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Quantum Conductance of Interacting Quantum Wire By Current Relaxing Backscattering and Unklapp Processes

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ABSTRACT We have studied the quantum conductance of interacting quantum wire by using current relaxing back scattering and Umklap process. We have derived a general formula for the conductance of interacting quantum wire with good contact and current relaxing processes in the wire. We have shown that for an interacting ballistic wire contacted to leads were generalized to an interacting wire with damping. We have calculated the resistance of an interacting quantum wire which has coexisting ballistic and diffusive channels. Such coexistence is expected for integrable modes where part of the current is protected by a local or quasilocal conservation law. We have found that in such a case the ballistic channel is small and completely dominates the transport so that the system shows ideal quantum conductance. Relevant back scattering at the contact were found and were neglected. We have calculated the resistance of single wall carbon nanotubes caused by a coupling to the phononic degrees of freedom of the tube. Three modes have been taken into account. We have found that there is damping of the phonons due to phonon-phonon interactions which modified the phonon propagator. Backscattering is created by impurities which are often relevant perturbations and completely suppress the conductance below a temperature scale. Irrelevant backscattering due to phonons dominated in clean samples. Taking the electrons in the carbon nanotubes as noninteracting it has been shown that acoustic phonon modes gave rise to resistivity that increases linearly with temperature. The conductance showed thermally activated behaviour. At every temperatures Umklapp scattering at half filling leaded to gaps both in the charge and in the spin sector and thus to thermally activated behaviour. In a device configuration the filling in the tube is usually tuned away from half filling so that the Umklapp term oscillates. In the calculation the electrons are treated as noninteracting. Calculation shows that one electron-electron interactions are included the interactions with phonon modes of the tube alone give resistivity of the right magnitude even at room temperature if standard parameters are used. The obtained results were found in good agreement with previously obtained results.

KEYWORDS Quantum Conductance, interacting quantum wire, back scattering, Umklapp process, relaxing process, carbon nanotube, coupling, acoustic phonon.

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INTRODUCTION

Maslov et al. [1] and Schulz et al. [2] studied that the when the contacts are adiabatic the conductance remained unnormalized by the electron-electron interactions within the wire. In any realistic system the conductance is affected by backscattering processes caused by phonons or impurities in the bulk if the wire [3-5]. Back scattering process at the junctions between the wire and the leads are also renormalized the conductacne [6-7]. Two types of systems have been used to study electronic transport in quantum wires. In semiconductor hetrostructures wide two dimensional electron gases are formed that are subsequently laterally confined by applying gate voltages. This approach makes it possible to obtain very clean wires that show clear signatures of conductance quantization [8]. The other systems are carbon nanotubes [9]. Single wall carbon nanotubes can either be semiconducting or metallic depending on their wrapping vector. The basic electronic properties of carbon nanotubes can be studied by viewing them as rolled up graphene sheets [10-11]. The momentum transverse to the tube direction is quantized leading to a finite number of bands. For a metallic tube two of these bands cross at the two Fermi points so that in low energy limit. Bockrath et al. [12] experimentally studied that rope of single wall carbon nano tubes on S_i / S_iO_2 substrates. In this system charging effects and resonant tunneling were observed. The conductance was found to be small and transport dominated by the probability to tunnel an electron between the contact and the carbon nanotube. Experimental data [13-14] for tunneling into bulk or an end of carbon nanotube were consistent with a single Luttinger Parameter $K_{C+} \approx 0.2 - 0.4$ for the total charge mode in response to theory [15] spin lattice relaxation rates $\frac{1}{T}$, have also been interpreted in terms of a Luttinger liquid with $K_{C+} \approx 0.2$ [16-17]. Single carbon nanotubes have contacted as well. In such experiment almost perfect contacts have been realized so that the conductance of short wires at low temperature was close to the ideal quantum conductance G_0. Contacting the same carbon nanotube at various distance Purewal et al. [18] were further able to measure the conductance not only as a function of temperature but also as a function of length. For $T >> \Delta_C$ Umklapp scattering can be treated perturbatively and leaded to resistivity that similarly to the phonon contribution increased linearly with temperature if the electrons are assumed to be non interacting [19]. If momentum relaxation by Umklapp or back scattering is neglected then a wire with electron-electron interactions can be described as a Luttinger liquid. In this case a renormalization of the conductance to $G = \frac{2nKe^2}{L}$ with a Luttinger parameter $K < 1$ for repulsive interactions might be expected [20]. Treating the electrons in the carbon nanotube as noninteracting it has been shown that acoustic phonon modes gave rise to a resistivity that increased linearly with temperature [21. For a finite quantum wire with contacts for the action with abrupt changes in the parameters, the relevant bosonic Green's function it was found by matching across the boundaries between the wire and leads [22-23]. There is a quasilocal conservation law [24] that protects parts of the current from decoying implying infinite dc conductivity even at finite temperatures. Within an effective field theory such a conservation law can be taken into account by using a memory matrix formalism and self energy of the boson propagator [25].

Pandey [26] has studied electron transmission through an impurity in quantum wire is strongly affected by the applied magnetic fields and the shape of the cross section. When the magnetic field entered along the small axis of cross section, electron transmission is strongly enhanced since the overlap between incident and reflected wave functions become smaller and back scattering is decreased. When the magnetic field entered along the large axis of the cross section the overlap between wave functions become larger backward scattering is enhanced and transmission is suppressed. The conductance strengths of magnetic fields decreases this is due to the decrease of the number of conducting channels upon increasing the strength of the applied magnetic field. As the cross sectional shape becomes symmetric switching of the conductance to different conductance levels occur. This is due to the shifting of the energy levels towards higher values resulting in a smaller number of contributing conducting channels to the conductance. Singh and Aparajita [27] have studied the origin of the electron energy relaxation in clean single channel quantum wires accounting for the scattering processes that involved three particle collisions. Thermal transport of single channel quantum wires, where a lower value of the thermal conductance that predicted by the Widemann Franzlaw as observed at the plateau of the electrical conductance. They have found that the thermal conductance is reduced by interactions which are qualitatively consistent with the experimental observation.

METHOD

We have considered electronic transport in a quantum wire described by the generic low energy Hamiltonian

$$H = -iv_F \sum_{r,\alpha,\sigma} r \int dx \Psi^{\dagger}_{r\alpha\sigma}(x) \partial_x \Psi_{r\alpha\sigma}(x)$$

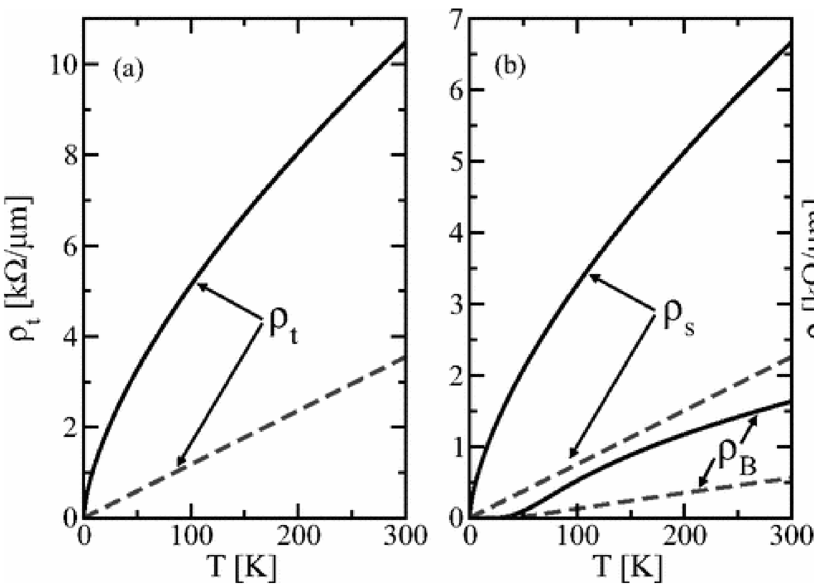
where v_F is the Fermi velocity and $\Psi^{\dagger}_{r\alpha\sigma}$ a fermionic annihilation creation operator with $\sigma = \pm$ being the spin and $\alpha = 1, ...n$ a band index. The fermionic field in the low energy limit is split into right movers $\Psi_{+,\alpha\sigma} = \Psi_{R,\alpha\sigma}$ and left movers $\Psi_{-,\alpha\sigma} \equiv \Psi_{L,\alpha\sigma}$. We have used standard Abelian basonization to express the fermionic operators in terms of bosonic fields

$$\Psi_{r\alpha\sigma}(x) = \frac{\eta_{r\alpha\sigma}}{\sqrt{2\pi\overline{a}}}.e^{i\left[k_F(r,\alpha)x + r\sqrt{2\pi}\phi_{r\alpha\sigma}(x)\right]}$$

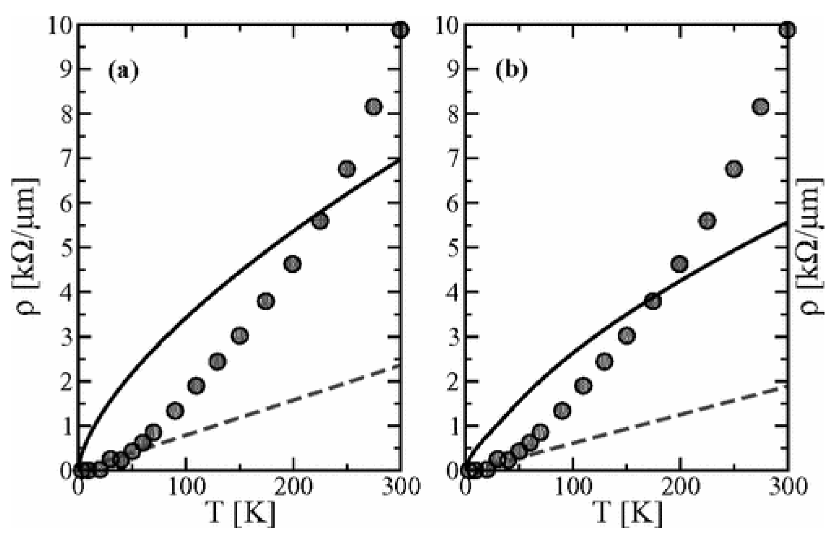
where $\eta_{r\alpha\sigma}$ are Klein factors ensuring ther fermionic commutation rules and $\overline{a}$ is a cutoff of the order of the lattice constant a. The charge density is expresses as $\rho = e\sqrt{\frac{2n}{\pi}}\partial_x \phi_j \delta$ through one of the bosonic fields, which is denoted by $\phi_j \delta$. The current density $j(x)$ is then obtained by the continuity equation $\partial_t \rho(x) = -i[\rho(x), H] = -\partial_x j(x)$ leading to $j = -e\sqrt{\frac{2n}{\pi}}\partial_t \varphi_j \delta$. We have generalized to approach by Maslov and Stone to derive formula for dc conductance of a quantum wire with good contacts and some form of damping in the bulk of the wire. We have used a self energy approach to consider damping by Umklapp scattering. We have taken the general case of damping due to backscattering assisted by other degrees of freedom such as phonons. For carbon nanotubes all microscopic parameters relevant for back scattering by phonons relatively allowed to obtain results for the dc conductance. The conductance of a finite end contacted quantum wire in the linear response regime with some damping in the bulk of the wire was studied. The damping in the wire made stem from electron-electron, electron-phonon or electron impurity interactions. We have considered the case where the damping is either caused by irrelevant interactions or cases where the interaction is relevant but in the temperature regime where the renormalized coupling constant for this interaction is small so that perturbation theory is applicable. Using Green's function we have perturbatively included the backward scattering to obtain the damping in the system. As the system is no longer homogeneous so we have considered the regime where the coherence length of the considered scattering process is much smaller than the length of the wire. The obtained results were compared with previously obtained results.

RESULTS AND DISCUSSION

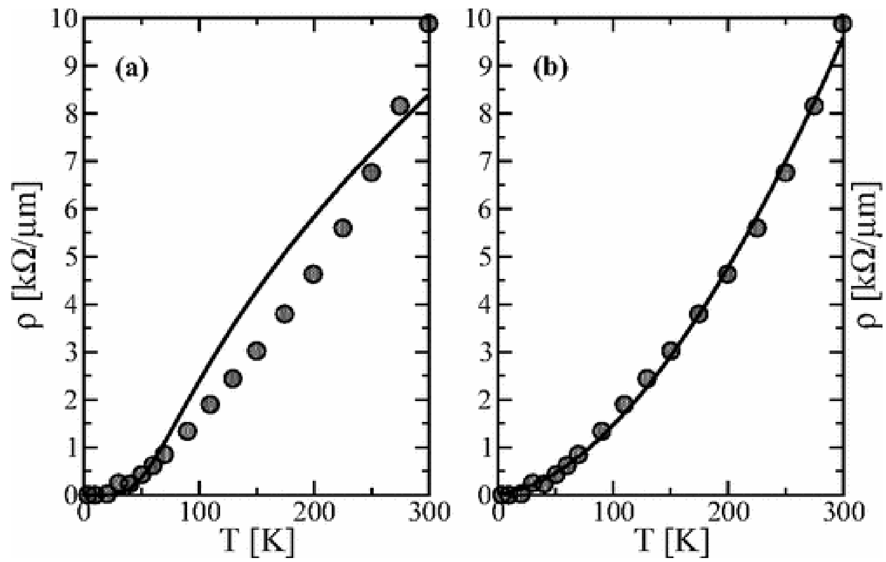
We have obtained the result for the resistivity of a carbon nanotube due to phonon assisted back scattering that included the electron-electron interactions of density-density type and depends only on microscopic parameters that can be theoretically estimated. The prefactor of the two acoustic modes is inversely proportional to the radius of the tube so that this scattering process becomes less important the wide tube. Graph (1) shows the result for armchair tube. The chiral angle and twist on mode only contributed. In graph (1) (b) the resistivity of a zigzag tube is shown when chiral angle η is zero. The Stretching mode contribution and the breathing mode contribution to the resistivity have been shown separately. The interactions substantially increased the resistivity by about a factor of 3 at room temperature is shown in graph (1). The contribution of the breathing mode for the zigzag tube is significantly increased by electron-electron interactions. The resistivity of device as a function of temperature is shown in graph (2), where we have subtracted a small constant contribution due to nonideal contacts and impurities. Scattering at impurities and at the contacts are relevant and leaded to an increasing resistivity. Treating the electrons as noninteracting gave a deviation which increased with temperature and is of the order of $7K\Omega/\mu m$ at T=300K. A variation of the chiral angle η for a tube with fixed diameter has a substantial effect on the resistivity as shown in graph (2) (a) and (2) (b). We have found that the acoustic modes are quenched and the resistivity is caused purely by a coupling to the breathing tube. The result is shown in graph (3) (a) but the electron-phonon coupling was substantially larger than the largest estimates. The conductance of single wall carbon nanotubes has been obtained for a wide range of tube lengths and temperatures. The increase of resistance at intermediate temperatures has been attributed to electron-phonon scattering, while at low temperatures also impurities and localization effects played a important role. Resistance of carbon nanotubes due to electron-phonon scattering has been calculated by taking only one of the acoustic modes into account and by assuming that the electrons are noninteracting. As long as there is a ballistic channel we find ideal quantum conductance. The conductivity of an infinite wire is reduced by a Luttinger parameter while the conductance of a finite wire with Fermi liquid contacts remained unchanged. Drude weight was reduced in comparison to a fully ballistic wire. We find that the dc conductance was not affected. We have found that ignoring the scattering at the contacts it was relevant and suppressed the conductance at low temperatures. The scattering involved the transfer of two electrons from on Fermi point to the other. Backscattering on the other hand, where only one electron scatterers between the Fermi points is Kinematically allowed if the transferred momentum is picked up by some other degree of the system. The obtained results were compared with previously obtained results of theoretical and experimental works and were found in good agreement.



Graph 1: (a) Resistivity ρ_t of a (10, 10) armchair tube caused by coupling to the twiston mode in the noninteracting case $(K_{c+} = 1)$ and in the interacting case $(K_{c+} = 0.3)$ with $g_2 = 1.5\,eV$. (b) ρ_s and ρ_B for a (18,0) zigzag tube in the noninteracting case $(K_{c+} = 1)$.



Graph 2: Results for (a) a (15, 15) armchair and (b) a (27,0) zigzag tube. Results with $g_2 = 1.5eV$ in the noninteracting case $K_{c+} = 1$ and in the interacting case with $K_{c+} = 0.3$.



Graph 3: The theoretical results for the breathing mode of a (27, 0) zigzag tube assuming $K_{c+} = 0.4$, $g_2 = 4.5$ eV and $\hbar\omega_B / k_B = 250\ K$.

CONCLUSION

We have calculated the conductance of metallic single wall carbon nanotubes in an energy range where back scattering due to phonon dominated. We have found that for a system where only a part of the current is protected by a conservation law, unreduced ideal quantum conductance for a fully wire was observed. The calculation show that a quantum wire which has ballistic transport channel can display a conductance that stay close to ideal quantum conductance over a wide temperature range. There is a damping of phonon due to phonon-phonon interactions, which modified the phonon propagator. This does not have any effect on the calculations. The phonon modes are always heavily populated and the phonon propagator became leading order even if damping is included. The splitting of the current into diffusive channel with finite relaxation rate and a ballistic channel as described by the self energy does not reduce the conductance compared to the purely ballistic case. As long as there is a ballistic channel we have found ideal quantum conductance. In some sense this is analogous to the case of repulsive electron-electron interactions; the conductivity of an infinite wire was reduced by the Luttinger parameter while the conductance of a finite wire with Fermi liquid contacts remains unchanged. If the transport is purely diffusive and the self energy is available, then Umklapp scattering did not lead to a length dependent conductance. The obtained results were compared with previously obtained results and were found in good agreement.

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