

Mechanical Resonator Linearly Coupled to a Normal State Single Electron Transistor and Electron Transport in Mesoscopic Conductor

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Received on 15.07.2021 Accepted on 30.10.2021	

ABSTRACT	We have studied the coupling of normal state single electron transistor and electron transport in mesoscopic conductor. For this we have used a simple model system consisting of a mechanical resonator linearly coupled to a normal state single electron transistor to explore the nonlinear dynamics which arise in non electromechanical systems. We have found that very weak linear electromechanical coupling gave rise to a strongly nonlinear response when the resonator was driven close to resonance. In the weak coupling limit and in the absence of driving, the single electron transistor acts on the resonator like a thermal bath with an effective temperature proportional to the bias voltage; it also damps the mechanical motion and renormalized the frequency of the resonator. We have found that for drive a certain threshold, the mechanical response as a function of frequency becomes strongly nonlinear and the mechanical system displayed many of the characteristics of the Duffing oscillator, frequency pulling, a strongly asymmetric line shape, hysteresis and bistability. The electromechanical coupling was found weak. We have described the effect of the single electron transistor on the resonator in terms of simple model which included damping and frequency renormalization terms which are both amplitude dependent. We have found that a calculation of the average mechanical response as a function of drive frequency using these two quantities led to results which were found in good agreement with a Monte Carlo simulation of the coupled dynamics. At large amplitudes the effect of the resonator on the single electron transistor charge dynamics can no longer be accounted for by a linear correction to the tunnel rates and the charge transport was strongly modified. The modified charge dynamics led to changes in the damping and frequency shift induced by the single electron transistor on the resonator leading in general to an amplitude dependence of these quantities. Such amplitude dependence is generic in non linear oscillators and led to the familiar phenomena of asymmetric frequency response hysteresis and bistability. The obtained results were found in good agreement with previously obtained results.
KEYWORDS	Coupling, Single Electron Transistor, Mesoscopic Conductor, Mechanical Resonator, Non Linear Dynamics, Montecarlo Simulation, Tunnel Rates.

How to cite this article: Amar, A., Aparajita, & Singh, B.K. (2021). Mechanical Resonator Linearly Coupled to a Normal State Single Electron Transistor and Electron Transport in Mesoscopic Conductor. *Bulletin of Pure and Applied Sciences- Physics*, 40D (2), 83-87.

INTRODUCTION

Armour et al. [1] presented for a resonator coupled to a normal state single electron transistor, the electrons always damp the mechanical motion on average but the resonator probability distribution nevertheless gradually changed from a Gaussian to a bimodal form as the coupling was increased [2-3]. If the conductor tends to transfer energy to the mechanical resonator increasing the coupling leads to a transition in the resonator dynamics which changed from fluctuations about a fixed point to a state of self sustaining oscillations [4-6], when the intrinsic damping of the mechanical resonator is no longer sufficient to balance the energy transferred by the current flowing through the conductor. Chutchev et al. [7] and Labadze et al. [8] studied that when the mechanical component of a nanoelectromechanical system with weak electromechanical coupling is driven to large amplitudes its influence on the charge dynamics of the conductor was greatly enhanced. The resulting change in the charge transport affected the way in which the charges acted back on the mechanical system leading to a feedback process which generated strongly nonlinear mechanical dynamics. Such effects have been studied in suspended carbon nanotube systems [9-12] by several investigators both theoretically and experimentally. Bennett et al. [13] presented that mechanical dynamics can be used to infer information about the electronic structure of the dots. Mozyrsky et al. [14] and Flowers et al. [15] studied resonators coupling to tunnel junctions, single electron transistors [16-17] or quantum dots [18-20]. The electromechanical interaction in nanoelectromechanical systems is weak and non linear coupling plays a significant role [21-23] in many cases as linear description. When the electromechanical coupling is weak, the effect of the resonator on average current flowing through the conductor provide an extremely sensitive measure for the mechanical displacement. Sanjiv kumar et al. [24] presented a microscopic model of interacting electrons with a magnetic ion spin localized in the centre of a self assembled quantum dot. They have found that the electrons occupying finite angular momentum orbitals interact with the localized spin through an effective exchange interaction mediated by electron-electron interactions with a localized spin placed in the centre of the dot, only the spins of electrons occupying the zero angular momentum states of the s, d shells couple directly to the localized spin via a contact exchange interaction. Singh et al. [25] studied the transport properties in periodic chemically nitrogen doped metallic nanotubes. They have found that the ballistic properties of carbon nanotubes remained for some doping configurations. It was also found that the resonant effect associated with specific symmetry of the wave function was close to the Fermi level. They have shown that both axial and screw predictions gave rise to such a behavior and that specific but realistic disorder preserved this ballistic transport in doped metallic carbon nanotube.

METHOD

The single electron transistor island was coupled to the left and right leads by tunnel junctions with equal capacitances and a bias voltage was applied symmetrically. A gate electrode was used to tune the operating point of the island. The island capacitor contained one plate which was mechanically compliant giving rise to a position dependent capacitance. The plate moved the operating point of the single electron transistor charged. As the charge on the island fluctuated the electrostatic force on the plate changed, giving rise to electromechanical coupling. We have used linear approximation for the dependence of the gate capacitance on the position of the resonator. For that we have used

$$C_g(x) = C_g^0 \left(1 - \frac{x}{d} \right)$$

Where x is the displacement of the resonator, d is the distance between the resonator and the single electron transistor island when two are uncoupled, C_g^0 is the capacitance when $x = 0$. The charging energy of the island

$$E_C = \frac{e^2}{2C_\Sigma},$$

where C_Σ is the total capacitance of the island the thermal energy $k_B T$ and the energy scale of the bias voltage eV . The Hamiltonian for the system with n charges on the single electron transistor island is written as

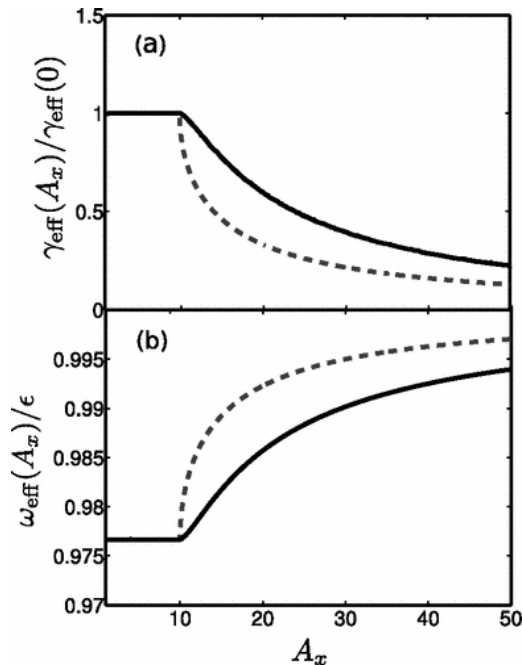
$$H_n = E_C \left[n^2 - 2nn_g^0 \left(1 - \frac{x}{d} \right) \right] + \frac{p^2}{2m} + \frac{m\omega_0^2 x^2}{2} - xF(t).$$

Where $n_g^0 = C_g^0 V_g / e$, p is the resonator momentum, ω_0 is the frequency of resonator, m is the mass of the resonator and $F(t)$ is the external drive. The bias voltage term $\frac{eV}{2}$ accounted for the change in energy of the leads associated with the tunneling of an electron. The damping and thermal fluctuations of the mechanical

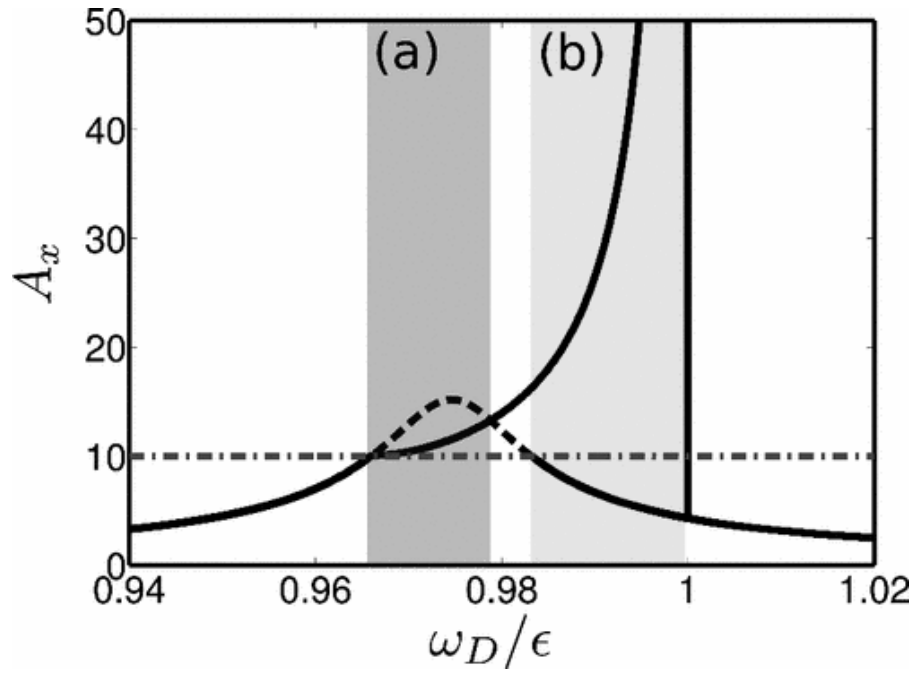
resonator due to its interactions with its surroundings apart from the single electron transistor were taken into account for the purpose of our work.

RESULTS AND DISCUSSION

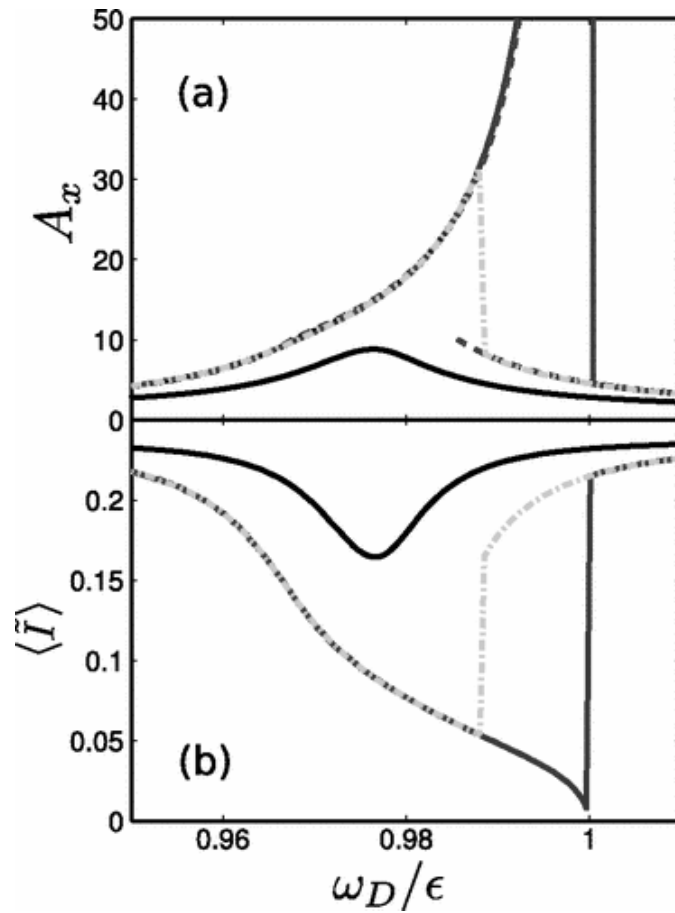
The amplitude dependence is shown in graph (1), these functions have been shown as dashed lined in graph (1). In the region where the amplitude is seen linear region, the damping and frequency shift remain their values. We have found that the damping decreased towards zero and the frequency moved towards the bare frequency. In the large amplitude limit the system spends less and less time inside the linear region where the single electron transistor electrons damp the motion. At sufficiently large amplitudes the damping of the resonator due to sources other than the single electron transistor electrons stabilized the dynamics even if as we assumed such damping is very small compared to $\gamma_{eff}(0)$. We have found that the small amplitude solution in the linear region is stable, the intermediate amplitude solution is unstable and the large amplitude solution is new stable fixed points for the systems where the damping and frequency shift are reduced form the linear values. The presence of more than one stable amplitude is common in driven nonlinear systems such as the Duffing oscillator. The nonlinear response arising in carbon nanotube experiments explained the observed behaviour using only conservative nonlinearities. Graph (2) shows the resulting curve using the nonlinear damping and frequency shift along with the conditions on the stability of the solutions as a function of derive frequency. At low frequencies below the shaded region and at high frequencies the system remained in the linear regime. The response to the derive become stronger closer to resonance. In the shaded region in graph (2) the amplitude grows beyond A_c and so the linear and nonlinear calculations gave different results. The linear calculations leaded to a Lorentzian peak centered around $\omega_{eff}(0)$. In the nonlinear case, the frequency shift became smaller, leading to a larger effective frequency. Thus derived frequency is farther from resonance than in the linear case, and hence the amplitude is smaller in shaded region. In the shaded region of graph (2) the derived frequencies are close to but below ϵ at high amplitude solution exists because of a positive feedback mechanism. In this regime as the amplitude grows beyond A_c , the enhancement in the effective frequency brings the system closer to resonance with the derive and the damping decreased as the amplitude grown. The system does eventually stabilize when the amplitude becomes larger enough that the system starts to move from resonance, when $\omega_{eff}(A_x)$ starts to increase beyond ω_D .



Graph 1: Amplitude-dependent (a) damping and (b) frequency of the resonator.



Graph 2: Stable solutions to the self-consistency expression.



Graph 3: Amplitude of the resonator (a) and average current (b) as a function of drive frequency.

Graph (3) shows the current as a function of drive frequency for both linear and non linear cases obtained from Monte Carlo simulations. For the linear case the current suppressed in the region where the resonator is resonantly driven as the amplitude of the oscillations increases one of the tunnel rates is suppressed leading to an overall reduction in current. The obtained results were compared with previously obtained results of theoretical and experimental works and were found in good agreement.

CONCLUSION

We have studied the interaction between the single electron transistor and the mechanical resonator. We have found that the average dynamics of the resonator described by a simple effective model which incorporated damping and frequency renormalization terms which are amplitude dependent. In the weak coupling limit and in the absence of driving, the single electron transistor acts on the resonator like a thermal bath with an effective temperature proportional to the bias voltage; it also damps the mechanical motion and renormalized the frequency of the resonator. We have found that for drives above a certain threshold the mechanical response as a function of frequency becomes strongly non-linear and the mechanical system displays many of the characteristics of the Duffing oscillator. At large amplitudes the effect of the resonator on the single electron transistor charge dynamics no longer be accounted for by a linear correction to the tunnel rates and the charge transport was strongly modified. We have found that a calculation of the average mechanical response as a function of drive frequency led to results which are in good agreement with a Monte Carlo simulation of the coupled dynamics. The obtained results were also found in good agreement with previously obtained results.

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