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Rotating MHD Flow: An Exact Solution by Hodograph Transformation

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Abstract

The motion of a homogenous, rotating, incompressible, second grade fluid with finite electrical conductivity through porous media in the presence of transverse magnetic field has been considered. For steady plane transverse flow the equations of motion have been recast in hodograph plane. Further the flow equations have been obtained in terms of Legendre transform function of the stream function. Lastly some examples have been taken to illustrate the developed theory and their streamlines are also plotted.

Keywords: MHD; exact solution; rotating frame; hodograph transformation; legendre transform function

1. Introduction

There are many phenomenon both naturally occurring as well as closer to our daily life like atmospheric or oceanic circulations, hurricanes and tornados, bath tub vorticities, stirring tea in a cup etc. that occur due to the rotating fluids. As such the theory of rotating fluids has become very important and many studies [1-15] have been made on the rotating fluid and several investigations have been carried out on various types of flows both non-MHD and MHD in a rotating system.

Also the study of non-Newtonian fluids is gaining importance day by day as many technological and industrial applications are based on them. In study of non-Newtonian fluids it is mostly found that the governing equations are second and third order non-linear differential equations and are very difficult to solve. As such many techniques have been used by researchers to transform the equations in solvable form and get the exact solutions. One such transformation technique is the hodograph transformation. Ames [16] has given an excellent survey of this method, which has been successfully used by many researchers [17-39].

In this paper we have studied steady plane rotating incompressible flow of an electrically conducting second grade fluid in the presence of transverse magnetic field through porous media and have applied the hodograph transformation for solving the system of non-linear partial differential equations governing the flow. Further we have determined exact solutions for special type of flows as illustration.

2. Basic Equations

The basic equations governing the motion of a rotating homogenous, electrically conducting, incompressible, second-grade fluid in the presence of magnetic field through porous media are

$$\nabla\cdot\mathbf{V} = \mathbf{0}, \tag{1}$$

$$\rho\left[\left(\mathbf{V}\cdot\nabla\right)\mathbf{V} + 2\mathbf{\Omega}\times\mathbf{V} + \mathbf{\Omega}\times\left(\mathbf{\Omega}\times\mathbf{r}\right)\right] = \operatorname{div}\mathbf{T} + \mu(\operatorname{curl}\mathbf{H})\times\mathbf{H} - \frac{\eta}{k}\mathbf{V}, \tag{2}$$

$$\frac{\partial\mathbf{H}}{\partial t} = \operatorname{curl}(\mathbf{V}\times\mathbf{H}) + \frac{1}{\mu\sigma}\nabla^2\mathbf{H}, \tag{3}$$

$$\nabla\cdot\mathbf{H} = 0, \tag{4}$$

and the constitutive equation for the Cauchy stress **T**,

$$\mathbf{T} = -p\mathbf{I} + \eta\mathbf{A}_1 + \alpha_1\mathbf{A}_2 + \alpha_2\mathbf{A}_1^2,$$

where **V** = velocity field vector, **H** = magnetic vector field , $p =$ dynamic pressure function, $\rho =$ the constant fluid field density, $\mathbf{\Omega} =$ angular velocity vector, $\mathbf{r} =$ radius vector $\eta =$ coefficient of dynamic viscosity, $\mu =$ constant magnetic permeability, $k =$ permeability of the medium, $\sigma =$ the electrical conductivity and α_1, α_2 are the normal stress moduli. $-p\mathbf{I}$ denotes the deterninate spherical stress, $\mathbf{I} =$ isotropic tensor, so that $-p\mathbf{I}$ becomes

$$-p\mathbf{I} = -\begin{pmatrix} p & 0 & 0 \\ 0 & p & 0 \\ 0 & 0 & p \end{pmatrix}.$$

The Rivlin-Ericksen tensors $\mathbf{A_1}$ and $\mathbf{A_2}$ are defined as

$$\mathbf{A}_1 = \nabla\mathbf{V} + (\nabla\mathbf{V})^T, \ \mathbf{A}_2 = \dot{\mathbf{A}}_1 + (\nabla\mathbf{V})^T\mathbf{A}_1 + \mathbf{A}_1(\nabla\mathbf{V}),$$

where a dot over $\mathbf{A_1}$ denotes the material time derivative.

Equations (1) to (3) form a system of three equations in three unknowns **V**, **H**, p. Equation (4) is an additional condition on **H** expressing the absence of magnetic poles in the flow.

Let us define the two-dimensional vorticity function $\omega(x, y)$ and a generalized energy function h(x, y) as:

$$\omega(x, y) = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y},$$

$$h(x,y)=\frac{1}{2}\rho V^2+p'+\frac{1}{2}\rho\,|\mathbf{\Omega}\times\mathbf{r}|^2-\alpha_1(u\nabla^2u+v\nabla^2v)-\frac{1}{4}(3\alpha_1+2\alpha_2)|\mathbf{A}_1|^2+\mu\frac{H^2}{2}\;, \tag{5}$$

where $V^2=u^2+v^2$, p' is the reduced pressure given by $p'=p-\frac{1}{2}\rho\,|\mathbf{\Omega}\times\mathbf{r}|^2$ and the last term being the centrifugal contribution of pressure, u, v are the components of velocity vector.

∇^2 is the Laplacian and

$$|\mathbf{A}_1|^2=4\left(\frac{\partial u}{\partial x}\right)^2+4\left(\frac{\partial v}{\partial y}\right)^2+2\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^2.$$

Here we have considered steady plane transverse flow. A steady plane flow in the (x, y) plane is said to be a transverse flow if the magnetic field vector is perpendicular to (x, y) plane which contains the fluid flow vector field and all the flow variables are functions of x and y. Thus, we take **V**=(u(x, y), v(x, y), 0), **H**=(0, 0, H(x, y)) and $\partial/\partial z=0$.

Introducing ω, h and the definition of **V** and **H** into the above system of equations we obtain the following system of equations:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \qquad \text{(continuity)}$$

$$\frac{\partial h}{\partial x}=\rho v\omega-\eta\frac{\partial \omega}{\partial y}-\alpha_1 v\nabla^2\omega-\frac{\eta}{k}u+2\rho v\Omega, \qquad \text{(linear momentum)}$$

$$\frac{\partial h}{\partial y}=-\rho u\omega+\eta\frac{\partial \omega}{\partial x}+\alpha_1 u\nabla^2\omega-\frac{\eta}{k}v-2\rho u\Omega, \tag{6}$$

$$u\frac{\partial H}{\partial x}+v\frac{\partial H}{\partial y}-\nu_H\left(\frac{\partial^2 H}{\partial x^2}+\frac{\partial^2 H}{\partial y^2}\right)=0, \qquad \text{(diffusion)}$$

$$\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}=\omega, \qquad \text{(vorticity)}$$

where $\nu_H=(\mu\sigma)^{-1}$ .The above system of five partial differential equations in five unknown functions u, v, ω, H and h as functions of (x, y) govern steady plane transverse flows of an incompressible second-grade fluid of finite electrical conductivity through porous media. Once a solution of these equations are found, the pressure function is determined from the expression for h(x, y) given in (5).

3. Equations in hodograph plane

Letting the function u= u(x, y) and v= v(x, y) to be such that, in the region of flow, the Jacobian

$$J(x,y)=\frac{\partial(u,v)}{\partial(x,y)}\neq 0, \quad 0<|J|<\infty, \tag{7}$$

we may consider x and y as functions of u and v. By means of x = x(u, v) ,y =y(u, v), we derive the following relations:

$$\frac{\partial u}{\partial x} = J\frac{\partial y}{\partial v}\ ,\ \frac{\partial u}{\partial y} = -J\frac{\partial x}{\partial v}\ ,$$

$$\frac{\partial v}{\partial x} = -J\frac{\partial y}{\partial u}\ ,\ \frac{\partial v}{\partial y} = J\frac{\partial x}{\partial u}\ . \tag{8}$$

We also obtain the relations

$$\frac{\partial g}{\partial x} = \frac{\partial(g,y)}{\partial(x,y)} = j\frac{\partial(g,y)}{\partial(u,v)}\ ,$$

$$\frac{\partial g}{\partial y} = -\frac{\partial(g,x)}{\partial(x,y)} = j\frac{\partial(x,g)}{\partial(u,v)}\ . \tag{9}$$

Where $g = g(x,\ y) = g\left(x(u,\ v),\ y(u,\ v)\right) = g(u,\ v)$ is any continuously differentiable function and

$$J = J(x,y) = \frac{\partial(u,v)}{\partial(x,y)} = \left[\frac{\partial(x,y)}{\partial(u,v)}\right]^{-1} = j(u,v)\,. \tag{10}$$

Employing these transformation relations for the first order partial derivatives and the transformation equations for the functions $\boldsymbol{\omega}, \mathbf{H}, \mathbf{h}$, defined by

$$\omega(x,y) = \omega\left(x(u,v), y(u,v)\right) = \boldsymbol{\omega}(u,v),$$

$$H(x,y) = H\left(x(u,v), y(u,v)\right) = \mathbf{H}(u,v),$$

$$h(x,y) = h\left(x(u,v), y(u,v)\right) = \mathbf{h}(u,v),$$

the system of equations (6) is replaced by the following system in the hodograph plane (u, v) :

$$\frac{\partial x}{\partial u} + \frac{\partial y}{\partial v} = 0\ , \tag{11}$$

$$j\frac{\partial(\mathbf{h},y)}{\partial(u,v)} = \rho v\boldsymbol{\omega} - \eta j w_1 - \alpha_1 v j\left[\frac{\partial(x, jw_1)}{\partial(u,v)} + \frac{\partial(jw_2, y)}{\partial(u,v)}\right] - \frac{\eta}{k}u + 2\rho v\Omega\,, \tag{12}$$

$$j\frac{\partial(x,\mathbf{h})}{\partial(u,v)} = -\rho u\boldsymbol{\omega} + \eta j w_2 + \alpha_1 u j\left[\frac{\partial(x, jw_1)}{\partial(u,v)} + \frac{\partial(jw_2, y)}{\partial(u,v)}\right] - \frac{\eta}{k}v - 2\rho u\Omega\,, \tag{13}$$

$$uG_1 + vG_2 - \nu_H\left[\frac{\partial(jG_1, y)}{\partial(u,v)} + \frac{\partial(x, jG_2)}{\partial(u,v)}\right] = 0\,, \tag{14}$$

$$j\left(\frac{\partial x}{\partial v} - \frac{\partial y}{\partial u}\right) = \boldsymbol{\omega}\,, \tag{15}$$

where

$$G_1 = G_1(u,v) = \frac{\partial(\mathbf{H},y)}{\partial(u,v)},\quad G_2 = G_2(u,v) = \frac{\partial(x,\mathbf{H})}{\partial(u,v)},$$

$$w_1 = w_1(u,v) = \frac{\partial(x,\boldsymbol{\omega})}{\partial(u,v)}, \quad w_2 = w_2(u,v) = \frac{\partial(\boldsymbol{\omega},y)}{\partial(u,v)}. \tag{16}$$

System of equations (11) to (15) is a system of five equations for the five unknown functions x(u, v), y(u, v), $\boldsymbol{\omega}$ (u, v), **H**(u, v) and **h** (u, v).

The equation of continuity implies the existence of a stream-function ψ (x, y) such that

$$d\psi = -v\,dx + u\,dy \ \text{ or } \ \frac{\partial\psi}{\partial x} = -v, \ \frac{\partial\psi}{\partial y} = u. \tag{17}$$

Likewise equation (11) implies the existence of a function L(u, v) , called the Legendre transform function of the stream-function Ψ (x, y) , so that

$$dL = -y\,du + x\,dv \ \text{ or } \ \frac{\partial L}{\partial u} = -y, \ \frac{\partial L}{\partial v} = x, \tag{18}$$

and the two functions ψ (x, y) , L(u, v) are related by

$$L(u,v) = vx - uy + \psi(x,y). \tag{19}$$

Introducing L(u, v) in the system (11)-(15), with j, w_1, w_2, G_1, G_2, given by (10), (16) respectively, it follows that (11) is identically satisfied and the system may be replaced by

$$j\frac{\partial\left(\frac{\partial L}{\partial u},\mathbf{h}\right)}{\partial(u,v)} = \rho v\boldsymbol{\omega} - \eta j w_1 - \alpha_1 v j\left[\frac{\partial\left(\frac{\partial L}{\partial v}, j w_1\right)}{\partial(u,v)} + \frac{\partial\left(\frac{\partial L}{\partial u}, j w_2\right)}{\partial(u,v)}\right] - \frac{\eta}{k}u + 2\rho v\Omega, \tag{20}$$

$$j\frac{\partial\left(\frac{\partial L}{\partial v},\mathbf{h}\right)}{\partial(u,v)} = -\rho u\boldsymbol{\omega} + \eta j w_2 + \alpha_1 u j\left[\frac{\partial\left(\frac{\partial L}{\partial v}, j w_1\right)}{\partial(u,v)} + \frac{\partial\left(\frac{\partial L}{\partial u}, j w_2\right)}{\partial(u,v)}\right] - \frac{\eta}{k}v - 2\rho u\Omega, \tag{21}$$

$$uG_1 + vG_2 - \nu_H\left[\frac{\partial\left(\frac{\partial L}{\partial u}, jG_1\right)}{\partial(u,v)} + \frac{\partial\left(\frac{\partial L}{\partial v}, jG_2\right)}{\partial(u,v)}\right] = 0, \tag{22}$$

$$j\left[\frac{\partial^2 L}{\partial v^2} + \frac{\partial^2 L}{\partial u^2}\right] = \boldsymbol{\omega}, \tag{23}$$

where $j = \left[\frac{\partial^2 L}{\partial v^2}\frac{\partial^2 L}{\partial u^2} - \left(\frac{\partial^2 L}{\partial u \partial v}\right)^2\right]^{-1}$,

$$w_1 = \frac{\partial\left(\frac{\partial L}{\partial v},\boldsymbol{\omega}\right)}{\partial(u,v)}, \ w_2 = \frac{\partial\left(\frac{\partial L}{\partial u},\boldsymbol{\omega}\right)}{\partial(u,v)},$$

$$G_1 = \frac{\partial\left(\frac{\partial L}{\partial u}, \mathbf{H}\right)}{\partial(u,v)}, \; G_2 = \frac{\partial\left(\frac{\partial L}{\partial v}, \mathbf{H}\right)}{\partial(u,v)}. \quad (24)$$

By using the integrability condition

$$\left[j\frac{\partial^2 L}{\partial u \partial v}\frac{\partial}{\partial v} - j\frac{\partial^2 L}{\partial v^2}\frac{\partial}{\partial u} \right]\left(j\frac{\partial\left(\frac{\partial L}{\partial u}, \mathbf{h}\right)}{\partial(u,v)} \right) = \left[j\frac{\partial^2 L}{\partial u^2}\frac{\partial}{\partial v} - j\frac{\partial^2 L}{\partial u \partial v}\frac{\partial}{\partial u} \right]\left(j\frac{\partial\left(\frac{\partial L}{\partial v}, \mathbf{h}\right)}{\partial(u,v)} \right),$$

i.e $\partial^2 h / \partial x \partial y = \partial^2 h / \partial y \partial x$ in the (x,y) plane, we eliminate $\mathbf{h}$(u, v) from (20) and (21) and obtain

$$\eta\left[\frac{\partial(\partial L/\partial v, jw_1)}{\partial(u,v)} + \frac{\partial(\partial L/\partial u, jw_2)}{\partial(u,v)} \right]$$
$$+\alpha_1 \left[v \frac{\partial(\partial L/\partial v, j\{\partial/\partial(u,v)(\partial L/\partial v, jw_1) + \partial/\partial(u,v)(\partial L/\partial u, jw_2)\})}{\partial(u,v)} \right.$$
$$\left. +u \frac{\partial(\partial L/\partial u, j\{\partial/\partial(u,v)(\partial L/\partial v, jw_1) + \partial/\partial(u,v)(\partial L/\partial u, jw_2)\})}{\partial(u,v)} \right]$$
$$+\frac{\eta}{k}\left[\frac{\partial(\partial L/\partial v, u)}{\partial(u,v)} - \frac{\partial(\partial L/\partial u, v)}{\partial(u,v)} \right] + 2\rho\Omega\left[\frac{\partial(\partial L/\partial v, v)}{\partial(u,v)} + \frac{\partial(\partial L/\partial u, u)}{\partial(u,v)} \right]$$
$$-\rho(uw_2 + vw_1) = 0, \quad (25)$$

where j, w_1, w_2 are given in (24) . Summing up we have the following theorem:

Theorem I: If L(u, v) is the Legendre transform function of a stream-function of steady, plane, transverse, rotating, incompressible, finitely conducting second-grade fluid flows through porous media and **H**(u, v) is the transformed magnetic vector component function then L(u, v) and **H**(u, v) must satisfy equations (25) and (22) where ω (u, v), j(u, v), w_1(u, v), w_2 (u, v), G_1(u, v), G_2(u, v) are given by (23) and (24).

If the fluid has infinite electrical conductivity, then the transformed diffusion equation becomes

$$uG_1 + vG_2 = 0, \quad (26)$$

where G_1, G_2 are given in (24). Then, we have the following theorem:

Theorem II: If L(u, v) is the Legendre transform function of a stream-function of steady, plane, transverse, rotating, incompressible, infinitely conducting second-grade fluid flows through porous media and **H**(u, v) is the transformed magnetic vector component function then L(u, v) and **H**(u, v) must satisfy equations (25) and (26) where ω (u, v), j(u, v), w_1(u, v), w_2 (u, v), G_1(u, v), G_2(u, v) are given by (23) and (24).

We now develop the flow equations in polar co-ordinates (q, θ) in the hodograph plane. We have

$$u = q\cos\theta, \quad v = q\sin\theta \;, \quad (27)$$

$$\frac{\partial}{\partial u} = \cos\theta\frac{\partial}{\partial q} - \frac{\sin\theta}{q}\frac{\partial}{\partial \theta} \;,$$

$$\frac{\partial}{\partial v} = \sin\theta\frac{\partial}{\partial q} + \frac{\cos\theta}{q}\frac{\partial}{\partial \theta} \;,$$

$$\frac{\partial(F,G)}{\partial(u,v)}=\frac{\partial(F^*,G^*)}{\partial(q,\theta)}\frac{\partial(q,\theta)}{\partial(u,v)}=\frac{1}{q}\frac{\partial(F^*,G^*)}{\partial(q,\theta)} \quad . \tag{28}$$

Where $F(u,v)=F^*(q,\theta),\ G(u,v)=G^*(q,\theta)$ are continuously differentiable functions.

Denoting $L^*(q,\theta),\ \omega^*(q,\theta),\ j^*(q,\theta),\ w_1^*(q,\theta),\ w_2^*(q,\theta),\ G_1^*(q,\theta),\ G_2^*(q,\theta)$ to be respectively transformed functions of L (u,v), $\omega(u,v),\ j(u,v), w_1(u,v), w_2(u,v), G_1(u,v), G_2(u,v)$ in (q,θ) coordinates and we can write them as follows:

$$j^*(q,\theta)=q^4\left[q^2\frac{\partial^2L^*}{\partial q^2}\left(q\frac{\partial L^*}{\partial q}+\frac{\partial^2L^*}{\partial\theta^2}\right)-\left(\frac{\partial L^*}{\partial\theta}-q\frac{\partial^2L^*}{\partial q\partial\theta}\right)^2\right]^{-1},$$

$$\omega^*(q,\theta)=j^*\left[\frac{\partial^2L^*}{\partial q^2}+\frac{1}{q^2}\frac{\partial^2L^*}{\partial\theta^2}+\frac{1}{q}\frac{\partial L^*}{\partial q}\right],$$

$$w_1^*(q,\theta)=\frac{1}{q}\frac{\partial\left(\sin\theta\frac{\partial L^*}{\partial q}+\frac{\cos\theta}{q}\frac{\partial L^*}{\partial\theta},\omega^*\right)}{\partial(q,\theta)},$$

$$w_2^*(q,\theta)=\frac{1}{q}\frac{\partial\left(\cos\theta\frac{\partial L^*}{\partial q}-\frac{\sin\theta}{q}\frac{\partial L^*}{\partial\theta},\omega^*\right)}{\partial(q,\theta)},$$

$$G_1^*(q,\theta)=\frac{1}{q}\frac{\partial\left(\cos\theta\frac{\partial L^*}{\partial q}-\frac{\sin\theta}{q}\frac{\partial L^*}{\partial\theta},H^*\right)}{\partial(q,\theta)},$$

$$G_2^*(q,\theta)=\frac{1}{q}\frac{\partial\left(\sin\theta\frac{\partial L^*}{\partial q}+\frac{\cos\theta}{q}\frac{\partial L^*}{\partial\theta},H^*\right)}{\partial(q,\theta)}. \tag{29}$$

Equations (25) and (22) are transformed to the (q,θ) plane as

$$\eta\chi^*+\alpha_1\left[\sin\theta\frac{\partial\left(\sin\theta\frac{\partial L^*}{\partial q}+\frac{\cos\theta}{q}\frac{\partial L^*}{\partial\theta},j^*\chi^*\right)}{\partial(q,\theta)}+\cos\theta\frac{\partial\left(\cos\theta\frac{\partial L^*}{\partial q}-\frac{\sin\theta}{q}\frac{\partial L^*}{\partial\theta},j^*\chi^*\right)}{\partial(q,\theta)}\right.$$

$$+\frac{\eta}{k}\frac{1}{q}\left[\frac{\partial\left(\sin\theta\frac{\partial L^*}{\partial q}+\frac{\cos\theta}{q}\frac{\partial L^*}{\partial\theta}, q\cos\theta\right)}{\partial(q,\theta)}-\frac{\partial\left(\cos\theta\frac{\partial L^*}{\partial q}-\frac{\sin\theta}{q}\frac{\partial L^*}{\partial\theta}, q\sin\theta\right)}{\partial(q,\theta)}\right]$$

$$+\frac{2\rho\Omega}{q}\left[\frac{\partial\left(\sin\theta\frac{\partial L^*}{\partial q}+\frac{\cos\theta}{q}\frac{\partial L^*}{\partial\theta}, q\sin\theta\right)}{\partial(q,\theta)}-\frac{\partial\left(\cos\theta\frac{\partial L^*}{\partial q}-\frac{\sin\theta}{q}\frac{\partial L^*}{\partial\theta}, q\cos\theta\right)}{\partial(q,\theta)}\right]$$

$$-\rho q\left(\cos\theta w_2^* + \sin\theta w_1^*\right) = 0, \tag{30}$$

$$q\left(\cos\theta G_1^* + \sin\theta G_2^*\right) - \frac{\nu_H}{q}\left[\frac{\partial\left(\cos\theta\frac{\partial L^*}{\partial q}-\frac{\sin\theta}{q}\frac{\partial L^*}{\partial\theta}, j^* G_1^*\right)}{\partial(q,\theta)}\right.$$

$$\left.+\frac{\partial\left(\sin\theta\frac{\partial L^*}{\partial q}+\frac{\cos\theta}{q}\frac{\partial L^*}{\partial\theta}, j^* G_2^*\right)}{\partial(q,\theta)}\right] = 0, \tag{31}$$

where χ^* is defined as

$$\chi^*(q,\theta) = \frac{1}{q}\left[\frac{\partial\left(\sin\theta\frac{\partial L^*}{\partial q}+\frac{\cos\theta}{q}\frac{\partial L^*}{\partial\theta}, j^* w_1^*\right)}{\partial(q,\theta)}+\frac{\partial\left(\cos\theta\frac{\partial L^*}{\partial q}-\frac{\sin\theta}{q}\frac{\partial L^*}{\partial\theta}, j^* w_2^*\right)}{\partial(q,\theta)}\right]. \tag{32}$$

From equations (30)-(31) we can have the following corollaries:

Corollary I: If $L^*(q,\theta)$ and $H^*(q,\theta)$ are the Legendre transform function of a stream function and the magnetic field vector function, respectively, of the equations governing the motion of steady, plane, rotating, transverse flows of incompressible finitely conducting second grade fluids through porous media, then $L^*(q,\theta)$ and $H^*(q,\theta)$ must satisfy equations (30) and (31) where $j^*, \omega^*, w_1^*, w_2^*, G_1^*, G_2^*, \chi^*$, are given by (29) and (32).

Corollary II: If $L^*(q,\theta)$ and $H^*(q,\theta)$ are the Legendre transform function of a stream function and the magnetic field vector function, respectively, of the equations governing the motion of steady, plane, rotating, transverse flows of incompressible infinitely conducting second grade fluids through porous media, then $L^*(q,\theta)$ and $H^*(q,\theta)$ must satisfy equations (30) and

$$\cos\theta G_1^* + \sin\theta G_2^* = 0 \,. \tag{33}$$

where $j^*, \omega^*, w_1^*, w_2^*, G_1^*, G_2^*, \chi^*$, are given by (29) and (32).

Once a solution $L^*(q,\theta), H^*(q,\theta)$ is known we employ

$$x = \sin\theta \frac{\partial L^*}{\partial q} + \frac{\cos\theta}{q}\frac{\partial L^*}{\partial \theta},$$

(34)

$$y = \frac{\sin\theta}{q}\frac{\partial L^*}{\partial \theta} - \cos\theta \frac{\partial L^*}{\partial q},$$

and (27) to obtain $u = u(x,y), v = v(x,y)$ in the physical plane. After obtaining these velocity components, we get $H(x,y)$ in the x, y plane from $H^*(q,\theta)$. The remaining flow variables are then determined by using the flow equations in the physical plane.

4. Applications

We now consider some flow problems as application of Theorems I and II and Corollaries I and II

Application I:

Let $L(u, v) = Au^2 + Bv^2$, (35)

be the Legendre transform function, where A, B are arbitrary constants and A, B are non zero. Using (35) in equations (23) and (24) we obtain

$$j = \frac{1}{4AB},\ \omega = \frac{A+B}{2AB},\ w_1 = w_2 = 0,\ G_1 = 2A\frac{\partial \mathbf{H}}{\partial v},\ G_2 = -2B\frac{\partial \mathbf{H}}{\partial u} \,.$$

(36)

Now employing (35) and (36) in equation (22), $\mathbf{H}(u, v)$ must satisfy

$$\frac{u}{B}\frac{\partial \mathbf{H}}{\partial v} - \frac{v}{A}\frac{\partial \mathbf{H}}{\partial u} - \nu_H\left[\frac{1}{2B^2}\frac{\partial^2 \mathbf{H}}{\partial v^2} + \frac{1}{2A^2}\frac{\partial^2 \mathbf{H}}{\partial u^2}\right] = 0. \tag{37}$$

Assuming $\mathbf{H}(u, v) = F(u) + G(v)$ to be the form of a possible solution for $\mathbf{H}(u, v)$, we find that (37) becomes

$$\frac{u}{B}G'(v) - \frac{v}{A}F'(u) - \nu_H\left[\frac{1}{2B^2}G''(v) + \frac{1}{2A^2}F''(u)\right] = 0.$$

(38)

Differentiating twice with respect to u, we get

$$\frac{F'''(u)}{A}v + \frac{\nu_H}{2A^2}F^{(\mathbf{iv})}(u) = 0.$$

The above equation holds true for all v if

$$\frac{F'''(u)}{A} = 0,\ \frac{\nu_H}{2A^2}F^{(\mathbf{iv})}(u) = 0. \tag{39}$$

Therefore we have

$F(u) = \frac{C_1}{2}u^2 + C_2 u + C_3$, where C_1, C_2, C_3, are arbitrary constants.

Using F(u) so obtained in equation (38), we find that G(v) must satisfy

$$\left[\frac{G'}{B} - \frac{C_1 v}{A}\right]u - \left[\frac{C_2}{A}v + \frac{\nu_H}{2B^2}G'' + \frac{\nu_H C_1}{2A^2}\right] = 0.$$

This equation holds true for all u if

$$\frac{G'}{B}-\frac{C_1 v}{A}=0,\qquad \frac{C_2}{A}v+\frac{\nu_H}{2B^2}G''+\frac{\nu_H C_1}{2A^2}=0. \tag{40}$$

Solving equations (40) we obtain

$$G(v)=-C_1\frac{v^2}{2}+C_4, C_2=0 \text{ and } A=-B. \tag{41}$$

Therefore we have

$$L(u,v)=A(u^2-v^2)\ ,\ \mathbf{H}(u,v)=\frac{C_1}{2}(u^2-v^2)+C_5\ , \tag{42}$$

where $C_5=C_3+C_4$.

Now using $L(u, v) = A(u^2 - v^2)$ in equation (25) we find that (25) is identically satisfied.
Using $L(u, v) = A(u^2 - v^2)$ in (18) and solving the resulting equations simultaneously we get

$$u(x,y)=\frac{-y}{2A},\ v(x,y)=\frac{-x}{2A}\quad . \tag{43}$$

Employing (35) in the solution for **H** (u, v) we obtain

$$H(x,y)=\frac{C_1}{8A^2}(y^2-x^2)+C_5\ . \tag{44}$$

Using $\omega = 0$ and equations (43) in the linear momentum equations in system (6) and integrating , we obtain h(x, y). Employing h(x, y) and (44) in (5), the pressure function is determined to be

$$p(x,y)=\frac{\eta}{kA}xy+\frac{\rho\Omega}{2A}\left(y^2-x^2\right)-\frac{\rho}{8A^2}(x^2+y^2)+\frac{(3\alpha_1+2\alpha_2)}{2A^2}-\frac{\mu}{2}\left[\frac{C_1}{8A^2}(y^2-x^2)+C_5\right]^2+C_6\ , \tag{45}$$

where C_6 is a constant. And the streamlines are given by

$x^2 - y^2$ = Constant.

This shows that the streamlines of the flow equations are concentric hyperbolas.

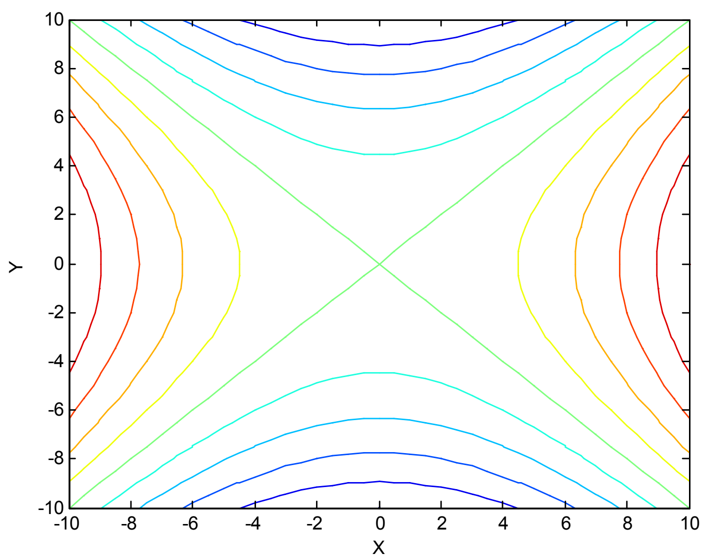

Figure 1: Concentric hyperbolic streamlines

Theorem III: If $L(u,v) = A(u^2 - v^2)$ is the Legendre transform function of a stream function for steady, plane, rotating, incompressible, finitely conducting second-grade fluid through porous media in the presence of transverse magnetic field, then the flow in the physical plane is a flow with concentric hyperbolic streamlines with flow variables given by (43) to (45).

Infinitely conducting fluid

Using $L(u,v) = A(u^2 - v^2)$ the equations (25) is identically satisfied and taking $A = -B$ in (36), (26) takes the form

$$\frac{1}{v}\frac{\partial \mathbf{H}}{\partial v} + \frac{1}{u}\frac{\partial \mathbf{H}}{\partial u} = 0 \ , \tag{46}$$

Solving (46) for $\mathbf{H}(u,v)$ we get

$$\mathbf{H}(u,v) = \phi\left(\frac{v^2 - u^2}{2}\right) , \tag{47}$$

where ϕ is an arbitrary function of its argument.

Proceeding as in the case of finitely conducting we get

$$u(x,y) = \frac{-y}{2A}, \ v(x,y) = \frac{-x}{2A} \quad , \tag{48}$$

$$H(x,y) = \phi\left[\frac{x^2 - y^2}{8A^2}\right] \quad , \tag{49}$$

and

$$p(x,y) = \frac{\eta}{kA}xy + \frac{\rho\Omega}{2A}\left(y^2 - x^2\right) - \frac{\rho}{8A^2}(x^2 + y^2) + \frac{(3\alpha_1 + 2\alpha_2)}{2A^2} - \frac{\mu}{2}\phi\left[\frac{x^2 - y^2}{8A^2}\right]^2 + C_7 , \tag{50}$$

where C_7 is an arbitrary constant.

We have the following theorem:

Theorem IV: If $L(u,v) = A(u^2 - v^2)$ is the Legendre transform function of a stream function for steady, plane, rotating, incompressible, infinitely conducting second-grade fluid through porous media in the presence of transverse magnetic field, then the flow in the physical plane is a flow with flow variables given by (48) to (50).

Application II:

Let $L(u,v) = uv$. (51)

Employing $L(u,v) = uv$ in equation (25) we find that (25) is identically satisfied. Using (51) in equations (23) and (24) we obtain

$$j = -1 \ , \ \omega = 0 \ , \ w_1 = w_2 = 0 \ , \ G_1 = -\frac{\partial \mathbf{H}}{\partial u}, \ G_2 = \frac{\partial \mathbf{H}}{\partial v}. \tag{52}$$

Now using (51) and (52) in equation (22), $\mathbf{H}(u, v)$ must satisfy

$$-u\frac{\partial \mathbf{H}}{\partial u}+v\frac{\partial \mathbf{H}}{\partial v}+\nu_H\left[\frac{\partial^2 \mathbf{H}}{\partial u^2}+\frac{\partial^2 \mathbf{H}}{\partial v^2}\right]=0 \ . \tag{53}$$

Assuming **H**(u, v) = F(u) G(v) to be the form of a possible solution for **H**(u, v), we find that (53) becomes

$$-uF'(u)\ G(v)+vF(u)\ G'(v)+\nu_H\left[F''(u)\ G(v)+F(u)\ G''(v)\right]=0,$$

dividing the whole equation by F(u) G(v) we get

$$-u\frac{F'(u)}{F(u)}+v\frac{G'(v)}{G(v)}+\nu_H\left[\frac{F''(u)}{F(u)}+\frac{G''(v)}{G(v)}\right]=0 \tag{54}$$

Above equation holds true if

$$-u\frac{F'(u)}{F(u)}+v\frac{G'(v)}{G(v)}=0,\ \nu_H\left[\frac{F''(u)}{F(u)}+\frac{G''(v)}{G(v)}\right]=0 \tag{55}$$

Solving equations (55) we get

$$F(u)=D_1u \text{ and } G(v)=D_2v \ ,$$

where D_1 and D_2 are arbitrary constants. Therefore we have

$$L(u,v)=uv \ , \ \mathbf{H}(u,v)=Duv \ , \tag{56}$$

Now proceeding as in the previous cases we obtain

$$u(x,y)=x \ , \quad v(x,y)=-y \ ,$$

$$H(x,y)=-Dxy \ ,$$

and

$$p(x,y)=\frac{\eta}{2k}\left(y^2-x^2\right)-4\rho\Omega xy-\frac{1}{2}\rho\left(x^2+y^2\right)+2\left(3\alpha_1+2\alpha_2\right)-\frac{\mu}{2}\left[-Dxy\right]^2+D_3 \tag{57}$$

where D_3 is an arbitrary constant. And the streamlines are given by

$xy = \text{Constant}$.

This shows that the streamlines of the flow equations are rectangular hyperbolae.

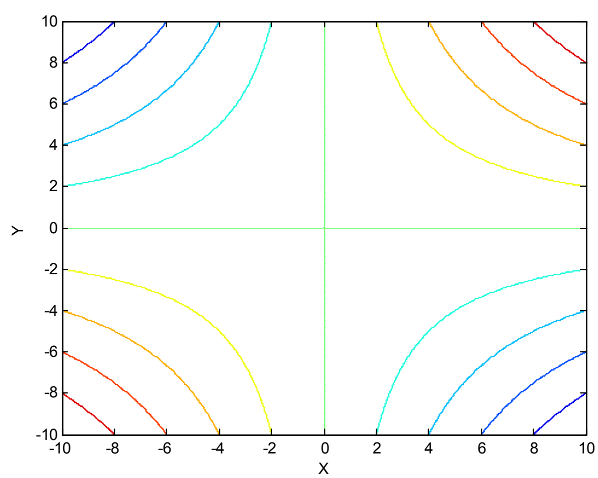

Figure 2: Rectangular hyperbolae streamlines

Theorem V: If $L(u,v)=uv$ is the Legendre transform function of a stream function for steady, plane, rotating, incompressible, finitely conducting second-grade fluid through porous media in the presence of transverse magnetic field, then the flow in the physical plane is a flow with rectangular hyperbolic streamlines with flow variables given by (57) .

Infinitely conducting fluid

Here in this case, only the diffusion equation is replaced by

$$-u\frac{\partial \mathbf{H}}{\partial u}+v\frac{\partial \mathbf{H}}{\partial v}=0, \tag{58}$$

Solving equation (58), $\mathbf{H}(u,v)$ is obtained as

$$\mathbf{H}(u,v)=\varphi(uv),$$

where φ is an arbitrary function of its argument.

On proceeding as before we obtain

$$u(x,y)=x, \quad v(x,y)=-y,$$

$$H(x,y)=\varphi(-xy),$$

$$p(x,y)=\frac{\eta}{2k}\left(y^2-x^2\right)-4\rho\Omega xy-\frac{1}{2}\rho\left(x^2+y^2\right)+2\left(3\alpha_1+2\alpha_2\right)-\frac{\mu}{2}\left[\varphi(-xy)\right]^2+D_4 \tag{59}$$

where D_4 is an arbitrary constant.

Theorem VI: If $L(u,v)=uv$ is the Legendre transform function of a stream function for steady, plane, rotating, incompressible, infinitely conducting second-grade fluid through porous media in the presence of transverse magnetic field, then the flow in the physical plane is a flow with rectangular hyperbolic streamlines with flow variables given by (59) .

In the absence of rotating reference frame i.e. $\mathbf{\Omega}=\mathbf{0}$ we recover the results of Sayantan Sil, Manoj Kumar and C. Thakur [30]. Also when porous media is absent i.e. the term $\frac{\eta}{k}\to 0$ our result will tally with and O.P Chandna and P.V Nguyen [20].

Application II:

Let

$$L^*(q,\theta)=F(q),\ F'(q)\neq 0,\ F''(q)\neq 0. \tag{60}$$

By using (60) in equations (29) we obtain

$$\begin{gathered}
j^*=\frac{q}{F'(q)F''(q)},\quad \omega^*=\frac{qF''(q)+F'(q)}{F'(q)F''(q)},\\
w_1^*=-\frac{1}{q}\omega^{*\prime}\cos\theta F'(q),\quad w_2^*=\frac{1}{q}\omega^{*\prime}\sin\theta F'(q),\\
G_1^*=\frac{1}{q}\left[\cos\theta F''(q)\frac{\partial H^*}{\partial\theta}+\sin\theta F'(q)\frac{\partial H^*}{\partial q}\right],\\
G_2^*=\frac{1}{q}\left[\sin\theta F''(q)\frac{\partial H^*}{\partial\theta}-\cos\theta F'(q)\frac{\partial H^*}{\partial q}\right],\\
x=F'(q)\sin\theta,\quad y=-F'(q)\cos\theta.
\end{gathered} \tag{61}$$

We now study finitely conducting fluid flow and infinitely conducting fluid flow as applications of Corollaries I and II.

Finitely conducting fluid

Using (60) and (61) in equations (30) and (31), we find that $F(q)$ and $H^*(q,\theta)$ must satisfy

$$\frac{\eta}{q}\left[\omega^{*'} + F'(q)\left\{\frac{\omega^{*'}}{F''(q)}\right\}'\right] - \frac{\eta}{k}\left[F'(q) + \frac{1}{q}F'(q)\right] = 0 \ , \tag{62}$$

$$F''(q)\frac{\partial H^*}{\partial \theta} - \frac{\nu_H}{q}\left[\cos\theta F''(q)(j^*G_1^*)_\theta + \sin\theta F'(q)(j^*G_1^*)_q + \sin\theta F''(q)(j^*G_2^*)_\theta - \cos\theta F'(q)(j^*G_2^*)_q\right] = 0 \ , \tag{63}$$

so that $F(q)$ is the Legendre transform function of a stream function.

For constant vorticity $\omega^* = \omega_o = \text{constant}$, equation (62) becomes

$$\frac{\eta}{k}\left[qF''(q) + F'(q)\right] = 0 \ , \tag{64}$$

Solving (64) we get

$$F'(q) = \frac{E_1}{q} \ , \tag{65}$$

where E_1 is an arbitrary constant.

By the expression of x and y in (61)

$$r = \sqrt{x^2 + y^2} = \pm F'(q) \ , \tag{66}$$

and from (65) and (66)

$$q = \pm\frac{E_1}{\sqrt{x^2 + y^2}} \ . \tag{67}$$

Using (66) and (67) in equation (34) and (27), we get

$$u(x,y) = -\frac{E_1 y}{x^2 + y^2}$$
$$v(x,y) = \frac{E_1 x}{x^2 + y^2} \ . \tag{68}$$

Transforming the diffusion equation (63) back to (x,y) plane we find that $H(x,y)$ must satisfy

$$\frac{E_1}{x^2 + y^2}\left[x\frac{\partial H}{\partial y} - y\frac{\partial H}{\partial x}\right] - \nu_H\left[\frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2}\right] = 0 \ . \tag{69}$$

A solution for $H(x,y)$ is

$$H(x,y) = E_2 \ln\left(\frac{x^2 + y^2}{2}\right) + E_3 \ , \tag{70}$$

where E_2 and E_3 are arbitrary constants.

As $\quad \omega(x,y) = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$, we find that

$$\omega(x,y)=\omega_0=0 \tag{71}$$

Using (68) and (71) in the linear momentum equations of system (6) and integrating, we get $h(x,y)$. Using this solution for $h(x,y)$ and (71) in (5), we get the pressure function as

$$p(x,y)=\frac{\eta}{k}E_1\left(\tan^{-1}\frac{x}{y}-\tan^{-1}\frac{y}{x}\right)-\frac{1}{2}\rho\frac{E_1^2}{\left(x^2+y^2\right)}+\frac{2\left(3\alpha_1+2\alpha_2\right)E_1^2}{\left(x^2+y^2\right)^2}-2E_1\rho\Omega\ln\left(x^2+y^2\right)$$
$$-\frac{\mu}{2}\left[E_2\ln\left(\frac{x^2+y^2}{2}\right)+E_3\right]^2+E_4\ , \tag{72}$$

where E_4 is an arbitrary constant.

Infinitely conducting fluid

In this case the transformed diffusion equation reduces to

$$\frac{\partial H^*}{\partial\theta}=0\ .$$

Therefore we have

$$H(x,y)=\phi(q)\ \ ,$$

where ϕ is an arbitrary function of its argument. In the (x,y) plane, we have

$$H(x,y)=\phi(q)\ , \tag{73}$$

where q is given by (67).

$u(x,y), v(x,y), \omega(x,y), H(x,y)$ are given by (68), (71) , (70) and $p(x,y)$ by

$$p(x,y)=\frac{\eta}{k}E_1\left(\tan^{-1}\frac{x}{y}-\tan^{-1}\frac{y}{x}\right)-\frac{1}{2}\rho\frac{E_1^2}{\left(x^2+y^2\right)}+\frac{2\left(3\alpha_1+2\alpha_2\right)E_1^2}{\left(x^2+y^2\right)^2}-2E_1\rho\Omega\ln\left(x^2+y^2\right)$$
$$-\frac{\mu}{2}\left[\phi(q)\right]^2+E_5\ , \tag{74}$$

where q is given by (67) and E_5 is an arbitrary constant.

Theorem VII: If $L^*(q,\theta)=F(q)$ is the Legendre transform function of a stream function for a steady, plane, rotating, transverse, incompressible, finitely conducting second-grade fluid flow through porous media then the flow in the physical plane is given by equations (68), (70), (71) and (72) with $\ln\left(x^2+y^2\right)=$ constant as its streamlines.

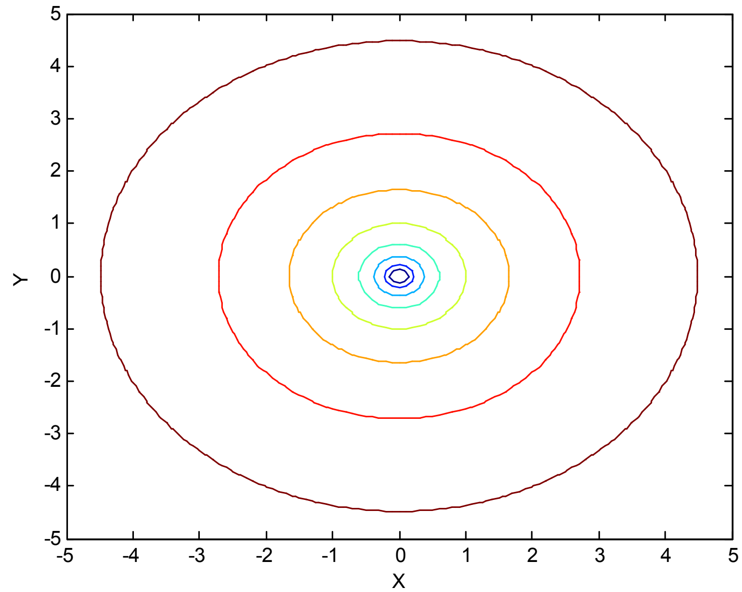

Figure 3: Concentric Circular streamlines

Theorem VIII: If $L^*(q,\theta)=F(q)$ is the Legendre transform function of a stream function for a steady, plane, transverse, incompressible, infinitely conducting second-grade fluid flow through porous media then the flow in the physical plane is given by equations (68), (71), (73) and (74) with $\ln(x^2+y^2)=$ constant as its streamlines.

Application III:

Let

$$L^*(q,\theta)=A\theta+B\ , \tag{75}$$

be the Legendre transform function, where A, B are arbitrary constants and $A\neq 0$.

Using (75) in equations (29), we obtain

$$j^*=\frac{-q^4}{A^2}\ ,\quad \omega^*=w_1^*=w_2^*=0\ ,$$

$$G_1^*=\frac{1}{q}\left[\frac{A}{q^2}\sin\theta\frac{\partial H^*}{\partial\theta}+\frac{A}{q}\cos\theta\frac{\partial H^*}{\partial q}\right]\ , \tag{76}$$

$$G_2^*=\frac{1}{q}\left[\frac{A}{q}\sin\theta\frac{\partial H^*}{\partial q}-\frac{A}{q^2}\cos\theta\frac{\partial H^*}{\partial\theta}\right]\ .$$

Finitely conducting fluid

Using (60), (76) in equations (29) and (30) we find that (29) is identically satisfied and $H^*(q,\theta)$ must satisfy

$$A\frac{\partial H^*}{\partial q}-\frac{\nu_H}{q}\Bigg[\frac{A}{q^2}\sin\theta\left(j^*G_1^*\right)_\theta+\frac{A}{q}\cos\theta\left(j^*G_1^*\right)_q$$
$$+\frac{A}{q}\sin\theta\left(j^*G_2^*\right)_q-\frac{A}{q^2}\cos\theta\left(j^*G_2^*\right)_\theta\Bigg]=0\ . \quad (77)$$

Using $L^*(q,\theta)=A\theta+B$ in equation (34) and making use of (27) we obtain

$$u(x,y)=\frac{Ax}{x^2+y^2}\ ,\ v(x,y)=\frac{Ay}{x^2+y^2}\ , \quad (78)$$

Transforming (77) back to (x,y) plane $H(x,y)$ must satisfy

$$\frac{Ax}{x^2+y^2}\frac{\partial H}{\partial x}+\frac{Ay}{x^2+y^2}\frac{\partial H}{\partial y}-\nu_H\left(\frac{\partial^2 H}{\partial x^2}+\frac{\partial^2 H}{\partial y^2}\right)=0\,. \quad (79)$$

A solution for $H(x,y)$ satisfying (65) is found to be

$$H(x,y)=M_1\left[\frac{\left(\tan^{-1}\frac{y}{x}\right)^2}{2}+\frac{\nu_H}{A}\ln\left(x^2+y^2\right)^{1/2}\right]+M_2\left(\tan^{-1}\frac{y}{x}\right)$$
$$+M_3\frac{\nu_H}{A}\left(x^2+y^2\right)^{A/2\nu_H}+M_4\,, \quad (80)$$

where $M_1,M_2,M_3,$ and M_4 are arbitrary constants.

Using $\omega=0$ and (78) in the linear momentum equations of system (6) and integrating, we get $h(x,y)$. Using this solution for $h(x,y)$ and (80) in (5) the pressure function is determined to be

$$p(x,y)=M_5-\frac{\eta A}{k}\ln\left(x^2+y^2\right)+2\rho\Omega A\left(\tan^{-1}\frac{x}{y}-\tan^{-1}\frac{y}{x}\right)-\frac{1}{2}\frac{\rho A^2}{\left(x^2+y^2\right)}$$
$$+\left(3\alpha_1+2\alpha_2\right)\frac{2A^2}{\left(x^2+y^2\right)^2}$$
$$-\frac{\mu}{2}\left[M_1\left\{\frac{\left(\tan^{-1}\frac{y}{x}\right)^2}{2}+\frac{\nu_H}{A}\ln\left(x^2+y^2\right)^{1/2}\right\}+M_2\left(\tan^{-1}\frac{y}{x}\right)+M_3\frac{\nu_H}{A}\left(x^2+y^2\right)^{A/2\nu_H}+M_4\right]^2, \quad (81)$$

where M_5 is an arbitrary constant.

Infinitely conducting fluid

In this case, the transformed diffusion equation is replaced by

$$A\frac{\partial H^*}{\partial q}=0.$$

Therefore, we get

$$H^*(q,\theta)=\phi(\theta),$$

where ϕ is an arbitrary function of its argument.

In the (x,y) plane we have

$$H(x,y)=\phi\left(\tan^{-1}\left(\frac{y}{x}\right)\right). \tag{82}$$

Using $L^*(q,\theta)=A\theta+B$ and proceeding as before we get

$$u=\frac{Ax}{x^2+y^2}\quad,\quad v=\frac{Ay}{x^2+y^2}\quad, \tag{83}$$

and

$$p(x,y)=M_6-\frac{\eta A}{k}\ln\left(x^2+y^2\right)+2\rho\Omega A\left\{\tan^{-1}\left(\frac{x}{y}\right)-\tan^{-1}\left(\frac{y}{x}\right)\right\}$$
$$-\frac{\rho A^2}{2\left(x^2+y^2\right)}+\frac{(3\alpha_1+2\alpha_2)2A^2}{\left(x^2+y^2\right)^2}-\frac{\mu}{2}\left[\phi\left\{\tan^{-1}\left(\frac{y}{x}\right)\right\}\right]^2\quad, \tag{84}$$

where M_6 is an arbitrary constant.

Theorem XI: If $L^*(q,\theta)=A\theta+B$ is the Legendre transform function of a stream function for steady, plane, rotating, transverse, incompressible, finitely conducting second grade flow through porous media, then the flow in the physical plane is given by equations (78), (80) and (81) having $\tan^{-1}\left(y/x\right)-\tan^{-1}\left(x/y\right)=$ constant as its streamlines.

Theorem X: If $L^*(q,\theta)=A\theta+B$ is the Legendre transform function of a stream function for steady, plane, rotating, transverse, incompressible, infinitely conducting second grade flow through porous media, then the flow in the physical plane is given by equations (82), (83) and (84) having $\tan^{-1}\left(y/x\right)-\tan^{-1}\left(x/y\right)=$ constant as its streamlines.

When there is no rotating frame i.e. $\Omega=0$ the results of S.Sil, M. Kumar and C. Thakur [30] is recovered, while when the rotating frame as well as porous media is absent i.e $\Omega=0$ and $\frac{\eta}{k}\to 0$ the results tally with O.P Chandna and P.V. Nguyen [20]. In the absence of rotating frame and for non-MHD fluid through porous media the results of M. Kumar [31] is obtained and for non-MHD fluid through porous media in a rotating frame the results of S. Sil and M. Kumar [15] is regained. And for non MHD fluid in the absence of both rotating frame and porous media the results of A.M. Siddiqui, P.N. Kaloni and O.P. Chandna [19] is recovered.

6. Conclusion

The main theme of this work is to obtain analytical solution of nonlinear equations governing the flow of second grade fluid in a rotating frame through porous media in the presence of a transverse magnetic field by the application of hodograph transformation technique. The flow equations have been obtained in terms of Legendre transform function of the stream function. The expressions for velocity profile, streamline and pressure distribution are successfully obtained in each case.

Streamline patterns are also plotted. The results indicate that for second grade MHD fluid the pressure distributions are dependent upon material parameters α_1 and α_2. Several results of various authors (as already mentioned in the text) can be recovered in the limiting cases; as such the present analysis is more general.

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