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Internal Energy, Enthalpy and Gibb's Free Energy of Spinning Black Hole in Active Galactic Nuclei

[1]Dipo Mahto*, [2]Rakesh Paswan

Author's Affiliations:
[1]Professor & Head, Dept. of Physics, Bhagalpur College of Engineering under Department of Science and Technology, Patna, Bihar 812001, India.
And
[1]Former Head, Department of Physics, Marwari College Bhagalpur, (TMBU, Bhagalpur) Bihar 812007, India.
[2]Assistant Professor & Head, Department of Mathematics, DSM College (Munger University, Munger) Jhajha, Bihar 811308, India.

***Corresponding author:**
Dipo Mahto
Professor & Head, Dept. of Physics, Bhagalpur College of Engineering under Department of Science and Technology, Patna, Bihar 812001, India.
E-mail: dipomahto@hotmail.com

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ABSTRACT The present paper gives a model for the change in the internal energy and enthalpy (Mahto et al., 2016) and reviews it to propose a model for change in Gibb's free energy of the Reissnner-Nordstrom black hole. This also calculates their values in AGN concluding that the enthalpy and internal energy has the same values and they are just double to that of Gibb's free energy for the same mass.

KEYWORDS Hawking radiation, Black holes & Super Nova Explosion

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INTRODUCTION

Black hole is a gift of nature. This is created after super- nova explosion and belongs to a category of dead stars [1]. It has mass greater than 5 of the solar mass. As per classical theory, the black hole is a perfect absorber and emits nothing. But the quantum theory suggests that the black hole emits Hawking radiation. [2,3, 4]. Kanak et al. proposed a model for the energy of non-spinning black holes (E_{BH}) in terms of the radius of the event horizon [5]. Mahto et al. gave a model for the energy of spinning black holes in terms of the event horizon [6]. Mahto et al. gave a model for the change in energy and entropy of Non-spinning black holes by the use of first law of the black hole mechanics and well-known relation $E=mc^2$[7]. David Hochberg (1994), Shen & Chang-Jun (2001), David Kastor et al. (2011) and Beauchesne and Edery (2012) had done their works related to Gibb's free energy [8,9,10,11].

In the present work, we have used a model for the change in the internal energy and enthalpy

(Mahto et al., 2016) and reviewed it to propose a model for change in Gibb's free energy of the Reissnner-Nordstrom black hole and calculated their values in AGN.

THEORETICAL DISCUSSION

In the case of spinning black holes with half spin, the internal energy and enthalpy is given by the following equation [12].

$$dU_{BHs} = dH_{BHs} = \frac{0.2320}{M} R_{bhs}^2 \qquad \text{……(1)}$$

The change in the internal energy of black holes from first law of black hole thermodynamics is given by the following equation (Dolan et al., 2012).

$$dM = dU = TdS \qquad \text{…….(2)}$$

The change in free energy of the Reissnner-Nordstrom black hole due to change in the mass of black hole can be obtained by differentiating the above equation

$$dG = \frac{1}{2} dM \qquad \text{.............(3)}$$

Using equation (1) & (2) in the above equation and solving, we have

$$dG_{BHs} = \frac{0.1160}{M} R_{bhs}^2 \qquad \text{……...(4)}$$

For half spinning black holes, we have

$$R_{bhs} = M \qquad \text{……..(5)}$$

Putting above value in equation (1), the internal energy and enthalpy is given by the following equation

$$dU_{BHs} = dH_{BHs} = 0.2320M \qquad \text{………(6)}$$

Putting (5) in equation (4), we have

$$dG_{BHs} = 0.1160M \qquad \text{…...(7)}$$

The equation (6) represents the change in internal energy and enthalpy, while equation (7) represents the Gibb's free energy of spinning black holes. Hence, the required model for the change in internal energy and enthalpy & Gibb's free energy of spinning black hole is summarized in the following box.

$dU_{BHs} = dH_{BHs} = 0.2320M$ for internal energy and enthalpy
$dG_{BHs} = 0.1160M$ for Gibb's free energy

Data in support of mass of black holes in XRBs and AGN and Sun:

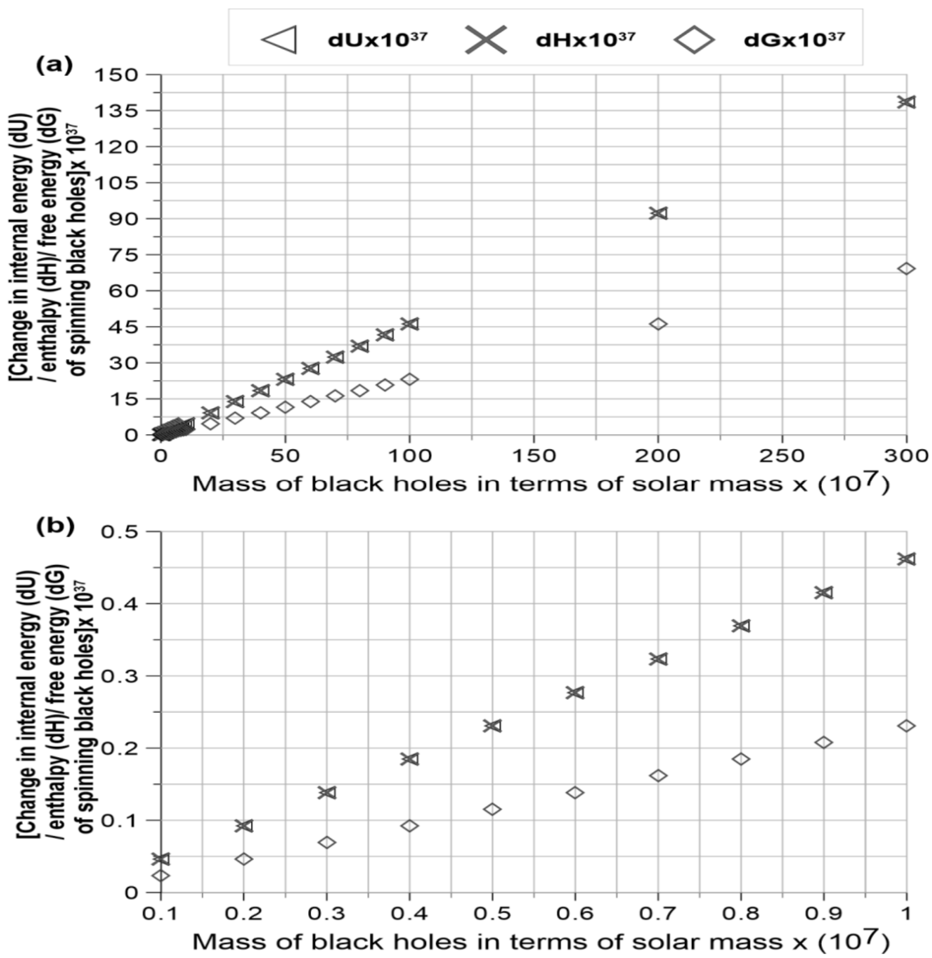
M ~ 5 - 20 M_Θ in XRBs, M ~ 10^6 – $10^{9.5}$ M_Θ in AGN and Mass of sun (M_Θ) = 1.99 x 10^{30} kg. [13]

Calculations:
dU=0.2320M= 0.2320x10^6 solar mass= 0.2320x10^6x1.99x10^{30}kg =0.46168x10^{36} Joule
dH=0.2320M= 0.2320x10^6 solar mass= 0.2320x10^6x1.99x10^{30}kg =0.46168x10^{36} Joule
dG=0.1160M= 0.1160x10^6 solar mass= 0.1160x10^6x1.99x10^{30}kg =0.23084x10^{36} J/mole
lly other values of internal energy/enthalpy and Gibb's Free energy can be calculated

Table 1: Change in enthalpy/internal and free energy of spinning black holes in AGN.

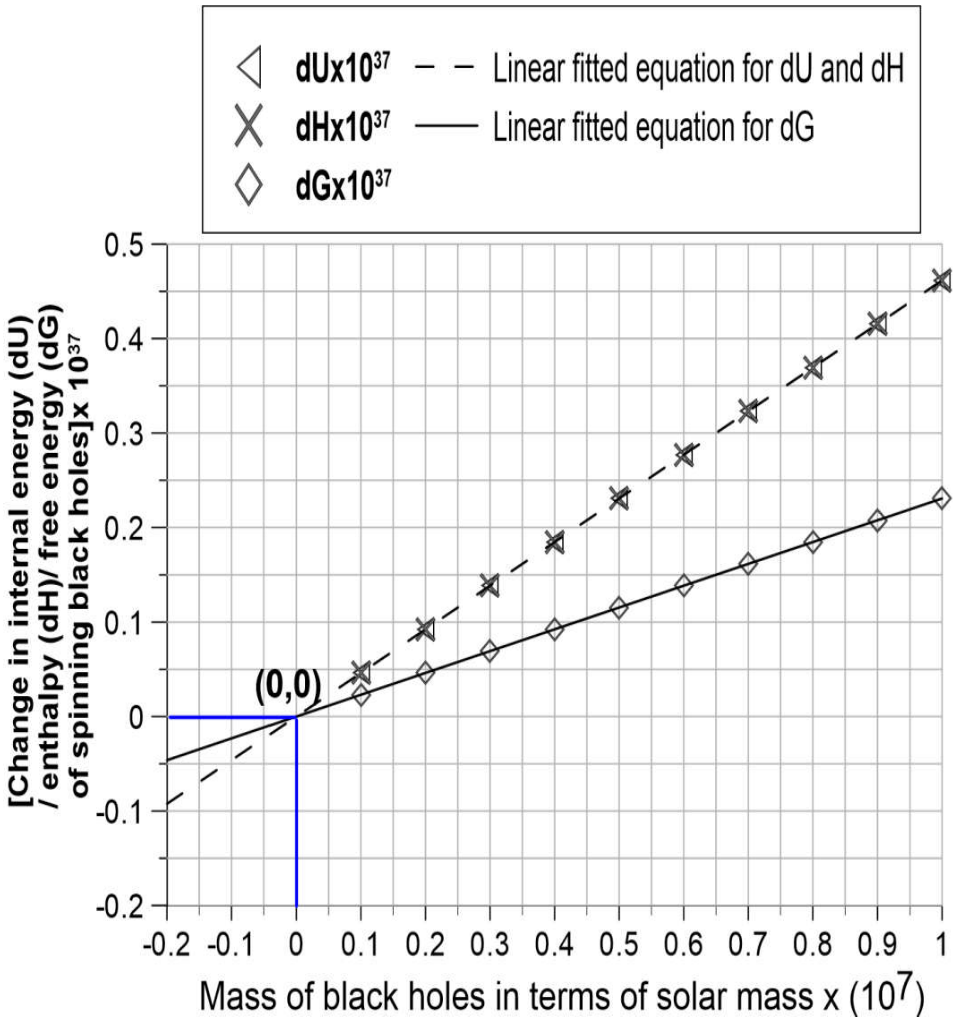
Sl. No.	Mass of BHs (M)	Mass of BHs in terms of solar mass (M_Θ x 10^7)	dU_{BHsb}(in Joule)	dH_{BHs} (in Joule)	dG_{BHs} (in Joule/mole)
1	1 x 10^6 M_Θ	0.1	0.046168x 10^{37}	0.046168x 10^{37}	0.023084 x 10^{37}
2	2 x 10^6 M_Θ	0.2	0.092336x 10^{37}	0.092336x 10^{37}	0.046165 x 10^{37}
3	3 x 10^6 M_Θ	0.3	0.138504x 10^{37}	0.138504x 10^{37}	0.069252 x 10^{37}
4	4 x 10^6 M_Θ	0.4	0.184672x 10^{37}	0.184672x 10^{37}	0.092336 x 10^{37}
5	5 x 10^6 M_Θ	0.5	0.230840x 10^{37}	0.230840x 10^{37}	0.115420 x 10^{37}
6	6 x 10^6 M_Θ	0.6	0.277008x 10^{37}	0.277008x 10^{37}	0.138504 x 10^{37}
7	7 x 10^6 M_Θ	0.7	0.323176x 10^{37}	0.323176x 10^{37}	0.161588 x 10^{37}
8	8 x 10^6 M_Θ	0.8	0.369344x 10^{37}	0.369344x 10^{37}	0.184672 x 10^{37}
9	9 x 10^6 M_Θ	0.9	0.415512x 10^{37}	0.415512x 10^{37}	0.207756 x 10^{37}
10	1 x 10^7 M_Θ	1	0.461680x 10^{37}	0.461680x 10^{37}	0.230840 x 10^{37}
11	2 x 10^7 M_Θ	2	0.923360x 10^{37}	0.923360x 10^{37}	0.461650 x 10^{37}
12	3 x 10^7 M_Θ	3	1.385040x 10^{37}	1.385040x 10^{37}	0.692520 x 10^{37}
13	4 x 10^7 M_Θ	4	1.846720x 10^{37}	1.846720x 10^{37}	0.923360 x 10^{37}
14	5 x 10^7 M_Θ	5	2.30840x 10^{37}	2.30840x 10^{37}	1.154200 x 10^{37}
15	6 x 10^7 M_Θ	6	2.77008x 10^{37}	2.77008x 10^{37}	1.385040 x 10^{37}
16	7 x 10^7 M_Θ	7	3.23176x 10^{37}	3.23176x 10^{37}	1.615880 x 10^{37}
17	8 x 10^7 M_Θ	8	3.69344x 10^{37}	3.69344x 10^{37}	1.846720 x 10^{37}
18	9 x 10^7 M_Θ	9	4.15512x 10^{37}	4.15512x 10^{37}	2.077560 x 10^{37}
19	1 x 10^8 M_Θ	10	4.61680x 10^{37}	4.61680x 10^{37}	2.308400 x 10^{37}
20	2 x 10^8 M_Θ	20	9.23360x 10^{37}	9.23360x 10^{37}	4.616500 x 10^{37}
21	3 x 10^8 M_Θ	30	13.85040x 10^{37}	13.85040x 10^{37}	6.925200 x 10^{37}
22	4 x 10^8 M_Θ	40	18.46720x 10^{37}	18.46720x 10^{37}	9.233600 x 10^{37}
23	5 x 10^8 M_Θ	50	23.08400x 10^{37}	23.08400x 10^{37}	11.542000 x 10^{37}
24	6 x 10^8 M_Θ	60	27.70080x 10^{37}	27.70080x 10^{37}	13.850400 x 10^{37}
25	7 x 10^8 M_Θ	70	32.31760x 10^{37}	32.31760x 10^{37}	16.158800 x 10^{37}
26	8 x 10^8 M_Θ	80	36.93440x 10^{37}	36.93440x 10^{37}	18.467200 x 10^{37}
27	9 x 10^8 M_Θ	90	41.55120x 10^{37}	41.55120x 10^{37}	20.775600 x 10^{37}
28	1 x 10^9 M_Θ	100	46.16800x 10^{37}	46.16800x 10^{37}	23.084000 x 10^{37}
29	2 x 10^9 M_Θ	200	92.33600x 10^{37}	92.33600x 10^{37}	46.165000 x 10^{37}
30	3 x 10^9 M_Θ	300	138.50400x 10^{37}	138.50400x 10^{37}	69.252000 x 10^{37}

Graph 1: Fit 1: Linear, Equation, y = (0.46168) x + 4.440892099E-017, NDPU = 10, $\bar{X}$ = 0.55, $\bar{Y}$ =0.253924, RLSS = 1.40593E-032, RSS = 0.175847, COD = 1.000, Residual mean square, sigma-hat-squared = 1.75741E-033.



Graph 1: The Graph (a) shows that the change in internal energy/enthalpy/free energy of spinning black holes for different mass of black holes in AGN in ranging from 10^6 to $10^{9.5}$ solar mass, while the Graph (b) shows that the change in internal energy/enthalpy/free energy of spinning black holes for different mass of black holes in AGN in ranging from (0.1 to 1)x10^7 solar mass for better and clear visual representation.

Graph 2: Fit 1: Linear, Equation, y = (0.2308412727 x -1E-006, NDPU = 10, $\bar{X}$ = 0.55, Average $\bar{Y}$ = 0.126962, RLSS = 6.76364E-012, RSS = 0.0439623, COD = 1.000, Residual mean square, sigma-hat-squared = 8.45455E-013.



Graph 2: The figure shows that the change in internal energy/enthalpy/free energy of spinning black holes for different mass of black holes in AGN.

RESULT AND DISCUSSION

In the present work, we have used a model $dH / dR_{bh} = dU / dR_{bh} = 0.2320(R_{bhs}^2 / M)$ for the change in the internal energy and enthalpy of the spinning black holes and further this model is extended to obtain the change in the internal energy, enthalpy and Gibb's free energy of the spinning black holes in terms of mass. The new model for the change in the internal energy, enthalpy and Gibb's free energy of the spinning black holes is given below:

$$dU_{BHs} = dH_{BHs} = 0.2320M$$ for internal energy and enthalpy

$$dG_{BHs} = 0.1160M$$ for Gibb's free energy

After obtaining this model, we have calculated their values in AGN and plotted the graphs between the change in internal energy, enthalpy and free energy of spinning black holes for different values of mass of black holes.

From the graph plotted between the change in internal energy, enthalpy and free energy of spinning black holes & different values of mass in AGN, it is obvious that the change in internal energy and enthalpy for different masses are the same and the Gibb's free energy of the spinning black holes have half value to that of the internal energy and enthalpy.

From graph, it is clear that there is an uniform variation in the change of internal energy, enthalpy and Gibb's free energy of the spinning black holes with increasing the mass of black holes.

The Graph (a) shows that the change in internal energy/enthalpy/free energy of spinning black holes for different mass of black holes in AGN in ranging from 10^6 to $10^{9.5}$solar mass, while the Graph (b) shows that the change in internal energy/enthalpy/free energy of spinning black holes for different mass of black holes in AGN in ranging from (0.1 to 1)x10^7 solar mass for better and clear visual representation.

In this case, when the nature of the graph plotted be t^n the mass of spinning black holes and their corresponding internal energy/enthalpy in AGN is observed, we see that there is an uniform variation of internal energy/enthalpy with different masses followed by the linear equation represented by $dU \times 10^{37}J = 0.46168 \times 10^7 M_\Theta + 4.440892099E\text{-}017$ for internal energy & $dH \times 10^{37}J = 0.46168 \times 10^7 M_\Theta + 4.440892099E\text{-}017$ for enthalpy with slope 0.46168 and fitting accuracy equal to 1.000 showing that the proposed model is very good fit for the data and indicates that 100% of the variation in the outcome has been explained just by predicting the outcome using the covariates included in the model. Similarly there is also an uniform variation of free energy with different masses followed by the linear equation represented by $dG \times 10^{37}J = 0.2304812727 \times 10^7 M_\Theta - 1E\text{-}006$ with slope 0.2304812727 and fitting accuracy equal to 1.000 showing that the proposed model is very good fit for the data and indicates that 100% of the variation in the outcome has been explained just by predicting the outcome using the covariates included in the model.

CONCLUSION

1. The change in internal energy and enthalpy of spinning black holes for different values of mass are the same.
2. The change in enthalpy as well internal energy increase with increasing the mass of spinning black holes.
3. The enthalpy and internal energy has the same values and they are just double to that of Gibb's free energy for the same mass of black holes.
4. The enthalpy and internal energy of spinning black holes are the manifestation of the same thing.
5. There is an uniform variation in the change of internal energy, enthalpy and Gibb's free energy of the spinning black holes with increasing the mass of black holes.

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