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Propagation of Guided Electromagnetic Waves in the Carbon Nanotube Optical Wave Guide with TE and TM modes

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ABSTRACT The carbon-nanotube (CNT) is cylindrical-tube wave-guide in which the mono-chromatic guided-electromagnetic-waves propagate that manifested to the electro-magnetic fields with$TE\ and\ TM\ modes$. We have obtained the solution of wave-equation as $Bessel's\ functions$ that show the character of guided-electro-magnetic-wave՝ in the optical CNT-waveguide within the phase-ingredients. The electro-magnetic fields within transverse wave in $linearly - polarized$ are parallel՝ as well as orthogonal`. The normalizing $propagation - function`b'$ is found within the $frequency`parameter$ within the lowest-modes and the guided-linearly-polarized՝ (LP) mode-variant of electro-magnetic-wave՝ with $conductivity$ with the $propagated - frequency$ in the CNT-optical waveguide.

KEYWORDS Carbon-nanotube (CNT),$Bessel's\ function$, Guided-electromagnetic-wave, Normalizing $propagation - function$, $Propagated - frequency$.

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INTRODUCTION

The structure of carbon` nanotubes [1,3] is explained with chiral, zig-zag, and armchair tube, radius of nanotube and metallic properties with propagating surface Plasmon-wave that determined by the numerical outputs. A spectrum of optical-absorption is manifested with larger bands of immersion that transit optically [2]. For $energy - region$, the character of optical-spectra(3) is in zig-zag$(3n, o)$carbon-nanotube that has maximum splitted-breath and zero in metallic armchair carbon-nanotube. The range of the lowest-frequency [4] $10^7 H_z$ within infrared-region and $3TH_z\ frequency$ is used for di-electric and analyzing the electro-magnetic character in $25H_z\ to\ 100H_z$. The effective electrical-conductivity using Drude-Model [5] has been summarized by $Waterman - Truell\ Approach$ for lower frequency-peak.

The electronic-structure of CNTs manifested with quantum-optic-applications. Nakanishi [6] and Ando studied about optical character of CNTs for $finite\ length$ and it is summarized with induced rim charges that exhilarated of Plasmon-mode-variant with the wave-vector $Q\left(=\frac{\pi}{l}\right)$ in-dirty-tubes and arise stronger electric field due to rim charges.

Use of the equalized multi-shell-approach, an $antenna\ efficiency$, character of propagating electromagnetic-waves is manifested for same $finite\ length$ metallic-CNT for guided-electromagnetic-waves with $slow - wave\ coefficients$, [7]. The theoretical manifestation of propagating electromagnetic-wave [8] in the double-wall CNT is described within $propagation - frequency$ of $electromagnetic - wave$ and material-parameter and $wave\ number$. Kumar [10] and Shuba [9]) have analyzed the $symmetric\ guided\ wave$ propagated through $finite\ length$ multi-walled CNTs with $gold\ core$ as the antenna and $attenuation - coefficient$ in $10.00\ to\ 100.00\ GHz\ frequency\ range$ that re-presents $high\ attenuation$ of propagated$surface - wave$. The $optical\ interband$ didn't occur in the lowest frequency-regime and the guided-electromagnetic-wave` can be propagated in the multi-wall CNTs [14] at $low\ or\ high\ attenuation$ with axial $surface\ conductivity$ material character. The plane` transverse mono-chromatic wave` propagates through SWCNT as light`s speed and is manifested by $Gaussian\ wave$ and root of $Helmholtz\ partial\ differential\ equation$ [11, 15]. Kumar [16] has been also manifested the behavior` of SWCNT as wave-guide with $Helmholtz\ equation$ or surface Plasmon-polariton wave-guide and used the $electric\ hertz\ potential$ for propagated electromagnetic-wave through CNTs [12, 17].

Victor [9] manifested electromagnetic-field in wave-guide with $Helmholtz\ equation$ using $spectral - parameter\ power\ series\ method$ and obtains dispersion for wave-guide that leads $group\ velocity$ and $propagation\ constant$ by using numeric-approach with $Fourier\ transform$ and found $asymptotic\ formula$ for $TE`\ wave\ and\ TM\ wave$ [13].

THEORETICAL METHODS

Let us consider the $cylindrical\ co - ordinate\ (r, \phi, z) system$ and the $z - axis$is along the CNT-waveguide axis for wave-equation that manifested with the $Cartesian - system$ shown in $Figure$ 1 and the wave-equation` as

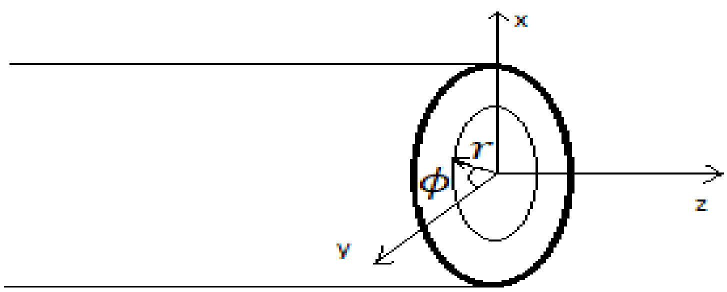

Figure 1: **Cylindrical co − ordinate (r, ϕ, z)of carbon nanotube with Cartesian system.**

$\frac{\delta^2 E_z}{\delta r^2}+\frac{1}{r}\frac{\delta E_z}{\delta r}+\frac{1}{r^2}\frac{\delta^2 E_z}{\delta \phi^2}+k_c^2 E_z=0$ (1)

And

$\frac{\delta^2 H_z}{\delta r^2}+\frac{1}{r}\frac{\delta H_z}{\delta r}+\frac{1}{r^2}\frac{\delta^2 H_z}{\delta \phi^2}+k_c^2 H_z=0$ (2)

Where, $k_c^2=\omega^2\mu\varepsilon-\beta^2=k^2-\beta^2 \; and \; k^2=\omega^2\mu\varepsilon$.

The solution of equations` (1) & (2) for plane-wave` [11] expressed as $E_z=e^{i\vec{k}.\vec{r}}\hat{e}_z$ and $H_z=e^{i\vec{k}.\vec{r}}\hat{e}_z$ that give the characteristics of transverse-wave propagated through the single wall cylindrical CNT. The electro-magnetic-wave is confining within metallic CNT-waveguide and it propagated either in$TE \; and \; TM$` $modes$ electric & magnetic field vectors lie in plane transverse along $z-direction$. Under certain conditions, we have $E_z=O$ that H_z is $finite$ & $H_z=o$ that E_z is $finite$. We have the utter root of sum (1) & (2) described as

$E_z` =[PJ_n(k_c r)+QK_n(k_c r)]Fe^{in\phi}$ (3)

And

$H_z=[LJ_n(k_c r)+MK_n(k_c r)]Fe^{in\phi}$ (4)

Here, $F,.P,.Q,.L,.and \; M$ are all arbitrary persistent. $J_n(k_c r)$ is the $Bessel's \; function$ and $K_n(k_c r)$ is the modified $Bessel's \; function$ that all infinite` at origin $(r=0)$. The functions$J_n(k_c r)$and$K_n(k_c r)$are with $k_c r$ for the tariff of $n=0,1,2,3,\dots\dots$. The arbitrary persistent $Q \; and \; M$ must be equal to zero if E_z and H_z is finite at $(r=O)$. Now we use the co-designation $J_n(ur)$and E_z and H_z including phase ingredients expressed as

$E_z=A_1 J_n(ur)e^{jn\phi}e^{j(\omega t-\beta z)}$ (5)

$H_z=B_1 J_n(ur)e^{jn\phi}e^{j(\omega t-\beta z)}$ (6)

Where, $A_{1.}=PF$, $B_{1.}=LF$ and$ur=k_c r$. The Eigen-value sum for $'\beta'$ expressed as

$(I_n+K_n)(K_1^2 I_n+K_2^2 K_n)=\left(\frac{\beta n}{r}\right)^2\left(\frac{1}{u^2}+\frac{1}{s^2}\right)^2$ (7)

Where, $I_n=\frac{J_n'(ur)}{uJ_n(ur)} \; and \; K_n=K_n'=\frac{K_n'(sr)}{sK_n(sr)}$. The discrete-values` of 'β' restricted to range` $K_2\le\beta\le K_1$. The modified` secondary $Bessel's \; function$ $K_n(sr)$ for larger tariff of $r$ is given as $K_n(sr)=\frac{e^{-sr}}{\sqrt{sr}}$ & $K_n(sr)\to 0$ as $sr\to\infty$ providing$'s'$ is a positive non-fictitious` quantity. The right side` of equation (7) fades and we` have

$I_0+K_0=0$ (8)

$$\frac{J_a'(ur)}{uJ_0(ur)}+\frac{K_n'(sr)}{sK_0(sr)}=0$$

Again $J_0'(ur)=-J_1(ur) \; and \; K_0'(sr)=-K_1(sr)$ so,

$\frac{J_1(ur)}{uJ_0(ur)}+\frac{K_1(sr)}{sK_0(sr)}=0$ (9)

That corresponds to transverse-magnetic` mode $TM_{op}(E_z=o)$ and

$K_1^2 I_0+K_2^2 K_0^2=0$ (10)

Or, $\frac{K_1^2 J_1(ur)}{uJ_0(ur)}+\frac{K_2^2 K_1(sr)}{sK_0(sr)}=0$ (11)

That corresponds to transverse-electric` mode $TE_{op}$ $(H_z=0)$. The parameters associated` with the cut-off condition and referred` to $N-number \; or \; N-parameter$ is given as$N^2=r^2(u^2+s^2)$ that exists in wave-guide function of N that representing the $normalizing \; propagated \; function$ 'b'given as $\frac{r^2 s^2}{N^2}=b$. The $N\text{-}parameter$ is related to mode-numbers$'M'$expressed as$M=\frac{N^2}{2}$. The mode$(n,p)is$ derived by adding $(n-1.)and \; (n+1.)$

$solutions \; in \; the \; core \; given \; by$

$E_z=E_0\{J_{n-1}(ur)\cos(n-1)\phi+J_{n+1}(ur)\cos(n+1)\phi\}$ (12)

$H_z=H_0\{J_{n-1}(ur)\cos(n-1)\phi+J_{n+1}(ur)\cos(n+1)\phi\}$ (13)

So, the $Bessel's \; function$ as

$J_{n-1}(ur)+J_{n+1}(ur)=\frac{2n}{ur}J_n(ur)$ (14)

From equation (17) and (18) we have $\frac{H_z}{E_z}=\frac{H_0}{E_0}$. We know the amplitude of $E \; and \; B$ as $\frac{B_0}{E_0}=\sqrt{\varepsilon\mu\sqrt{1+\left(\frac{\sigma}{\varepsilon\omega}\right)^2}}$ then we can calculate $\frac{H_z}{E_z}$ as

$\left(\frac{H_z}{E_z}\right)^4=\left(\frac{\mu}{\mu_0^2}\right)^2\left\{\varepsilon+\left(\frac{\sigma}{\omega}\right)^2\right\}$ (15)

The propagated-frequency` of electro-magnetic-wave, $\omega$, is summarized by a relation-ship for $m=o, and \; m\ne o$ respectively` and may written as

$\omega^2=4\left(\frac{e}{\pi}\right)^2\frac{v_F}{\varepsilon_0\hbar}\ln\left(\frac{1.123}{kR_c}\right)k$ (16)

And $\omega = \sqrt{\frac{\left(\alpha\, m^2 + m\frac{e^2 v_F}{\varepsilon_0 \pi^2 \hbar}\right)}{R_c^2}}$ (17)

RESULTS AND DISCUSSIONS

The guided-electromagnetic-wave energy travels in CNT-wave-guide if mode-variant is aside $cut - off\ frequency$. The electro-magnetic field models and propagation-constant for mode-variant are indistinguishable.

For $(n,p) = (O, 1)$ and $(2`, 1)$, the distinction with pair-variants TE_{01} and TM_{01} $reduces\ to\ zero$ i.e., limit of sickly-guiding shown in Figure2. We have initiated that the variants of longitudinal-field as E_z and T_z are smaller than that of the main transverse-variants of the guided-electromagnetic-wave solutions. Transverse electro-magnetic fields are parallel and orthogonal in the $linearly\ polarized\ modes$ $(LP_{11} = TE_{01}, TM_{01})$ $of\ waves$.

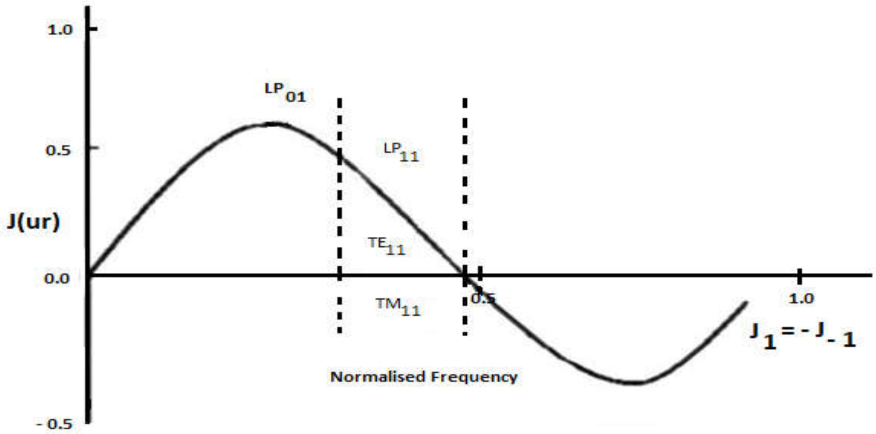

Figure 2: $\boldsymbol{TE\ and\ TM\ modes}$ **in carbon nanotube waveguide with** $\boldsymbol{linearly\ polarized\ modes\ (LP)}$**.**

There are analogous roots with the counter-field polarities for each $LP\ modes$ with degraded solutions. The sickly-guiding approximation relative to the boundary-condition given as

$$-u_{np}\frac{J_{n-1}(u_{np}r)}{J_n(u_{np}r)} = s_{np}\frac{s_{n-1}(s_{np}r)}{s_n(s_{np}r)} \quad (18)$$

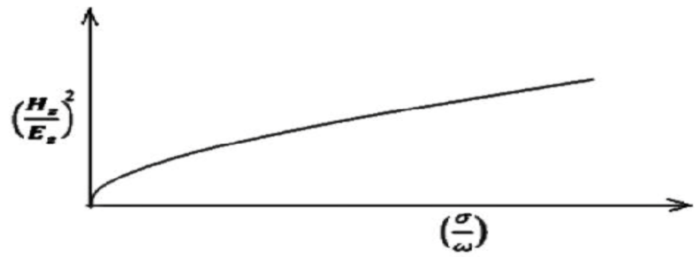

Figure3: Plot the magnetic field and electric field along z – direction to the conductivity and propagation frequency. As $\left(\frac{\sigma}{\omega}\right)$ **increases,** $\left(\frac{H_z}{E_z}\right)^2$ **will increases.**

So, we have u_{np} and s_{np} with numeric root and β_{np} for guided-mode-variant may be established. The disparity between $\left(\frac{H_z}{E_z}\right)^2$ and $\left(\frac{\sigma}{\omega}\right)$ with Mathway graphic software is parabola shown in Figure3 with and shows increasing the fields and conductivity. The structures of quasi transverse guided electromagnetic waves are with the low attenuation. Equation (16) is quasi-acoustic variant and sum (17) is sensitized to nanotube and they constitute when R_c will $increase$, ω $will\ decrease$ that shown in Figure 4.

The $parameter\ 'k'$ with region $\frac{\omega}{q} < c\ (speed\ of\ light), k^2 = q^2 - \frac{\omega^2}{c^2}$, it means we have the slow $TM\ wave. Consider\ \omega = vq$ and we have the $speed\ lines$ of the three electron-beams manifested by Figure5. We have maximum $phase\ and\ group\ velocity$ for the maximum $propagation\ frequency$ and contrariwise by the utterance, $v_p = v_g = \frac{\delta\omega}{\delta k}\ and\ v_p \cdot v_g = v^2$.

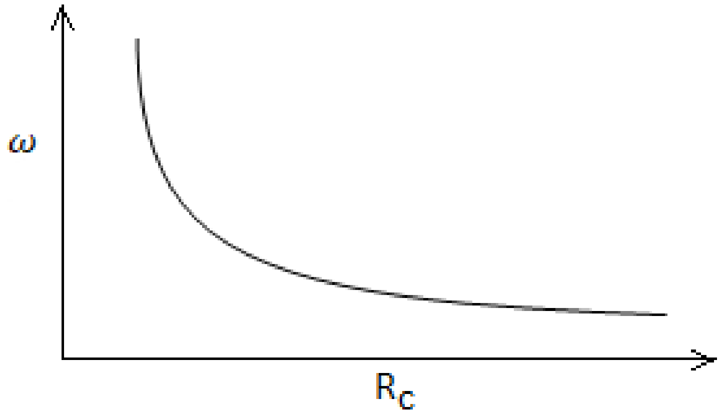

Figure 4: Variation of propagation frequency, ω , with $the\ radius, R_c$, of carbon nanotube.

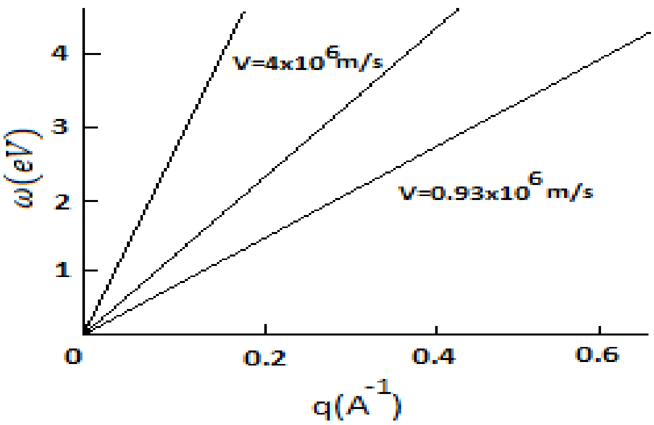

Figure 5: The $electron\ beams\ velocity$ in the $nanotubes\ for\ m = 0$. As increasing the electro-magnetic-wave $frequency$, decreasing the $radial\ penetration\ depth$ for $TM\ surface$. The $range\ of\ velocities$ are $0.93x10^6\ to\ 4x106\ m/s$.

$The\ propagation\ function$ 'b' $as\ a\ function\ of$ 'N' to the lower-order of variants is constitute in Figure6. All variants can subsist for $values\ of$ 'N' that outdoes the limit value. The tariff of $V(ur) = 2.405$ is the first root of lower-order $Bessel\ function\ J_0(ur) = 0$. If $V \leq 2.40\ as\ 2.3560\ for\ wavelength = 0.8\mu m$, the propagation is possible. So, the guided-wave can be propagated through optical CNT-waveguide.

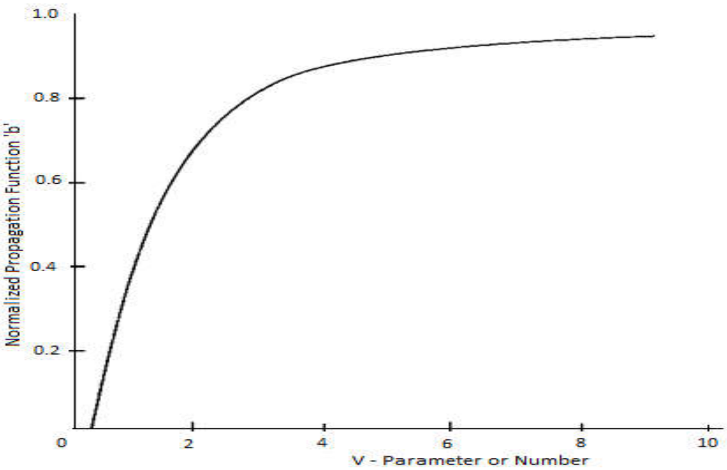

Figure 6: **b– the normalized propagation function as the function of V – parameter and the curves of TE_{op} and TM_{op} modes for $(0, 1)$.**

CONCLUSIONS

The $Bessel's\ functions$ interpret the nature of propagating electro-magnetic waves in $TE\ and\ TM\ modes$ in CNT as$guided\ waves$. We have initiated the $normalized\ propagation\ function$ 'b' and the $modes\ number$ within $N - parameter$. The electro-magnetic fields are parallel as well as orthogonal in the $linearly\ polarized$. The disparities of the electro-magnetic field along $z - direction$ are to the conductivity and propagated-frequency. For the lowest frequency, the guided-electromagnetic-wave is the highest. The radius of CNT is inversely proportional to the propagated-frequency i.e., for the smallest radius, we have the highest frequency. The electron-beam velocity in CNT-optical wave-guide is of range 10^6.

As results, the guided-electromagnetic-waves in CNT-optical wave-guide is verified by the $function$ 'b' with $parameter - N$ and expressed by $Bessel's\ function$ and $orthogonal\ wave$ that $linearly\ polarized$ in $TE\ and\ TM\ modes$. The guided-electromagnetic-wave velocity is interpreted with $phase\ and\ group\ velocities$ as the electron-beam speed in CNT-optical wave-guide within the propagated-frequency. Therefore, we have initiated the maximum $phase\ and\ group\ velocities$ for the maximum propagated-frequency and contrariwise. We have initiated the conclusion as the guided-electromagnetic-wave propagates with maximum velocity and frequency through the smallest radius of CNT-optical wave-guide.

REFERENCES

1. Moradi A. J. Electromagnetic Analysisand Applications, 2010; 2, 672-676.
2. Katawuca H. and et.al., 1999; vol. 103, no, 1-3, pp. 2555-2558
3. Saito R. and Katawea H.Appl. Phys. Springer- Verlog Berlin Heidelberg, 2001; 80,213-247
4. Mikhail V. Shuba and et.al. Scientific Reports, Nature Research, 2020; 10:9361.
5. A.Maffucci and et.al. Springar International publishing Switzerland, 2017.
6. Takeshi Nakanishi and T.Ando, Journal of the Physical Society of Japan, 2009; vol.78, No.11, 114708
7. Shuba M.V. and et.al. Phys. Rev. B, 2007; 76, 155407.
8. Basmaci A. N. Materials MDPI, 2021; 14, 4096.
9. M. V. Shuba and et.al. Phys. Rev. B 79, 2009; 155403.
10. Kumar J. S. and Kumar A. Bulletin of Pure and Applied Sciences, Vol.40D (1), 2021; 49-55.
11. Kumar J. S. and Kumar A. Kanpur Philosophers, IX [V(II)] , 2022; 127-131
12. Kumar J. S. and Kumar A. Carbon Nanotubes; Recent Advances, New Perspective and Potential Applications, IntechOpen, 2022.

13. A. Snyer, IEEE, 1969; 17(12).
14. Kumar J. S. and Kumar A. Bulletin of pure and Applied Sciences, Vol.40D (2), 2021; 88-93.
15. Kumar J. S. ICSRD, 42, 2023.
16. Kumar J. S. and Kumar A. Bulletin of pure and Applied Sciences, Vol.40D (2), 2022; 75-80.
17. Kumar J. S. and Kumar A. AIP Conference proceedings, 2965(1), 2024.

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