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Odd graceful labeling for the jewel graph and the extended jewel graph without the prime edge *

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Abstract R.B. Gnanajothi (Topics in Graph Theory, Ph.D. Thesis, Madurai Kamaraj University, Madurai, Tamil Nadu, India, 1991) introduced odd graceful labeling. A function f is called an odd graceful labeling of a graph G if $f : V(G) \to \{0, 1, 2, ..., 2q-1\}$ is injective and the induced function $f^* : E(G) \to \{1, 3, ..., 2q-1\}$ defined as $f^*(e = uv) = |f(u) - f(v)|$ is bijective. A graph which admits an odd graceful labeling is called an odd graceful graph. Many results exist on odd graceful labeling. The concept of odd graceful labeling is implemented in the areas of coding theory.
In this paper we prove that the jewel graph J_n^* and the extended jewel graph $EJ_{n,m}^*$ without the prime edge is odd graceful.

Key words Odd graceful labeling, Jewel graph, Extended Jewel graph.

2020 Mathematics Subject Classification 05C78.

1 Introduction

Graph theory is the study of graphs that are mathematical structures which can be used to model pairwise relation between objects. A graph is made up of vertices which are connected by edges. A graph labeling can be defined as an assignment f of labels to the vertices of G which induces for each edge xy a label depending on the vertex labels $f(x)$ and $f(y)$. Graph labeling is a prominent area of research in graph theory that has rigorous applications in coding theory, communication networks, optimal circuit's layouts and graph decomposition problems.
The first graph labeling method known as the graceful labeling was introduced by Rosa [6]. The graceful labeling of a graph G with q edges is an injection f from the vertices of G to the set $\{0, 1, 2, ..., q\}$ such that when each edge xy is assigned the label $|f(x) - f(y)|$, the resulting edges are distinct. In 1991, Gnanajothi [3] introduced another type of labeling called as the odd graceful labeling. An odd-graceful labeling is an injection f from $V(G)$ to $\{0, 1, 2, ..., (2q-1)\}$ such that, when each edge xy is assigned the

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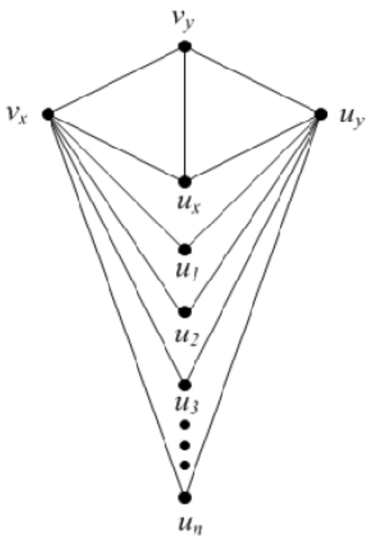

Fig. 1: Jewel graph.

label or weight $|f(x) - f(y)|$, the resulting edge labels are $\{1, 3, 5, ..., (2q - 1)\}$. The splitting graphs of path P_n and even cycle C_n are proved to be odd graceful by Sekar [7], while ladders and graphs obtained from them by subdividing each step exactly once are shown to be odd graceful by Kathiresan [4]. The subdivided shell flower graphs are proved to be odd graceful by Jeba Jesintha and Ezhilarasi Hilda [1] whereas Moussa and Badr [5] proved the odd graceful labeling of crown graphs. For more results on odd graceful labeling we refer to Gallian's survey [2].

2 Preliminary definitions

In this section we give the basic definitions which are relevant for developing the results in this paper.

Definition 2.1. The jewel graph J_n is the graph with the vertex set $V(J_n) = \{v_x, v_y, u_x, u_y, u_i : 1 \leqslant i \leqslant n\}$ and the edge set $E(J_n) = \{v_x u_x, v_x v_y, v_y u_x, u_x u_y, v_y u_y, v_x u_i, u_y u_i : 1 \leq i \leq n\}$. A jewel graph J_n is shown in Fig. 1. The prime edge in a jewel graph is defined to be the edge joining the vertices v_y and u_x.

Definition 2.2. The jewel graph J_n^* without the prime edge is defined as the graph in which the prime edge, that is the edge joining the vertices v_y and u_x is removed. It is shown in Fig. 2.

Definition 2.3. The extended jewel graph $EJ_{n,m}^*$ without the prime edge is the graph with the vertex set $V(EJ_{n,m}^*) = \{v_x, v_y, u_x, u_y, u_i, v_j : 1 \leq i \leq n, 1 \leq j \leq m\}$ and the edge set $E(EJ_{n,m}^*) = \{v_x u_x, v_x v_y, u_x u_y, v_y u_y, v_x u_i, u_y u_i, v_x v_j, u_y v_j : 1 \leq i \leq n, 1 \leq j \leq m\}$. The extended jewel graph $EJ_{n,m}^*$ without the prime edge is shown in Fig. 3.

3 Main results

In this section we deduce two theorems which are the main results developed in this paper.

Theorem 3.1. *The jewel graph J_n^* without the prime edge is odd graceful.*

Proof. Let J_n^* be the jewel graph without the prime edge where n denotes the number of jewels in the graph. Let p and q be the number of vertices and edges respectively. The prime edge which joins

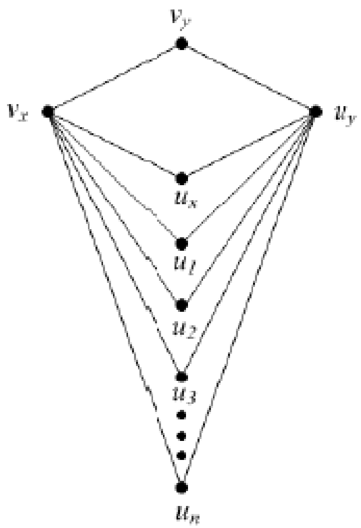

Fig. 2: The jewel graph without prime edge.

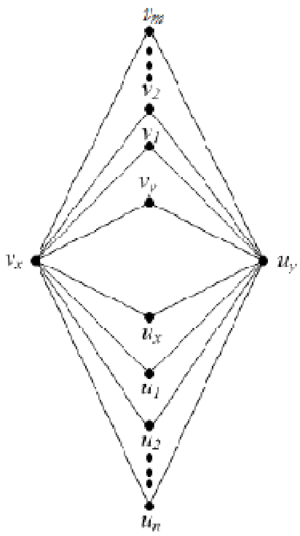

Fig. 3: The extended jewel graph without prime edge.

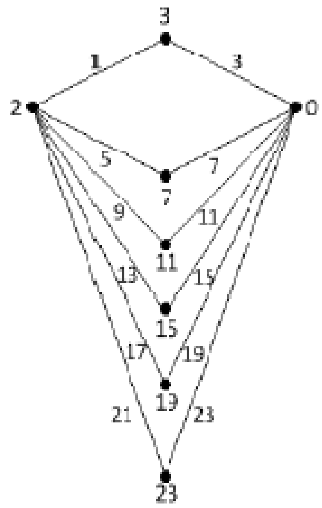

Fig. 4: The odd graceful labeling of the jewel graph J_4^* without prime edge.

the vertices v_y and u_x is removed.
The total number of vertices are given by $p = |V(J_n^*)| = n + 4$. The total number of edges are given by $q = |E(J_n{}^*)| = 2n + 4$.
Consider J_n^* with the vertex set $V(J_n^*) = \{v_x, v_y, u_x, u_y, u_i : 1 \leq i \leq n\}$ and the edge set $E(J_n^*) = \{v_x u_x, v_x v_y, v_y u_x, u_x u_y, v_y u_y, v_x u_i, u_y u_i : 1 \leq i \leq n\}$.
We define the vertex labeling $f : V(G) \to \{0, 1, 2, ..., 2q - 1\}$ as follows:

$$\begin{aligned} f(u_x) &= 7 \\ f(u_y) &= 0 \\ f(v_x) &= 2 \\ f(v_y) &= 3 \\ f(u_i) &= 4i + 7,\ 1 \leq i \leq n \end{aligned} \tag{3.1}$$

We compute the edge labels as follows:

$$\begin{aligned} |f(v_y) - f(v_x)| &= 1 \\ |f(v_y) - f(u_y)| &= 3 \\ |f(u_x) - f(v_x)| &= 5 \\ |f(u_x) - f(u_y)| &= 7 \\ |f(u_i) - f(u_y)| &= 4i + 7,\ 1 \leq i \leq n, \\ |f(u_i) - f(v_x)| &= 4i + 5,\ 1 \leq i \leq n. \end{aligned} \tag{3.2}$$

From the above computed edge labels we observe that the edge labels are distinct odd numbers from the set $\{1, 3, 5, \ldots, (2q - 1)\}$. Hence the jewel graph J_n^* without the prime edge is odd graceful. □

We illustrate the above Theorem 3.1 in Fig. 4.

Theorem 3.2. *The extended jewel graph $EJ_{n,m}^*$ without the prime edge is odd graceful.*

Proof. Let $EJ^*_{n,m}$ be the extended jewel graph without the prime edge. The total number of vertices are given by $|V(EJ^*_{n,m})| = m+n+4$. The total number of edges are given by $|E(EJ^*_{n,m})| = 2(m+n)+4$. Consider $EJ^*_{n,m}$ with the vertex set $V(EJ^*_{n,m}) = \{v_x, v_y, u_x, u_y, u_i, v_j : 1 \leq i \leq n, 1 \leq j \leq m\}$ and the edge set $E(EJ^*_{n,m}) = \{v_xu_x, v_xv_y, u_xu_y, v_yu_y, v_xu_i, u_yu_i, v_xv_j, u_yv_j : 1 \leq i \leq n, 1 \leq j \leq m\}$.
We define the vertex labeling $f : V(G) \rightarrow \{0, 1, 2, \ldots, 2q - 1\}$ as follows:

$$f(u_x) = 7$$

$$f(u_y) = 0$$

$$f(v_x) = 2$$

$$f(v_y) = 3$$

$$f(u_i) = 4i + 7,\ 1 \leq i \leq n$$

$$f(v_j) = 4(n + j) + 7,\ 1 \leq j \leq m \tag{3.3}$$

We compute the edge labels $f : E(G) \rightarrow \{1, 3, \ldots, 2q - 1\}$ as follows:

$$|f(v_y) - f(v_x)| = 1$$

$$|f(v_y) - f(u_y)| = 3$$

$$|f(u_x) - f(v_x)| = 5$$

$$|f(u_x) - f(u_y)| = 7$$

$$|f(u_i) - f(u_y)| = 4i + 7,\ 1 \leq i \leq n$$

$$|f(u_i) - f(v_x)| = 4i + 5,\ 1 \leq i \leq n$$

$$|f(v_j) - f(v_x)| = 4(n + j) + 5,\ 1 \leq j \leq m$$

$$|f(v_j) - f(u_y)| = 4(n + j) + 7,\ 1 \leq j \leq m \tag{3.4}$$

From the above computed edge labels we observe that the edge labels are distinct odd numbers from the set $\{1, 3, 5, \ldots, (2q - 1)\}$. Hence the extended jewel graph $EJ^*_{n,m}$ without the prime edge is odd graceful. □

We illustrate the above Theorem 3.2 in Fig. 5.

4 Conclusion

In this paper we proved that the jewel graph without the prime edge J^*_n and its extension $EJ^*_{n,m}$ satisfies odd graceful labeling.

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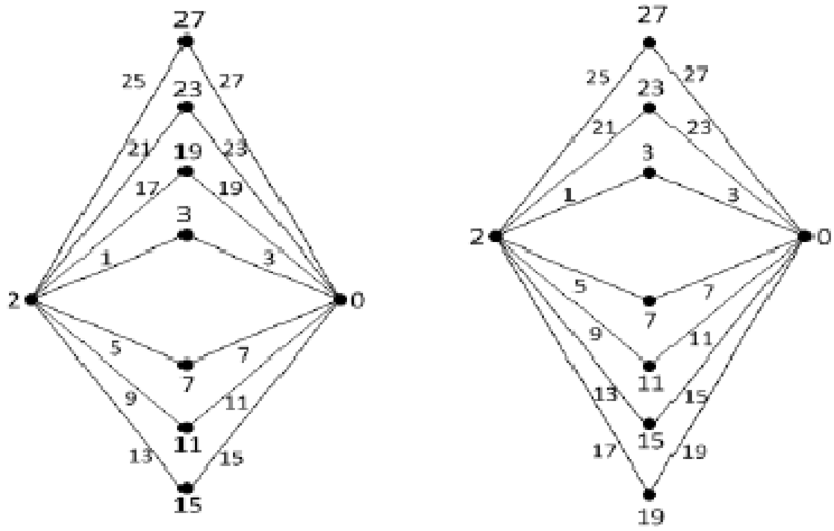

Fig. 5: The odd graceful labeling of $EJ_{2,3}^{*}$ and $EJ_{3,2}^{*}$ without the prime edge.

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