

## **Propagation of Surface Plasmon Waves along Multi Wall Carbon Nanotube with Gold Core**

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<b>ABSTRACT</b>	<p>We have studied the propagation of surface plasmon waves along multiwalled carbon nanotube with gold core. We have modeled the shells of multiwalled carbon nanotube as impedance sheets with axially directed surface conductivity incorporating inter shell coupling in an integral equation approach. We have found that in low frequency regime optical interband transitions did not occur and guided waves propagated with low attenuation in an multiwalled carbon nanotube which has metallic shells. In the same frequency range the axial polarizability of a finite length multiwalled carbon nanotube has a resonant behavior due the antenna length matching effect. The shells with surface conductivity due to interband transitions suppressed guided wave propagation. Surface Plasmon wave propagation in a multiwalled carbon nanotube with gold core showed that in the near infrared and visible regime the shells behaved as lossy dielectric materials and suppressed surface wave propagation along the gold core. The electromagnetic characteristics of carbon nanotube based antennas have been examined in different frequency regime ranging from the microwave to the visible regime. Carbon nanotube has been demonstrated to play a crucial role connected electrically to planner periodic structures of single wall carbon nanotubes, carbon nanotube bundles and carbon nanotube arrays. The inter shells interaction leaded to inter shell electron tunneling or hopping. Fermi momentum of two incommensurable shells do not coincide within the first Brillouin zone and the inter shell tunneling vanished. In the presence of localized defects inter shell conduction increased and defectfree shells were managed by neglecting intershell conduction. We have used microscopic model for multiwalled carbon nanotube and radiation characteristics were determined by its waveguiding properties, the dispersion equation for guided wave propagation on an infinitely long multiwall carbon nanotube. We have found that the guided wave has strong retardation and high attenuation so that the frequency of a geometric resonance is not connected to the free space wave length but to a shorter effective wavelength that depends on the material properties. The obtained results were compared with the previously obtained results and were found in good agreement.</p>
<b>KEYWORDS</b>	<p>Propagation, Surface Plasmon, Carbon Nanotube, Impedance, Surface Conductivity, Coupling, Attenuation, Polarizability.</p>

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## INTRODUCTION

Bandow et al. [1] presented that multiwalled carbon nanotube comprises  $N$  concentric shells or tube each contained by rolling a graphene sheet into a cylinder. The number of shells in a multiwalled carbon nanotube range from 2 to 200 and the distance between consecutive shells from 3.4 to 3.6 Å, which is close to the interlayer distance in graphene. The lattice structures of consecutive shells are generally uncorrelated with each other have different chiralities. Experiments on multiwalled carbon nanotubes have indicated that often the different shells have different periodicities. The consecutive shells of a multiwalled carbon nanotube are called commensurate/incommensurate if the ratio of their unit cell lengths along the carbon nanotube axis is rational/irrational indicating the presence or absence of translational symmetry. Incommensurability affects the transport and optical properties of multiwalled carbon nanotubes [2-9]. Two incommensurate shells interact differently than two commensurate cells [10]. Abrikosov et al. [11] considered a multiwalled carbon nanotube as a set of coaxial, continuous Kroning-Penny type potential in the radial direction. Dyachkov and Makaev [12] as well as Tunney and Cooper [13] assumed the intershell interaction to be so small that each shell could be considered to be in a perturbed eigenstate of a single wall carbon nanotube. A computer simulation with some input parameters extracted from experiment has also been reported [14]. A key element of the analysis is the formulation of the effective boundary conditions for the electromagnetic field on the carbon nanotube surface [15-17]. Any attempt to describe a multiwalled carbon nanotube with cross sectional radius between 25 and 150 nm as a perfectly conducting rod [18-19] can provide only rough estimates and is inapplicable to multiwalled carbon nanotubes of small radius. We have adopted microscopic model for multiwalled carbon nanotubes. This approach is well established in antenna theory and has been successfully applied to singlewall carbon nanotube antennas [20] and almost circular bundles of closely packed single wall carbon nanotubes [21]. Carbon nanotubes have been proposed to fabricate several different integrated circuits elements and electromagnetic devices such as transmission lines [22], interconnects [23] and nano antennas [24-25]. The fabrications of a carbon nanotube based amplitude modulator or demodulator and a fully integrated radio receiver [26] have been reported.

Batrakov et al. [27] demonstrated the potential of carbon nanotubes as emitters of terahertz and infrared radiation. Sattar and Kumar [28] derived expressions for impedance for wave guiding devices from the microwave frequency regime to optics and plasmonics. Their expressions were based on electromagnetic eigen modes that are excited at the interface of a structure. A plasmonic insulator-metal-insulator waveguide characterized by referential impedance illustrates another photonic structure. They observed the impedance for the reciprocity based overlap of eigen modes. They found that applicability of simple circuit parameters ends and how the impedance can be interpreted beyond any particular point. The unconjugated reciprocity framework was set up to solve for the interface reflection coefficient of the different electromagnetic eigen modes. Kumar and Ranjan [29] studied transmission through surface disordered waveguides in general and a solid basis. Their results showed that desired transmission properties on a waveguide through the roughness of its boundaries have been obtained. This surface scattering approaches predicted that how mode specific scattering lengths in waveguides depended on the details of system's surface roughness. They found that an observed shift of the amplitude scattering could be attributed to the nonvanishing disorder strength. They also found that short wavelengths can be exhibited effects predicted for systems with long range correlations leading to drastic changes in their transmission properties.

## METHOD

The cylindrical axis of the chosen multiwalled carbon nanotube has been taken parallel to the  $Z$ -axis of the cylindrical co-ordinate system  $(\rho, \phi, Z)$ . The centroid of the multiwalled carbon nanotube was assumed to coincide with the origin of the co-ordinate system. Considering the shells from 1 to  $N$  beginning from innermost shell so that their cross sectional radii comply with the condition  $R_N > R_{N-1} > \dots > R_1$ . The cross-sectional radius  $R_N$  of the outer most shell is assumed to be much smaller than the free space wavelength. The  $p$ -th shell  $p \in [1, N]$  possesses an axial conductivity per unit area as  $\sigma_p$ . The transverse current density in every shell is neglected. Intershell tunneling is also neglected. Electromagnetic fields with a azimuthal symmetry are excited in an multiwalled carbon nanotube by a uniform external field and we have neglected azimuthally symmetric fields. This is also applicable for finite length of multiwalled carbon nanotubes in the long wavelength regime. The intershell tunneling is small so that as a superposition of  $N$  independent functions, we have obtained the solution of guided wave

$$\Pi(\rho, z) = e^{i h z} \sum_{p=1}^N A_p \phi_p(\rho) \dots \dots \dots (1)$$

where  $\{A_p\}$  is the set of unknown coefficients and  $h$  is the unknown guide wave number. Taking  $K = \sqrt{h^2 - k^2}$  the basis function  $\phi_p(\rho)$  have been taken to satisfy the differential equation.

$$\frac{1}{\rho} \frac{d}{d\rho} \left[ \rho \frac{d}{d\rho} \phi_p(\rho) \right] + K^2 \phi_p(\rho) = 0, p \in [1, N] \dots\dots\dots(2)$$

Subject to the boundary conditions at the surface  $\rho = R_p$

$$\left. \frac{\partial}{\partial \rho} \phi_p(\rho) \right|_{\rho=R_{p+0}} - \left. \frac{\partial}{\partial \rho} \phi_p(\rho) \right|_{\rho=R_{p-0}} = \frac{4\pi}{ikc} \dots\dots\dots(3)$$

$$\phi_p(\rho) \Big|_{\rho=R_{p+0}} = \phi_p(\rho) \Big|_{\rho=R_{p-0}} \dots\dots\dots(4)$$

The solution of eqn (2) is

$$\phi_p(\rho) = \frac{4\pi i R_p}{Kc} \begin{cases} K_0(KR_p) I_0(k\rho), \rho < R_p \\ I_0(KR_p) K_0(K\rho), \rho > R_p \end{cases} \dots\dots\dots(5)$$

where  $I_0(\cdot)$  and  $K_0(\cdot)$  are modified Bessel functions of the zeroth order.

The dispersion relation

$$\det M = 0 \dots\dots\dots(6)$$

The elements  $M_{qp}$  of the  $N \times N$  matrix  $M$  are given by

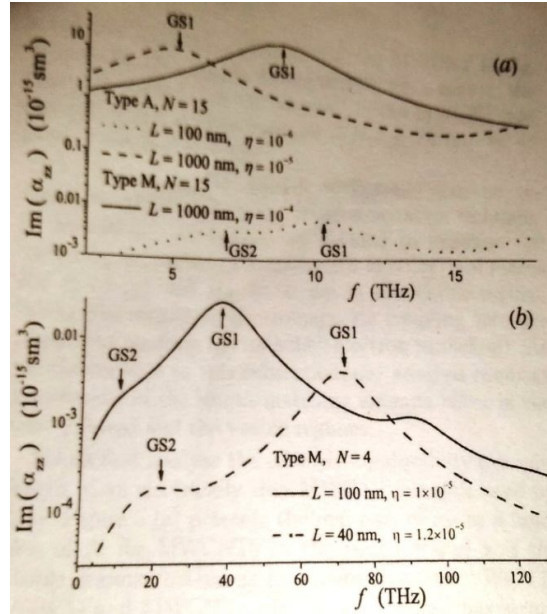
$$M_{qp} = \begin{cases} K_0(kR_q) I_0(kR_p), q > p \\ K_0(KR_p) I_0(kR_q) - \frac{i\omega\delta_{qp}}{4\pi R_q \sigma_q k^2}, q \leq p \end{cases} \dots\dots\dots(7)$$

where  $\delta_{qp}$  is the kronecker delta. The dispersion equation (6) has  $N$ ,  $K$ -roots, each corresponding to a guided wave in the multiwalled carbon nanotube. From each  $K$ -root we have determined  $h$  and the slow wave coefficient  $\beta = \frac{K}{h}$ .

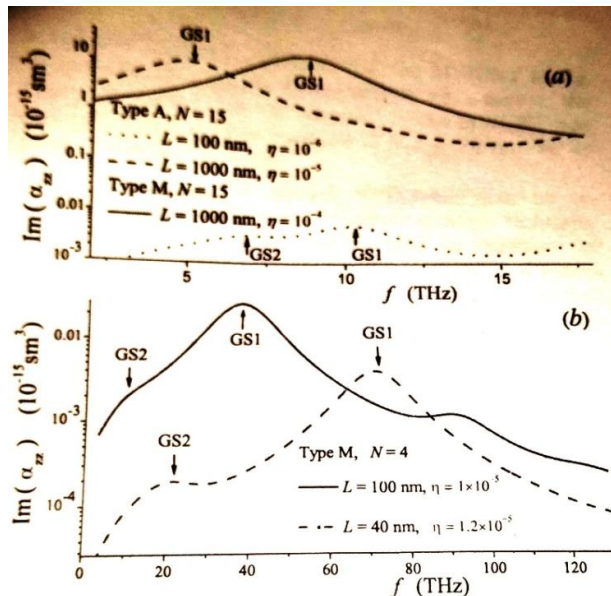
## RESULT AND DISCUSSION

Graph (1) shows the radial dependences of  $E_z$  and  $H_\phi$  inside the multiwalled carbon nanotube for the guided waves  $gw_1$   $gw_2$ . The axial component of the electric field is distributed over the entire cross section of the multiwalled carbon nanotube. The azimuthal component of the magnetic field is discontinuous across each metallic shell, in accordance with boundary condition. The degree of discontinuity decreased as the shell number  $p$  increased with decreasing magnitudes of  $J_p$  as shown in graph (1) (b). Outside the multiwalled carbon nanotube the radial distribution of the electric and magnetic field is governed by the agreement of the modified Bessel function and the electromagnetic field is highly localized to the multiwalled carbon nanotube. The axial surface current density  $J_p$ ,  $p \in (1, N)$  is shown in graph (1)(b) for guided waves  $gw_1$  and  $gw_2$ . In the opposition on the radial distributions of the axial electric field, the magnitude of the current density is maximal on the innermost shell and then strongly decreases with the increase of the shell number  $p$ . The energy flux density for  $gw_1$  is maximum near the surface of the innermost metallic shell and then slightly varies about some mean value between 4<sup>th</sup> and 13<sup>th</sup> shells. The energy flux density for  $gw_2$  is mostly concentrated between the two innermost metallic shells and then decreases very rapidly as  $\rho \rightarrow R_N$ . Then two innermost metallic shells are dominant contributors to the retardation of both  $gw_1$  and  $gw_2$ . Graph (2) shows the frequency dependence of the imaginary part of the polarizability scalar  $\alpha_{zz}$  in the long wavelength regime  $KL \ll 1$  for different lengths  $L$  and shell numbers  $N$  in multiwalled carbon nanotubes.  $G_{s1}$  and  $g_{s2}$  represents first geometric resonance of the guided waves  $gw_1$  and  $gw_2$ . Geometric resonances occur at  $f_s = s \left( \frac{c}{2L} \right) \text{Re}(\beta)$ ,  $s \in \{1, 3, 5, \dots\}$ . The first

geometric resonance for  $s=1$  for the multiwalled carbon nanotubes appears at a higher frequency. The resonance frequency for the  $gw_1$  and  $gw_2$  depend on  $L$  nonlinearly. Graph (2) shows the resonance frequency of the multiwalled carbon nanotubes of type A increases by a factor of about 2, from 5 to  $\sim 10$  THz, while the length  $L$  decreases by a factor of 10, from 1000 to 100 nm. The first resonance of the multiwalled carbon nanotube of type A and length  $L = 100$  nm at  $\sim 10$  THz frequency is not strong and the guided wave is strongly attenuated. Surface Plasmon waves in the infrared and the visible regimes propagate along a metal wire. Guided wave propagation be affected if the metallic wire were to be enclosed in an ensemble of concentric carbon shells. We have considered the propagation of surface plasmon waves in a multiwalled carbon nanotube with a gold core. The bulk volumetric conductivity  $\sigma_{\text{met}}$  of gold was taken to follow the Drude model with parameters.

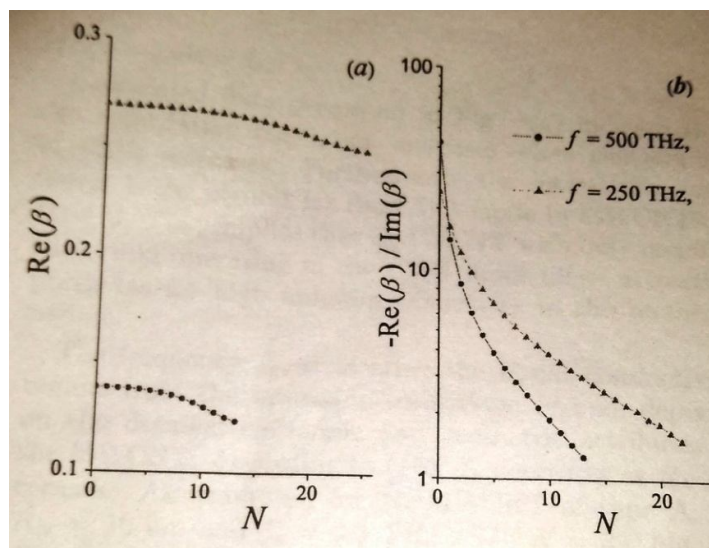


**Graph1:** The radial dependencies of (a)  $E_z$  and  $H_\phi$ , (b)  $J_p$  on the surface of the shells.



**Graph 2:** Frequency-dependence of  $\text{Im}(\alpha_{zz})$  of multiwalled carbon nanotubes of type A and M for different length  $L$  and number of shells  $N$ .

Graph (3) shows the dependences of  $\text{Re}(\beta)$  and  $-\text{Re}(\beta)$  of a surface Plasmon wave on the number  $N$  of shells in multiwalled carbon nanotube of type M with a gold core of cross-sectional radius  $R_0 = 3.5\text{nm}$ . where  $\text{Re}(\beta)$  is the retardation coefficient. The  $p$ -th shell of the multiwalled carbon nanotube has a  $(90+9p,0)$  zigzag configuration and radius  $R_p = \sqrt{3(90+9p)b}/2\pi$ ,  $p \in [1, N]$ . In the near infrared,  $f = 250\text{THz}$  and the visible  $f = 500\text{THz}$ , regimes. Graph (3) (a) shows that the retardation coefficient  $\text{Re}(\beta)$  depends only slightly on  $N$ . The values of  $-\text{Re}(\beta)/\text{Im}(\beta)$  is less than for an isolated metal wire  $N = 0$  and significantly decreased as  $N$  increased which indicated the enhancement of the power lost  $p_i$  to ohmic dissipation and consequently the decreasing of antenna efficiency  $\eta$  of an multiwalled carbon nanotube with a metal core. The obtained results were compared with previously obtained results of theoretical and experimental works and were found in good agreement.



**Graph 3:** Dependences of (a)  $\text{Re}(\beta)$  and (b) the ratio  $-\text{Re}(\beta)/\text{Im}(\beta)$  on the number  $N$  of carbon shells for a surface Plasmon wave in an multiwalled carbon nanotube with gold core.

## CONCLUSION

The radiation characteristics of an multiwalled carbon nanotube have been determined by its wave guiding properties and the dispersion equation for guided wave propagation on an infinitely long multiwalled carbon nanotube. The boundary value problem was solved numerically by an integral equation technique. We have found that slightly attenuated guided waves and antenna resonances due to the edge effect existed for not-too thick multiwalled carbon nanotube in the far and mid infrared regimes. The propagation of guided waves has dielectric effect on the performance of finite length multiwalled carbon nanotube as an antenna. We have also found that interband transitions suppress guided wave propagation in multiwalled carbon nanotubes. Guided wave propagation and geometric resonances of the guided waves are typical for macroscopic wire antennas. The guided waves have quasi transverse electromagnetic structure and are characterized by low retardation and low attenuation. The existence of guided surface waves and geometric resonances is also typical for nanowire antennas in the Drude conductivity regime. But the guided wave has strong retardation and high attenuation so that the frequency of a geometric resonance is not connected to the free space wavelength but to shorter effective wave length that depends on the material properties. The obtained results indicated that the retardation coefficient increases when the number of shells increases. The retardation coefficient is the highest in multiwalled carbon nanotubes of type M, which shows that a multiwalled carbon nanotube with only metallic shells and operating in guided wave one mode offers attractive prospects for high antenna efficiencies in the terahertz regime. The obtained results were compared with previously obtained results and were found in good agreement.

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