tfrac{1}{2!2!}\lambda(\phi^\dagger \phi)^2.$$

By contrast with φ^4 theory, the Lagrangian is invariant under the transformation $\varphi \rightarrow e^{i\alpha}\varphi$ for constant α. It can be made to be a local symmetry under certain conditions on $\alpha(x)$ and will give rise to a current. The conservation of this current and the constancy of the charge will be considered with respect to quantum corrections arising in the perturbative series for the matrix elements.

The field equations of the original noninteracting Lagrangian are

$$\begin{aligned}(\Box + m^2)\phi &= 0 \\ (\Box + m^2)\phi^\dagger &= 0\end{aligned} \tag{2.5}$$

Consider variation of the Lagrangian under a phase rotation $\varphi \rightarrow e^{i\alpha}\varphi$ with respect to the two fields φ and $\varphi^\dagger$ only,

$$\begin{aligned}\delta\mathcal{L} &= \frac{1}{2}\partial^\mu(e^{-i\alpha}\phi^\dagger)\partial_\mu(e^{i\alpha}\phi) - \frac{1}{2}m^2(e^{-i\alpha}\phi^\dagger)(e^{i\alpha}\phi) - \mathcal{L} \\ &= \frac{1}{2}[-i\partial^\mu\alpha\ \phi^\dagger + \partial^\mu\phi^\dagger][i\partial_\mu\alpha\ \phi + \partial_\mu\phi] - \frac{1}{2}m^2\phi^\dagger\phi - \mathcal{L} \\ &= \frac{1}{2}\partial^\mu\alpha\partial_\mu\alpha\phi^\dagger\phi + \frac{i}{2}[\partial_\mu\alpha\ \phi\partial^\mu\phi^\dagger - \partial^\mu\alpha\ \phi^\dagger\partial_\mu\phi].\end{aligned} \tag{2.6}$$

Since

$$\begin{aligned}
&\int d^4x \frac{i}{2}\partial^\mu\alpha(\phi\partial_\mu\phi^\dagger - \phi^\dagger\partial_\mu\phi) \\
&= \int d^4x \frac{i}{2}\partial^\mu[\alpha(\phi\partial_\mu\phi^\dagger - \phi^\dagger\partial_\mu\phi)] - \int d^4x \frac{i}{2}\alpha\partial^\mu(\phi\partial_\mu\phi^\dagger - \phi^\dagger\partial_\mu\phi) \\
&= \int d^4x \frac{i}{2}\partial^\mu[\alpha(\phi\partial_\mu\phi^\dagger - \phi^\dagger\partial_\mu\phi)] - \int d^4x \frac{i}{2}[\phi\partial^\mu\partial_\mu\phi^\dagger - \phi^\dagger\partial^\mu\partial_\mu\phi] \\
&= \int d^4x \frac{i}{2}\partial^\mu[\alpha(\phi\partial_\mu\phi^\dagger - \phi^\dagger\partial_\mu\phi)]
\end{aligned} \tag{2.7}$$

after integration by twice in the second integral, when α is an infinitesimal parameter,

$$\begin{aligned}
\delta'\int d^4x\mathcal{L} &= \int d^4x\ \partial_\mu K^\mu + \mathcal{O}(\alpha^2) \\
K^\mu &= \frac{i}{2}\alpha(\phi\partial_\mu\phi^\dagger - \phi^\dagger\partial_\mu\phi).
\end{aligned} \tag{2.8}$$

Now suppose that α is not infinitesimal. The integral

$$\frac{1}{2}\int d^4x\ \partial^\mu\alpha\partial_\mu\alpha\phi^\dagger\phi \tag{2.9}$$

does not consist of a total derivative in general, and it vanishes only when either $\partial_\mu\alpha = 0$ or $\partial^\mu\alpha\partial_\mu\alpha = 0$. Then α is constant, representing a global symmetry, or the gradient of the function α must be a null vector. Let $\partial^\mu\alpha = (\omega, 0, 0, \omega)$, with ω being a constant. Then $\alpha = (t + z) + \theta$, where θ is constant. If ω is a function of x^μ,

$$\alpha = \int\ dt\ \omega(x^\mu) + \int\ dz\ \omega(x^\mu) + \theta.$$

A similar conclusion follows for the other two coordinate directions.

It remains to be determined whether this degree of freedom in $\alpha(x)$ forms a group. When ω is constant, a rotation by $\alpha_1 = \omega_1(t + z) + \theta_1$ and then $\alpha_2 = \omega_2(t + z) + \theta_2$ gives a phase $\alpha_1+\alpha_2 = (\omega_1+\omega_2)(t+z)+(\theta_1+\theta_2)$ with gradient $\partial^\mu(\alpha_1+\alpha_2) = (\omega_1+\omega_2, 0, 0, \omega_1+\omega_2)$, which is a null vector. Similarly, if ω is variable, the combination of the two rotational angles is

$$\alpha_1(x^\mu) + \alpha_2(x^\mu) = \int\ dt\ (\omega_1(x^\mu) + \omega_2(x^\mu)) + \int\ dz\ (\omega_1(x^\mu) + \omega_2(x^\mu)) + (\theta_1 + \theta_2)$$

which has a null gradient vector $\partial^\mu(\alpha_1(x^\mu)+\alpha_2(x^\mu)) = (\omega_1(x)+\omega_2(x), 0, 0, \omega_1(x)+\omega_2(x))$.

Both the phase and the gradient may be evaluated modulo 2π. Therefore, the entire set of angles will be comprise the unit circle regardless of the functional dependence of $\omega(x^\mu)$. Consequently, the restriction of the angular rotations being added as local functions of the coordinates requires that the group is $U(1)$. Therefore, although the functional dependence of $\alpha(x^\mu)$ would be restricted by $\frac{\partial\alpha}{\partial t} = \frac{\partial\alpha}{\partial z}$ or two similar conditions for the other coordinates, together with Lorentz boosts of the gradient vector, the phases form a local $U(1)$ group at each point x^μ.

Setting

$$\alpha\ J'^\mu = \frac{\partial\mathcal{L}}{\partial\partial_\mu\phi}\delta\phi + \frac{\partial\mathcal{L}}{\partial\partial_\mu\phi^\dagger}\delta\phi^\dagger - K'^\mu, \tag{2.10}$$

where

$$\partial_\mu K'^\mu = \frac{\partial \mathcal{L}}{\partial \phi}\delta\phi + \frac{\partial \mathcal{L}}{\partial \phi^\dagger}\delta\phi^\dagger,$$

the divergence is

$$\begin{aligned}\partial_\mu(\alpha J'^\mu) &= \partial_\mu \frac{\partial \mathcal{L}}{\partial\partial_\mu\phi}\delta\phi + \frac{\partial L}{\partial\partial_\mu\phi}\partial_\mu(\delta\phi) + \partial_\mu\frac{\partial L}{\partial\partial_\mu\phi^\dagger}\delta\phi^\dagger + \frac{\partial \mathcal{L}}{\partial\partial_\mu\phi^\dagger}\partial_\mu(\delta\phi^\dagger) - \partial_\mu K'^\mu \\ &= \partial_\mu \frac{\partial \mathcal{L}}{\partial\partial_\mu\phi}\delta\phi + \frac{\partial L}{\partial\partial_\mu\phi}\partial_\mu(\delta\phi) + \partial_\mu\frac{\partial L}{\partial\partial_\mu\phi^\dagger}\delta\phi^\dagger + \frac{\partial \mathcal{L}}{\partial\partial_\mu\phi^\dagger}\partial_\mu(\delta\phi^\dagger) \\ &- \frac{\partial \mathcal{L}}{\partial\phi}\delta\phi - \frac{\partial \mathcal{L}}{\partial\phi^\dagger}\delta\phi^\dagger \\ &= \frac{\partial L}{\partial\partial_\mu\phi}\partial_\mu(\delta\phi) + \frac{\partial \mathcal{L}}{\partial\partial_\mu\phi^\dagger}\partial_\mu(\delta\phi^\dagger)\end{aligned} \tag{2.11}$$

which could be zero only when $\delta\varphi$ and $\delta\varphi^\dagger$ are constant. However, $\delta\varphi$ is a function of the coordinates generally and this condition will not be satisfied. Therefore, it is necessary to use a different vector K^μ such that

$$\frac{\partial \mathcal{L}}{\partial\phi}\delta\phi + \frac{\partial \mathcal{L}}{\partial\phi^\dagger}\delta\phi^\dagger + \frac{\partial \mathcal{L}}{\partial\partial_\mu\phi}\delta(\partial_\mu\phi) + \frac{\partial \mathcal{L}}{\partial\partial_\mu\phi^\dagger}\delta(\partial_\mu\phi^\dagger) = \partial_\mu K^\mu. \tag{2.12}$$

Then, since $\partial_\mu(\delta\varphi) = \delta(\partial_\mu\varphi)$ and $\partial_\mu(\delta\varphi^\dagger) = \delta(\partial_\mu\varphi^\dagger)$, the divergence of the current density given by

$$\alpha J^\mu = \frac{\partial \mathcal{L}}{\partial\partial_\mu\phi}\delta\phi + \frac{\partial \mathcal{L}}{\partial\partial_\mu\phi^\dagger}\delta\phi^\dagger - K^\mu \tag{2.13}$$

vanishes.

Given that

$$\begin{aligned}&\frac{\partial L}{\partial\phi}\delta\phi + \frac{\partial L}{\partial\phi^\dagger}\delta\phi^\dagger + \frac{\partial L}{\partial\partial_\mu\phi}\delta(\partial_\mu\phi) + \frac{\partial L}{\partial\partial_\mu\phi^\dagger}\delta(\partial_\mu\phi^\dagger) \\ &\quad = -\frac{1}{2}m^2\phi^\dagger(e^{i\alpha}-1)\phi - \frac{1}{2}m^2(e^{-i\alpha}-1)\phi^\dagger \\ &\quad + \frac{1}{2}\partial^\mu\phi^\dagger[i\partial_\mu\alpha e^{i\alpha}\phi + (e^{i\alpha}-1)\partial_\mu\phi] + \frac{1}{2}\partial^\mu\phi[-i\partial_\mu\alpha e^{-i\alpha}\phi^\dagger + (e^{-i\alpha}-1)\partial_\mu\phi^\dagger \\ &\quad = (\cos\,\alpha - 1)[\partial^\mu\phi^\dagger\partial_\mu\phi - m^2\phi^\dagger\phi] + \frac{i}{2}\partial_\mu\alpha[e^{i\alpha}\partial^\mu\phi^\dagger - e^{-i\alpha}\phi^\dagger\partial^\mu\phi],\end{aligned} \tag{2.14}$$

the integral of the first term is

$$\int d^4x\ (cos\ \alpha - 1)\partial^\mu\phi^\dagger\partial_\mu\phi = \int d^4x\ (cos\ \alpha - 1)[\partial^\mu(\phi^\dagger\partial_\mu\phi) - \phi^\dagger\partial^\mu\partial_\mu\phi]$$
$$= \int d^4x \bigg\{\frac{1}{2}\partial^\mu[(cos\ \alpha - 1)(\phi^\dagger\partial_\mu\phi + \phi\partial_\mu\phi^\dagger)] - \frac{1}{2}sin\ \alpha\ \partial_\mu\alpha(\phi^\dagger\partial_\mu\phi + \phi\partial^\mu\phi^\dagger)$$
$$- \frac{1}{2}(cos\ \alpha - 1)(\phi^\dagger\partial^\mu\partial_\mu\phi + \phi\partial^\mu\partial_\mu\phi^\dagger)\bigg\}. \tag{2.15}$$

The integral of the total derivative would vanish for scalar fields vanishing at spatial infinity. Therefore, the integral of the entire expression (2.12) is

$$\int d^4x \bigg\{ - \frac{1}{2}\frac{e^{i\alpha} - e^{-i\alpha}}{2i}\partial_\mu\alpha(\phi^\dagger\partial^\mu\phi + \phi\partial^\mu\phi^\dagger)$$
$$- \frac{1}{2}(cos\ \alpha - 1)((\phi^\dagger\partial^\mu\partial_\mu\phi + \phi\partial^\mu\partial_\mu\phi^\dagger) + m^2\phi^\dagger\phi) + \frac{i}{2}\partial_\mu\alpha\ [e^{i\alpha}\phi\partial^\mu\phi^\dagger - e^{-i\alpha}\phi^\dagger\partial^\mu\phi]\bigg\}. \tag{2.16}$$

For infintesimal α, and, by the equations of motion, it equals

$$\int d^4x\ \frac{i}{2}\partial_\mu\alpha[\phi\partial^\mu\phi^\dagger - \phi^\dagger\partial^\mu\phi] + \mathcal{O}(\alpha^2). \tag{2.17}$$

which is equivalent to the integral $\int d^4x\ \delta\mathcal{L}$ when $\partial^\alpha\partial_\mu\alpha = 0$. Therefore, $\partial_\mu K^\mu$ may be identified with δL. After integration by parts in $\int d^4x\ \partial_\mu K^\mu$, K^μ may be identified with $\frac{i}{2}\alpha(e^{i\alpha}\phi\partial^\mu\phi^\dagger - e^{-i\alpha}\phi^\dagger\partial^\mu\phi) + \mathcal{O}(\alpha^2)$, and

$$\alpha J^\mu = \frac{1}{2}\partial^\mu\phi^\dagger(e^{i\alpha} - 1)\phi + \frac{1}{2}\phi^\dagger(e^{-i\alpha} - 1)\partial_\mu\phi - \frac{i}{2}\alpha(e^{i\alpha}\phi\partial^\mu\phi^\dagger - e^{-i\alpha}\phi^\dagger\partial^\mu\phi) + \mathcal{O}(\alpha^2)$$
$$= \frac{i}{2}\alpha(\phi\partial^\mu\phi^\dagger - \phi^\dagger\partial^\mu\phi) - \frac{i}{2}\alpha(\phi\partial^\mu\phi^\dagger - \phi^\dagger\partial^\mu\phi) + \mathcal{O}(\alpha^2). \tag{2.18}$$

The cancellation between the two terms in Eq. (2.18) may be attributed to the contributions of both the scalar field and its conjugate giving rise to currents with charges of the opposite signs. Selecting only one of the terms yields the current density

$$\frac{i}{2}(\phi\partial^\mu\phi^\dagger - \phi^\dagger\partial^\mu\phi).$$

The charge then is

$$Q = \int d^3x\ Im(\phi^\dagger\overleftrightarrow{\partial^0}\phi).$$

Under the transformations $\phi \to e^{i\alpha}\phi,\ \phi^\dagger \to e^{-i\alpha}\phi^\dagger$, the Lagrangian derived from a generalized momentum operator transforms as

$$\begin{aligned}\mathcal{L} \to \frac{1}{2}\frac{\partial}{\partial t}(e^{-i\alpha}\phi^\dagger)\frac{\partial}{\partial t}(e^{i\alpha}\phi) + \frac{5}{6}\sum_i \frac{\partial}{\partial x_i}(e^{-i\alpha}\phi^\dagger)\frac{\partial}{\partial x_i}(e^{i\alpha}\phi) + \frac{1}{6}m^2(e^{-i\alpha}\phi^\dagger)(e^{i\alpha}\phi) \\ = \frac{1}{2}\left[\left(\frac{\partial\alpha}{\partial t}\right)^2\phi^\dagger\phi + i\frac{\partial\alpha}{\partial t}\frac{\partial\phi^\dagger}{\partial t}\phi - i\frac{\partial\alpha}{\partial t}\phi^\dagger\frac{\partial\phi}{\partial t} + \frac{\partial\phi^\dagger}{\partial t}\frac{\partial\phi}{\partial t}\right] \\ + \frac{5}{6}\left[\sum_j\left(\frac{\partial\alpha}{\partial x_j}\right)^2\phi^\dagger\phi + i\sum_j\frac{\partial\phi^\dagger}{\partial x_j}\frac{\partial\alpha}{\partial x_j}\phi - i\sum_j\frac{\partial\alpha}{\partial x_j}\phi^\dagger\frac{\partial\phi}{\partial x_j} + \sum_j\frac{\partial\phi^\dagger}{\partial x_j}\frac{\partial\phi}{\partial x_j}\right] \\ + \frac{1}{6}m^2\phi^\dagger\phi\end{aligned} \tag{2.19}$$

and

$$\begin{aligned}\delta\mathcal{L} = \left[\frac{1}{2}\left(\frac{\partial\alpha}{\partial t}\right)^2 + \frac{5}{6}\sum_j\left(\frac{\partial\alpha}{\partial x_j}\right)^2\right]\phi^\dagger\phi + \frac{i}{2}\frac{\partial\phi}{\partial t}\left(\phi\frac{\partial\phi^\dagger}{\partial t} - \phi^\dagger\frac{\partial\phi}{\partial t}\right) \\ + \frac{5}{6}i\sum_i\frac{\partial\alpha}{\partial x_j}\left(\phi\frac{\partial\phi^\dagger}{\partial x_j} - \phi^\dagger\frac{\partial\phi}{\partial x_j}\right).\end{aligned} \tag{2.20}$$

Setting this expression equal to

$$\partial_\gamma K^\gamma,$$

which equals

$$\frac{\partial K^0}{\partial t} + \sum_i \frac{\partial K^j}{\partial x_j}$$

in the Euclidean metric, and supposing that the function α satisfies the equation

$$\frac{1}{2}\left(\frac{\partial\alpha}{\partial t}\right)^2 + \frac{5}{6}\sum_i\left(\frac{\partial\alpha}{\partial x_i}\right)^2 = 0, \tag{2.21}$$

it follows that

$$\frac{\partial K^0}{\partial t} + \sum_j\frac{\partial K^j}{\partial x_j} = \frac{i}{2}\frac{\partial\alpha}{\partial t}\left(\phi\frac{\partial\phi^\dagger}{\partial t} - \phi^\dagger\frac{\partial\phi}{\partial t}\right) + \frac{5}{6}i\sum_j\frac{\partial\alpha}{\partial x_j}\left(\phi\frac{\partial\phi^\dagger}{\partial x_j} - \phi^\dagger\frac{\partial\phi}{\partial x_j}\right) \tag{2.22}$$

If

$$\begin{aligned}K^0 = \frac{1}{2}i\alpha\left(\phi\frac{\partial\phi^\dagger}{\partial t} - \phi^\dagger\frac{\partial\phi}{\partial t}\right) \\ K^j = \frac{5}{6}i\alpha\left(\phi\frac{\partial\phi^\dagger}{\partial x_j} - \phi^\dagger\frac{\partial\phi}{\partial x_j}\right),\end{aligned} \tag{2.23}$$

$$\frac{\partial K^0}{\partial t} = \frac{1}{2} i \frac{\partial \alpha}{\partial t} \left(\phi \frac{\partial \phi^\dagger}{\partial t} - \phi^\dagger \frac{\partial \phi}{\partial t} \right) + \frac{1}{2} i\alpha \left(\phi \frac{\partial^2 \phi^\dagger}{\partial t^2} - \phi^\dagger \frac{\partial^2 \phi}{\partial t^2} \right)$$
$$\frac{\partial K^j}{\partial x_j} = \frac{5}{6} i \frac{\partial \alpha}{\partial x_j} \left(\phi \frac{\partial \phi^\dagger}{\partial x_j} - \phi^\dagger \frac{\partial \phi}{\partial x_j} \right) + \frac{5}{6} i\alpha \left(\phi \frac{\partial^2 \phi^\dagger}{\partial x_j} - \phi^\dagger \frac{\partial^2 \phi}{\partial x_j^2} \right) \tag{2.24}$$

and Eq.(2.22) follows from the field equations

$$-\left(\frac{\partial^2 \phi}{\partial t^2} + \frac{5}{3} \sum_j \frac{\partial^2 \phi}{\partial x_j^2} \right) + \frac{m}{3} \phi = 0. \tag{2.25}$$

Then (K^0, K^j) is not a contravariant vector in four-dimensional Euclidean space, while K^j is a contravariant vector in three dimensions.

The current density will be defined by

$$\begin{aligned} \alpha J^0 &= \frac{\partial \mathcal{L}}{\partial \dot{\phi}} + \frac{\partial \mathcal{L}}{\partial \dot{\phi}^\dagger} \delta \phi^\dagger - K^0 \\ &= \frac{1}{2} \frac{\partial \phi^\dagger}{\partial t} (e^{i\alpha} - 1) + \frac{1}{2} \frac{\partial \phi}{\partial t} (e^{-i\alpha} - 1) \phi^\dagger - \frac{i}{2} \alpha \left(\phi \frac{\partial \phi^\dagger}{\partial t} - \phi^\dagger \frac{\partial \phi}{\partial t} \right) \end{aligned} \tag{2.26}$$

and

$$\begin{aligned} \alpha J^j &= \frac{\partial \mathcal{L}}{\partial \partial_j \phi} \delta \phi + \frac{\partial \mathcal{L}}{\partial \partial_j \phi^\dagger} \delta \phi^\dagger - K^j \\ &= \frac{5}{6} \frac{\partial \phi^\dagger}{\partial x_j} (e^{i\alpha} - 1) \phi + \frac{5}{6} \frac{\partial \phi}{\partial x_j} (e^{-i\alpha} - 1) \phi^\dagger - \frac{5}{6} i\alpha \left(\phi \frac{\partial \phi^\dagger}{\partial x_j} - \phi^\dagger \frac{\partial \phi}{\partial x_j} \right). \end{aligned} \tag{2.27}$$

It follows that

$$\begin{aligned} \alpha J^0 &= \frac{1}{2} \frac{\partial \phi^\dagger}{\partial t} (i\alpha + \mathcal{O}(\alpha^2)) \phi + \frac{1}{2} \frac{\partial \phi}{\partial t} (-i\alpha + \mathcal{O}(\alpha^2)) \phi^\dagger - \frac{1}{2} i\alpha \left(\phi \frac{\partial \phi^\dagger}{\partial t} - \phi^\dagger \frac{\partial \phi}{\partial t} \right) \\ \alpha J^j &= \frac{5}{6} \frac{\partial \phi^\dagger}{\partial x_j} (i\alpha + \mathcal{O}(\alpha^2)) + \frac{5}{6} \frac{\partial \phi}{\partial x_j} (-i\alpha + \mathcal{O}(\alpha^2)) \phi^\dagger - \frac{5}{6} i\alpha \left(\phi \frac{\partial \phi^\dagger}{\partial x_j} - \phi^\dagger \frac{\partial \phi}{\partial x_j} \right) \end{aligned} \tag{2.28}$$

and there is a cancellation at O(α) reflecting the contributions of both the field φ and $\varphi^\dagger$ to the charges. The first term then will be the current density for the scalar field

$$\begin{aligned} J_\phi^0 &= \frac{1}{2} i \left(\phi \frac{\partial \phi^\dagger}{\partial t} - \phi^\dagger \frac{\partial \phi}{\partial t} \right) \\ J_\phi^j &= \frac{5}{6} i \left(\phi \frac{\partial \phi^\dagger}{\partial x_j} - \phi^\dagger \frac{\partial \phi}{\partial x_j} \right). \end{aligned} \tag{2.29}$$

The generalization of Stokes' theorem to higher-dimensional manifolds has the form

$$\int_M dV\ \partial_\mu w^\mu = \int_\Sigma d\Sigma_\mu w^\mu, \tag{2.30}$$

where the manifold M is bounded by the hypersurface Σ and w^μ is a contravariant vector. Since J^μ is not a contravariant vector in four dimensions, the generalization to a curved space

$$\begin{aligned}\int_M d^4x\ \sqrt{-g}\ D_\mu w^\mu &= \int_M d^4x\ \sqrt{-g}\ \frac{1}{\sqrt{-g}}(\partial_\mu \sqrt{-g} w^\mu) = \int_M d^4x\ \partial_\mu(\sqrt{-g} w^\mu) \\ &= \int_\Sigma d^3x \sqrt{-g} w^0\end{aligned} \tag{2.31}$$

will be necessary. Consider a manifold with the metric $ds^2 = dt^2 + \frac{25}{9}\sum_j d\tilde{x}_j^2$, with $\tilde{x}_j = \frac{3}{5}x_j$. Then, in the $(t, \tilde{x}_j)$ coordinates

$$J_\phi^\mu = \frac{1}{2} i(\phi \partial^\mu \phi^\dagger - \phi^\dagger \partial^\mu \phi). \tag{2.32}$$

In the new metric, the conservation equation would be

$D_\mu J_\phi^\mu = 0$, and $\sqrt{-g} = \left(\frac{5}{3}\right)^3$.

integral in Eq.(2.32) equals

$$\begin{aligned}Q &= \int_M d^4x\ \partial_\mu J_\phi^\mu = \int_M d^4x\ \sqrt{-g}\ D_\mu J_\phi^\mu = \left(\frac{5}{3}\right)^3 \int_M dt\ d^3\tilde{x} \sqrt{-g}\ D_\mu J_\phi^\mu \\ &= \left(\frac{5}{3}\right)^3 \int_\Sigma d^3\tilde{x}\ \left(\frac{5}{3}\right)^3\ J^0 = \int_M d^3x\ \left(\frac{5}{3}\right)^3 J_\phi^0 \\ &= \frac{125}{54} i \int_\Sigma d^3x\ \left(\phi \frac{\partial \phi^\dagger}{\partial t} - \phi^\dagger \frac{\partial \phi}{\partial t}\right).\end{aligned} \tag{2.33}$$

Furthermore, the charge is constant since

$$\frac{\partial}{\partial t}\int\ d^3x\ J^0 = \int d^3x\ \frac{\partial J^0}{\partial t} = -\int\ d^3x\ \vec{\nabla}\cdot\vec{J} = -\int d\vec{S}\cdot\vec{J} = 0 \tag{2.34}$$

because the current would tend to zero at spatial infinity.

3. A PSEUDOSCALAR COMPONENT OF NUCLEON INTERACTIONS

The theory of the nuclear interactions began with the study of the β-decay and the binding of the protons and neutrons in the nucleus. The first is an example of decay through the weak interactions and the second requires the strong nuclear force. The range of the nuclear force initially had been explained through a Yukawa potential. It had been proven that two fermions also could interact through the mediation of a spin one particle. A resolution of the divergences that arose in the vector model [6] had been found in a theory with a mixture of vector and pseudoscalar particles of different masses [7]. The difference between the rapidity of the decay of the spin one intermediate vector boson in β-decay relative to the lifetime of the integer spin particle in cosmic rays [8] seemed to provide support for the two types of particles in the nuclear forces [9]. The value of quadrupole moment of the deuteron also required pseudoscalar meson with a reduces mass [10].

The constants in the mixed model may be adjusted to match the experimental data if the mass of the pseudoscalar is set equal to 1/3 of the spin one vector boson [9][11]. The modern theory of the weak nuclear force describes the interactions mediated by the Z^0 and $W^{\pm}$, with masses 91.1876 ± 0.0021 GeV/c^2 and 80.379 ± 0.012 GeV/c^2 respectively [12]. In the quark model, the process for β decay is

$$n = udd \to udu + W^- \to udu + e^- + \bar{\nu}_e. \tag{3.1}$$

Therefore, the spin one particle in this process is the W - boson. Since both the d and $\bar{u}$ quarks are spin-1/2 fermions, the composite system may have either total spin 1 or 0. Therefore, the spin state may be identified with a pseudoscalar with the predicted mass

$$M_{ps} = \frac{1}{3} M_W = 26.793 \pm 0.004\ GeV/c^2. \tag{3.2}$$

This value nearly coincides with the observed masses of particles in cosmic rays, which have experimentally measured masses close to 20 GeV/c^2 in lead detectors [8]. The equality of the two masses is precise since energy will be lost in the collision with the lead apparatus.

The pseudoscalar therefore may be regarded not as a different particle from the vector boson. Rather, it can be identified as another spin state of the same particle. The factor of 3 may explained by first interpreting the product of particles with these energies non- relativistic bosons instead of relativistic mesons. Therefore, the wavefunction will satisfy the Schrödinger equation.

The commutator between position and momentum operators will have real components

$$[x_i, p_j] = (-\kappa_0 \gamma^i + i\hbar)\delta_{ij} \tag{3.3}$$

The nonrelativistic formula for the energy

$$\kappa_0 \gamma^i - i\hbar\partial_i. \tag{3.4}$$

The nonrelativistic formula for the energy
$E = \frac{p^2}{2m} + V(\vec{x})$ yields

$$\left[-\sum_i \frac{(\kappa_0^2 + \hbar^2)}{2m} \partial_i^2 - \frac{i\kappa_0\hbar}{m} \gamma^i \partial_i^2 \right] \psi + V(\vec{x})\psi = E\psi \tag{3.5}$$

After setting κ_0 equal to $\hbar$, the coupled equations [5] for a four-component spinor are

$$\begin{aligned}
&-\frac{\hbar^2}{m}\nabla^2\psi_1 - i\frac{\hbar^2}{2m}\partial_3^2\psi_3 - i\frac{\hbar^2}{2m}\partial_1^2\psi_4 - \frac{\hbar^2}{2m}\partial_2^2\psi_4 + V(\vec{x})\psi_1 = E\psi_1 \\
&-\frac{\hbar^2}{m}\nabla^2\psi_2 - i\frac{\hbar^2}{2m}\partial_1^2\psi_3 + \frac{\hbar^2}{2m}\partial_2^2\psi_3 + i\frac{\hbar^2}{2m}\partial_3^2\psi_4 + V(\vec{x})\psi_2 = E\psi_2 \\
&-\frac{\hbar^2}{m}\nabla^2\psi_3 + i\frac{\hbar^2}{2m}\partial_4^2\psi_1 + i\frac{\hbar^2}{2m}\partial_1^2\psi_2 + \frac{\hbar^2}{2m}\partial_2^2\psi_2 + V(\vec{x})\psi_3 = E\psi_3 \\
&-\frac{\hbar^2}{m}\nabla^2\psi_4 + i\frac{\hbar^2}{2m}\partial_1^2\psi_1 + \frac{\hbar^2}{2m}\partial_2^2\psi_1 - i\frac{\hbar^2}{2m}\partial_3^2\psi_2 + V(\vec{x})\psi_4 = E\psi_4.
\end{aligned} \tag{3.6}$$

An equivalence can be established between a real, Majorana spinor and Klein-Gordon field. The classical limit of this Majorana spinor would be a spin zero wavefunction satisfying a generalized nonrelativistic Schrödinger equation. Therefore, setting $\bar{\psi} = \psi^T C$, where $C = i\gamma_2\gamma_0$ is the charge conjugation matrix, and requiring the components to be real, $\psi_1 = \psi_4,\ \psi_2 = -\psi_3$ and

$$-\frac{3\hbar^2}{2m}\partial_2^2\psi_i + V(\vec{x})\psi_i = E\psi_i \qquad i = 1,\ 2,\ 3,\ 4. \tag{3.7}$$

Consequently, the effective mass for a wave function near classical scales may be set equal to 1/3m. Since the spin-0 state formed from the combination of the *u* and *d* quarks satisfies a generalized nonrelativistic Schro¨dinger equation

$$-\frac{3\hbar^2}{2m}\partial_2^2\varphi + V(\vec{x})\varphi = E\varphi. \tag{3.8}$$

At these scales, its effective mass will be one third of the mass of the W^- boson.

The electric quadrupole moment of the deuteron is an intrinsic characteristic which is likely to be described by the strong nuclear force. The conventional method for deriving its values begins with a nucleon pion interaction term. Since the pion can be viewed as the mediating particle of strong interactions at low energies. The above investigations into the mixed theory of the vector boson and pseudoscalar particle include a theoretical explanation of the quadrupole moment based on the M_{ps} = $1/3M_V$. The rest mass of the neutron is much less than that of the W^- boson, and it would not be expected that the reaction (3.1) has an effect on properties of the nucleus such as the quadrupole moment. Nevertheless, a neutron at rest could decay into a a proton and W^-, with the latter producing and electron and an anti-neutrino by balancing the momentum vectors. A compatible theoretical prediction would establish a principle for the verification of the mass hierarchy in the particle spectrum.

4. RENORMALIZABILITY OF THE CHARGED SCALAR FIELD THEORY

Labelling the particles as π^+ and π^-, for example, there are diagrams now describing the interaction of π^+ and π^-, with charge conservation at each vertex. At second order in perturbation theory, the *s* and *t* channel diagrams are identical to those in φ^4 theory.

The propagator [13]

$$i\Delta_F = \frac{15i}{5E^2 + 3|\vec{p}|^2 + 15m^2} \tag{4.1}$$

has the same dimensions as the momentum-space propagator in standard quantum field theory.

The *s*-channel diagram is described by the amplitude

$$\begin{aligned} &\frac{1}{2}\left(\frac{\lambda}{2!2!}\right)^2 \int \frac{d^4k}{(2\pi)^4} \frac{15i}{5E_k^2 + 3|\vec{k}|^2 + 15m^2} \frac{15i}{5E_{p-k}^2 + 3|\vec{p} - \vec{k}|^2 + 15m^2} \\ &= -\frac{3^2 5^2}{2^5}\lambda^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{(5E_k^2 + 3|\vec{k}|^2 + 15m^2)(5E_{p-k}^2 + 3|\vec{p} - \vec{k}|^2 + 15m^2)} \\ &\qquad \delta(p_1 + p_2 - p)\delta(p_3 + p_4 - p) \end{aligned} \tag{4.2}$$

Writing the three-momentum integral in spherical polar coordinates

$$\begin{aligned} &-\frac{3^2 5^2}{2^5(2\pi)^3}\frac{1}{|\vec{p}|}\lambda^2 \int \frac{dE_k d|\vec{k}||\vec{k}|}{5E_k^2 + 3|\vec{k}|^2 + 15m^2} \int_0^\pi \frac{d\theta\ sin\ \theta}{5(E_p - E_k)^2 + 3(|\vec{p}|^2 + |\vec{k}|^2 - 2|\vec{p}||\vec{k}|cos\ \theta} \\ &= \frac{3\cdot 5^4}{2^6(2\pi)^3}\frac{1}{|\vec{p}|}\lambda^2 \int \frac{dE_k d|\vec{k}||\vec{k}|}{5E_k^2 + 3|\vec{k}|^2 + m^2} ln\left[\frac{5(E_p - E_k)^2 + 3(|\vec{p}| - |\vec{k}|)^2 + 15m^2}{5(E_p - E_k)^2 + 3(|\vec{p}| + |\vec{k}|)^2 + 15m^2}\right] \end{aligned} \tag{4.3}$$

it follows that

$$I(0) = lim_{|\vec{p}|\to 0}\frac{3\cdot 5^2}{2^6(2\pi)^3}\lambda^2\int\frac{dE_k d|\vec{k}||\vec{k}|}{5E_k^2+3|\vec{k}|^2+15m^2}$$
$$\frac{d}{d|\vec{p}|}\left\{ln\left[\frac{5(E_p-E_k)^2+3(|\vec{p}|-|\vec{k}|)^2+15m^2}{5(E_p-E_k)^2+3(|\vec{p}|+|\vec{k}|)^2+15m^2}\right]\right\}\Bigg|_{p=0} \quad arc\ tan\left(\frac{\Lambda}{\sqrt{\frac{5}{3}E_k^2+5m^2}}\right)$$
$$=\frac{3\cdot 5^2}{2^5(2\pi)^3}\lambda^2\int_0^{\Lambda_E}dE_k\int_0^{\Lambda}d|\vec{k}|\frac{1}{5E_k^2+3|\vec{k}|^2+15m^2}$$
$$=\frac{3\cdot 5^2}{2^5(2\pi)^3}\lambda^2\int_0^{\Lambda_E}dE_k\frac{1}{\frac{\sqrt{\frac{5}{3}E_k^2+5m^2}}{3}} \tag{4.4}$$

If $E_p=\sqrt{|\vec{p}|^2+m^2}$ or $E_p=f(\sqrt{|\vec{p}|^2+m^2})$, where f is monotonically increasing. The integration region can be partitioned into two intervals $\left[0,\sqrt{\frac{3}{5}\Lambda^2-3m^2}\right]$ and $\left[\sqrt{\frac{3}{5}\Lambda^2-3m^2},\Lambda_E\right]$. Then

$$\frac{3\cdot 5^2}{2^5(2\pi)^3}\lambda^2\int_0^{\sqrt{\frac{3}{5}\lambda^2-3m^2}}\frac{dE_k}{\frac{5}{3}E_k^2+5m^2}\ arc\ tan\left(\frac{\Lambda}{\sqrt{\frac{5}{3}E_k^2+5m^2}}\right)$$
$$=\frac{3\cdot 5^2}{2^5(2\pi)^3}\lambda^2\int_0^{\sqrt{\frac{3}{5}\lambda^2-\frac{m^2}{5}}}\frac{dE_k}{\frac{5}{3}E_k^2+5m^2}\left\{\frac{\pi}{2}-\frac{\sqrt{\frac{5}{3}E_k^2+5m^2}}{\Lambda}+\frac{1}{3}\frac{\left(\frac{5}{3}E_k^2+5m^2\right)^{\frac{3}{2}}}{\Lambda^3}-...\right\}$$
$$=\frac{\sqrt{3}\cdot 5^{\frac{3}{2}}\lambda^2}{2^9\pi^2}arc\ sinh\left(\frac{3\Lambda^2-15m^2}{\sqrt{15}m}\right)-\frac{3\cdot 5^{\frac{7}{2}}\lambda^2}{2^5(2\pi)^3}\frac{3\Lambda^2-15m^2}{\Lambda}+\frac{5^{\frac{7}{2}}\lambda^2}{2^8 3^{\frac{1}{2}}\pi^3}\frac{(3\Lambda^2-15m^2)}{\Lambda^3}$$
$$+\frac{5^{\frac{3}{2}}\lambda^2}{2^5 3^{\frac{1}{2}}\pi^3}\frac{\sqrt{3\Lambda^2-m^2}}{\Lambda^3}+...\ . \tag{4.5}$$

Since
$arc\ sinh\left(\frac{\sqrt{3\Lambda^2-15m^2}}{\sqrt{15}m}\right)=ln\left(2\frac{\sqrt{3\Lambda^2-15m^2}}{\sqrt{15}m}\right)+\frac{1}{4}\left(\frac{\sqrt{3\Lambda^2-15m^2}}{\sqrt{15}m}\right)^{-2}-\frac{3}{32}\left(\frac{\sqrt{3\Lambda^2-15m^2}}{\sqrt{15}m}\right)^{-4}$
+..., this integral is a logarithmic function of Λ. In the second range,

$$arc\ tan\left(\frac{\Lambda}{\sqrt{\frac{5}{3}E_k^2+5m^2}}\right)=\frac{\Lambda}{\left(\frac{5}{3}E_k^2+5m^2\right)^{\frac{1}{2}}}-\frac{1}{3}\frac{\Lambda^3}{\left(\frac{5}{3}E_k^2+5m^2\right)^{\frac{3}{2}}}+... \tag{4.6}$$

and

$$\frac{3\cdot 5^2}{2^5\pi^3}\lambda^2\int_{\sqrt{\frac{3}{5}\Lambda^2-3m^2}}^{\Lambda_E}\frac{dE_k}{\sqrt{\frac{5}{3}E_k^2+5m^2}}\left\{\frac{\Lambda}{\sqrt{\frac{5}{3}E_k^2+5m^2}}-\frac{1}{3}\frac{\Lambda^3}{\left(\frac{5}{3}E_k^2+5m^2\right)^{\frac{3}{2}}}+...\right\}$$
$$=\frac{3^2\cdot 5^2}{2^2\pi^3}\Lambda\lambda^2\int_{\sqrt{\frac{3}{5}\Lambda^2-3m^2}}^{\Lambda_E}\frac{dE_k}{E_k^2+3m^2}-\frac{3^2 5^2}{2^5\pi^3}\int_{\sqrt{\frac{3}{5}\Lambda^2-3m^2}}^{\Lambda_E}\frac{dE_k}{E_k^4\left(1+\frac{3m^2}{E_k^2}\right)^2}+...$$
$$=\frac{3^2 5}{2^5\pi^3}\Lambda\lambda^2\frac{1}{\sqrt{3}m}\left[arc\ tan\left(\frac{\sqrt{5}1\Lambda_E}{\sqrt{3}m}\right)-arc\ tan\left(\frac{\sqrt{3\Lambda^2-15m^2}}{\sqrt{15}m}\right)\right]$$

$$-\frac{3^2 5^2}{2^5\pi^3}\Lambda^3\lambda^2\left\{\frac{\frac{1}{3}}{\left(\frac{3}{5}\Lambda^2-3m^2\right)^{\frac{3}{2}}}-\frac{\frac{1}{3}}{\Lambda_E^{\frac{3}{2}}}-6m^2\left(\frac{\frac{1}{5}}{\left(\frac{3}{5}\Lambda^2-\frac{m^2}{5}\right)^{\frac{5}{3}}}-\frac{\frac{1}{5}}{\Lambda_E^{\frac{3}{2}}}\right)+\ldots\right\}. \qquad (4.7)$$

Setting Λ_E equal to

$\sqrt{\Lambda^2+m^2}$,

it equals

$$\begin{aligned}
&\frac{3^2 5^2}{2^5\pi^3}\Lambda\lambda^2\frac{1}{\sqrt{3}m}\left[\frac{m}{\sqrt{3\Lambda^2-15m^2}}-\frac{\sqrt{15}m}{\sqrt{5}(\Lambda^2+15m^2)^{\frac{1}{2}}}+\mathcal{O}\left(\frac{1}{\Lambda^3}\right)\right]\\
&\quad-\frac{3\cdot 5^2}{2^5\pi^3}\Lambda^3\lambda^2\left\{\frac{1}{\left(\frac{3}{5}\Lambda^2-3m^2\right)^{\frac{3}{2}}}-\frac{1}{(\Lambda^2+15m^2)^{\frac{3}{2}}}+\mathcal{O}\left(\frac{1}{\Lambda^5}\right)\right\}+\ldots\\
&=\frac{3^2 5^2}{2^5\pi^3}\Lambda\lambda^2\left[\sqrt{\frac{5}{3}}\left(1-\sqrt{\frac{3}{5}}\right)\frac{1}{\Lambda}+\mathcal{O}\left(\frac{1}{\Lambda^3}\right)\right]\\
&\quad-\frac{3\cdot 5^2}{2^5\pi^3}\Lambda^3\lambda^2\left[\left(\frac{5}{3}\right)^{\frac{3}{2}}\left(1-\left(\frac{3}{5}\right)^{\frac{3}{2}}\right)\frac{1}{\Lambda^3}+\mathcal{O}\left(\frac{1}{\Lambda^5}\right)\right]+\ldots \qquad (4.8)\\
&=\frac{3^2 5^2}{2^5\pi^3}\lambda^2\left[arc\ tan\left(\sqrt{\frac{5}{3}}\right)-\frac{\pi}{4}\right]+\mathcal{O}\left(\frac{1}{\Lambda^2}\right)
\end{aligned}$$

which tends to a constant as $\Lambda \to \infty$, and $I(0)$ diverges logarithmically. The *t*-channel diagram will be given by a similar integral with the product of delta functions $\delta(p_1+p-p_4)\delta(p_2+p-p_3)$ and the counterterm will be $-\frac{3\sqrt{3}5^{\frac{3}{2}}}{2^9\pi^2}ln\left(2\sqrt{3}\frac{\Lambda}{m}\right)$.

The renormalization procedure is similar to that of φ^4 theory, except for a separate evaluation of the *u*-channel amplitudes. Designating the momenta of the incoming π and π^- as p_1 and p_2, and the outgoing π^+ and π^- as p_3 and p_4 respectively, the *s*-channel describes a direct interaction of positively and negatively charged pions. By contrast, there is an intermediate scalar propagator mediating the charged pions in the *t*-channel. In the *u*-channel, the incoming π^+ would be scattered into an outgoing π^-, while the incoming π^- is transformed into an outgoing π^+. The interchange of the charge does not occur in φ^4 theory. Nevertheless, the evaluation of the amplitudes would be similar yielding a logarithmic function of the cut-off Λ_E multiplying the product of delta functions $\delta(p_1 + p - p_4)\ \delta(p_2 + p - p_3)$. It has been established that a logarithmic divergence may be absorbed into a renormalization of the coupling in φ^4 theory [14]. Similarly, the sum of the $ln\ \Lambda_E$ terms in the *s*, *t* and *u* channels may be cancelled by a diagram generated by a renormalization of the coupling.

The two-loop sunset diagram in $(\varphi^\dagger\varphi)^2$ theory is superficially quadratically divergent [15]. Other additional logarithmic factors arise when the limit Λ_E for the energy integral is dependent on the upper bound for the momentum space integrals through $\Lambda_E = \sqrt{\Lambda^2+m^2}$. The technique of integration by parts may be used to render the integral a sum of a logarithmically and linearly divergent expression, after the surface term is discarded [16].

The surface term is known to contain the quadratic divergence. The other bubble diagram at two loops is logarithmically divergent. For a quadratic divergence, the kinetic terms would generate the counterterm, whereas, the renormalization of the mass and coupling is sufficient to cancel linear and logarithmic divergences after dimensional regularization [16].

Since the superficial degree of divergence of both φ^4 and $(\varphi^\dagger\varphi)^2$ theory equals $\omega = 4 - E$, where E is the number of external lines, the same types of divergences occur at each order in perturbation theory. The parameters in the Lagrangian can be renormalized to generate counterterms to cancel the divergences, and $(\varphi^\dagger\varphi)^2$ theory shall be renormalizable.

By contrast with φ^4 theory, the global $U(1)$ transformation $\varphi(x) \to \varphi\hat{}(x) = e^{i\alpha}\varphi(x)$ generates a Ward identity by requiring the invariance of the path integral. Since

$$\int D[\phi]D[\phi^\dagger]\hat{\phi}(x_{i_1})....\hat{\phi}^\dagger(x_{j_1})..\hat{\phi}(x_{i_n})...\hat{\phi}^\dagger(x_{j_m})e^{iS[\hat{\phi},\hat{\phi}^\dagger]}$$
$$= \int D[\phi]D[\phi^\dagger]\phi(x_{i_1})...\phi^\dagger(x_{j_1})...\phi(x_{i_n})...\phi^\dagger(x_{j_m})e^{iS[\phi,\phi^\dagger]}$$
$$+ \int D[\phi]D[\phi^\dagger]\bigg\{\bigg(\sum_{k=1}^{n}\phi(x_{i_1})...\delta\phi(x_{i_k})...\phi^\dagger(x_{j_1})...\phi(x_{i_n})...\phi^\dagger(x_{j_m})$$
$$+\sum_{\ell=1}^{m}\phi(x_{i_1})...\phi^\dagger(x_{j_1})...\delta\phi^\dagger(x_{j_\ell})...\phi(x_{i_n})...\phi^\dagger(x_{j_m})\bigg)e^{iS[\phi,\phi^\dagger]}$$
$$+\phi(x_{i_1})...\phi^\dagger(x_{j_m})\ exp\bigg(iS[\phi,\phi^\dagger] + \int d^4x\bigg\{\bigg[\frac{1}{2}\left(\frac{\partial\alpha}{\partial t}\right)^2 + \frac{5}{6}\sum_j\left(\frac{\partial\alpha}{\partial x_j}\right)^2\bigg]$$
$$+\frac{i}{2}\frac{\partial\phi}{\partial t}\left(\phi\frac{\partial\phi^\dagger}{\partial t} - \phi^\dagger\frac{\partial\phi}{\partial t}\right) + \frac{5}{6}i\sum_j\frac{\partial\alpha}{\partial x_j}\left(\phi\frac{\partial\phi^\dagger}{\partial x_j} - \phi^\dagger\frac{\partial\phi}{\partial x_j}\right)\bigg\}\bigg)\bigg\} + \mathcal{O}(\delta\alpha^2). \tag{4.9}$$

For α satisfying Eq. (2.21),

$$\int D[\phi]D[\phi^\dagger]e^{iS[\phi,\phi^\dagger]}\bigg(i\int d^4x\ \partial_\mu K^\mu(x)\phi(x_{i_1})...\phi^\dagger(x_{j_1})...\phi(x_{i_n})...\phi^\dagger(x_{j_m})$$
$$- i(n-m)\alpha\phi(x_{i_1})...\phi^\dagger(x_{j_1})....\phi(x_{i_n})...\phi^\dagger(x_{j_m}) + \mathcal{O}(\alpha^2)\bigg) = 0. \tag{4.10}$$

where K^μ is given in Eq.(2.23). It can be seen from Eqs.(2.26) and Eq.(2.27) that

$$K^\mu = \ \alpha J^\mu_\phi.$$

By Stokes' theorem

$$\left\langle \int_\Sigma d\Sigma_\mu J^\mu_\phi\ \phi(x_{i_1})...\phi^\dagger(x_{j_1})...\phi(x_{i_n})...\phi^\dagger(x_{j_m})\right\rangle = (n-m)\langle\phi^\dagger(x_{j_1})....\phi(x_{i_n})...\phi^\dagger(x_{j_m})\rangle. \tag{4.11}$$

Since the space like hyper surface Σ may be selected to represent constant time, the quantum expectation value of the charge

$$\langle Q\rangle = n - m \tag{4.12}$$

is equal to the number of particles minus the number of anti-particles. This number is constant throughout any interaction

$$\frac{d\langle Q\rangle}{dt} \ = \ 0.$$

The constancy of charge is preserved throughout the summation of perturbation series in the quantization of the theory.

The spin coupling formula for the masses of the pseudoscalar mesons

$$m_{q\bar{q}}m_q + m_{\bar{q}} - \frac{3}{4}\frac{A}{m_q m_{\bar{q}}}, \tag{4.13}$$

With

$m_u \sim 0.305\ GeV/c^2,\ m_d \sim 0.308\ GeV/c^2,\ m_s \sim 0.487\ GeV/c^2,\ A \sim 0.06\ (GeV/c^2)^3$

gives

$$m_{\pi^0} = \frac{1}{2}\left[m_u + m\bar{u} - \frac{3}{4}\frac{A}{m_u m_{\bar{u}}} + m_d + m_{\bar{d}} - \frac{3}{4}\frac{A}{m_d m_{\bar{d}}}\right] \simeq 0.133985\ GeV/c^2 \quad (4.14)$$

for the neutral pion

$\pi^0 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$,

which is nearly the experimental value 134.97657 ± 0.00050 MeV/c^2 [17]. It is not necessary, therefore, to determine the masses through sum rules and QCD perturbation theory, which give lesser values for the light quarks [18].

Pion scattering is characterized by crossing symmetry, unitarity, a fixed-angle, high- energy limit and linear Regge trajectories that also occur for string amplitudes [19]. The pointlike limit of the string scattering matrix would yield the cross-sections, which may be verified by solving the dispersion relations or integral equations for the *S*-state and *P* -state in the partial wave expansion [2][20].

5. CONCLUSION

The description of a quantum theory would have commutation relations for the position and momentum operators with a well-defined classical limit coinciding with the Poisson bracket if there is a real component. Then, the momentum operator must be generalized to have a gamma matrix term by hermiticity. A similar conclusion is valid for the energy where a γ_5 matrix is necessary. The relativistic formula relating the energy and momentum may be translated into a new wave equation for a scalar field after tracing of the matrices. The Lagrangian of a charged scalar field satisfying the modified Klein-Gordon equation still has a global U (1) and a local U (1) symmetry with conditions on the phase variable.

The current is found to vanish as a result of cancelling contributions from the charged scalar field φ and the conjugate $\varphi^\dagger$. Restricting the current to only φ, a formula for the charge may be given. It is larger than the conventional formula by a factor of $(5/3)^3$. The theory may be quantized with a modified propagator. An expansion of the one-loop diagrams in the *s* and *t* channels yields a logarithmic divergence which can be cancelled by a counter term. The model therefore is renormalizable in four dimensions.

The theory of a mixed vector and pseudoscalar mediating the nuclear force received experimental confirmation from the β-decay and the observation of cosmic rays. The resolution of singularities and the sign of the quadrupole moment required that the masses of the two states must be different. It was established further that there would be a match with the experimental data if the mass of the pseudoscalar particle equalled one third of the mass of the vector boson.

The reduction in the effective mass is explained theoretically in Section 3 through a generalized Schrödinger equation. Since the intermediate vector boson of the weak interactions describing β-decay is the W^-, the mass of the pseudoscalar state would be 26.793 MeV/c^2, which is consistent with the detection in the lead counters of particles with an energy of 20 MeV . This experimental confirmation of the model is indicative of a fundamental relation between the weak and strong interactions in the nucleus.

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