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Cordial labeling on different types of nested triangular graphs *

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Abstract A function $f : V(G) \to \{0,1\}$ is called the binary vertex labeling of a graph G and $f(v)$ are called the labels of the vertex v of G under f. For an edge $e = (u,v)$, the induced function $f : E(G) \to \{0,1\}$ is defined as $f(e) = |f(u) - f(v)|$. Let $v_f(0), v_f(1)$ be the number of vertices of G having labels 0 and 1 respectively under f and $e_f(0), e_f(1)$ be the number of edges of G having labels 0 and 1 respectively under f. A binary vertex labeling f of a graph G is called cordial labeling if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph which admits cordial labeling is called a cordial graph. In this paper we prove the cordial labeling for the Nested Triangle graph, the Shadow graph of the Nested Triangle graph and the double graph of the Nested Triangle graph.

Key words Cordial labeling, Nested Triangle graph, Shadow graph, Double graph.

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1 Introduction

Cahit [2] introduced the concept Cordial Labeling as a weaker version of the graceful and harmonious labeling. Rokad et al. [1] proved that the shadow graph of a star, the splitting graph of a star and the degree splitting graph of star, the Jewel graph and the Jelly fish graph are all cordial. Cahit [2] proved that the complete graph is cordial iff $n \leq 3$, the ladders, the friendship graphs, the paths, the wheels and the pinwheels are cordial. Raj and Koilraj [4] proved that the splitting graph of the Path, the Cycle, the complete bipartite graph and the Wheel graphs are cordial. Madhubala and Rajakumari [5] proved that the Bistar, the shadow of the Bistar and the double graph of the Bistar is divisor cordial. Meena et al. [6] proved that Shell graphs, the Star of a Shell graph, the Multiple Shell graph and the Cycle of the Shell graphs are cordial. In this paper we prove that a Nested Triangular graph NT_n is cordial. Moreover, we also show that the shadow graph of NT_n and the double graph of NT_n also admit cordial labeling. An extensive report on graph labeling can be found in the survey of Gallain [3].

2 The preliminary definitions

We state below the preliminary definitions required to achieve our results in this paper.

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Definition 2.1. An arbitrary vertex labeling f of a graph G is called a cordial labeling [1] if $|e_f(0) - e_f(1)| \leq 1$ and $|v_f(0) - v_f(1)| \leq 1$. A graph G is called a cordial graph if it admits cordial labeling.

Definition 2.2. A Nested Triangle graph [3] is a planar graph with n vertices, for $n = 3i$, where $i = 1, 2, 3, \ldots$ forms a sequence of $n/3$ triangles, when we join the pairs of corresponding vertices on consecutive triangles in the sequence.

Definition 2.3. A Shadow graph $D_2(G)$ [3] of a connected graph G is obtained by taking two copies of G say, G' and G'' and by joining each vertex u' in G' to the neighbors of the corresponding vertex v' in G''.

Definition 2.4. Let G' be a copy of a simple graph G and for each vertex v_i of G let u_i be the vertex of G' corresponding to the vertex v_i. The Double graph [3] is the graph with vertex set $V(G) \cup V(G')$ and the edge set $E(G) \cup E(G') \cup / \{ u_i v_j / u_i \in V(G); v_j \in V(G')$ and $u_i u_j \in E(G)\}$.

3 Main results

In this section the Cordial labeling for certain nested triangle graphs is discussed.

Theorem 3.1. *The Nested Triangle graph NT_n admits cordial labeling.*

Proof. Consider G to be a Nested Triangle graph having the vertex set $\{a_i, b_i, c_i : 1 \leq i \leq n\}$. Then $|V(G)| = 3n$ and $|E(G)| = 6n - 3$. We describe the nested triangle graph as follows:
The inner most triangle has vertices denoted by a_1, b_1, c_1. The next triangle symmetric to the inner most triangle has vertices denoted by a_2, b_2, c_2; and so on. Similarly the outermost triangle has vertices represented by a_n, b_n, c_n. The generalized graph for the theorem is shown in Fig. 1.

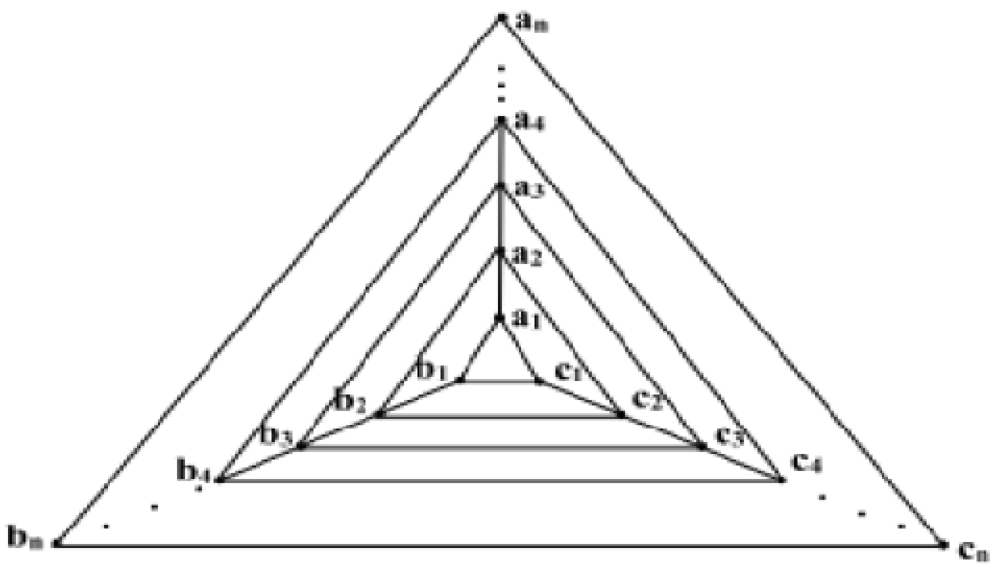

Fig. 1: The cordial labeling for NT_n.

Let us define the vertex labeling $h : V(G) \to \{0, 1\}$ as follows:

$$h(a_i) = 0, \quad \text{for} \quad 1 \leqslant i \leqslant n,$$
$$h(b_i) = 1, \quad \text{for} \quad 1 \leqslant i \leqslant n,$$
$$h(c_i) = \begin{cases} 0, & i \equiv 1(\text{mod}2) \\ 1, & i \equiv 0(\text{mod}2) \end{cases} \quad \text{for} \quad 1 \leqslant i \leqslant n.$$

The following two cases arise for discussion:

Case 1: If n is odd,

$$v_h(0) = \frac{3n+1}{2}, \; v_h(1) = \frac{3n-1}{2}.$$

Case 2: If n is even,

$$v_h(0) = \frac{3n}{2} = v_h(1).$$

The edge labeling is defined as below:
The edges with 0's are defined as

$$h(a_i a_j) = n-1, \text{ for } 1 \leq i \leq n-1, \; 2 \leq j \leq n,$$
$$h(b_i b_j) = n-1, \text{ for } 1 \leq i \leq n,$$
$$h(a_i c_i) = \frac{n+1}{2}, \text{ for } 1 \leq i \leq n,$$
$$h(b_i c_i) = \frac{n-1}{2}, \text{ for } 1 \leq i \leq n.$$

Therefore,

$$e_h(0) = 3n-2.$$

The edges with 1's are defined as below:

$$h(c_i c_j) = n-1, \text{ for } 1 \leq i \leq n-1, \; 2 \leq j \leq n,$$
$$h(a_i b_i) = n, \text{ for } 1 \leq i \leq n,$$
$$h(a_i c_i) = \frac{n-1}{2}, \text{ for } 1 \leq i \leq n,$$
$$h(b_i c_i) = \frac{n+1}{2}, \text{ for } 1 \leq i \leq n.$$

Therefore,

$$e_h(1) = 3n-1.$$

The vertices and edges of the nested triangular graph are labeled according to the above pattern, we obtain that $e_h(0) = 3n-2$ and $e_h(1) = 3n-1$. Thus, $|e_h(0) - e_h(1)| \leq 1$. Therefore, the nested triangle graph NT_n is cordial. □

Theorem 3.2. *The shadow of the nested triangle, $D_2(NT_n)$ admits cordial labeling.*

Proof. Let us consider two copies, say, G' and G'' of the nested triangular graph. Let $\{a_i, b_i, c_i; 1 \leq i \leq n\}$ and $\{a'_i, b_i', c_i'; 1 \leq i \leq n\}$ be the vertex sets of G' and G''. We obtain the shadow of the nested triangle graph denoted as $D_2(NT_n)$ by joining a_n of G' with a'_n of G'', a_n of G' with b'_n of G'', c_n of G' with a'_n of G'', c_n of G' with b'_n of G''. Then $|V(D_2(NT_n))| = 6n$ and $|E(D_2(NT_n)| = 12n-2$. The graph of $D_2(NT_n)$ is as shown in Fig. 2.
The vertex labeling of this graph is defined as follows:

$$h(a_i) = 0, \text{ for } 1 \leq i \leq n,$$
$$h(b_i) = 1, \text{ for } 1 \leq i \leq n,$$
$$h(c_i) = \begin{cases} 0, & i \equiv 1(\text{mod } 2) \\ 1, & i \equiv 0(\text{mod } 2) \end{cases} \text{ for } 1 \leq i \leq n,$$
$$h(a'_i) = 0, \text{ for } 1 \leq i \leq n,$$
$$h(b'_i) = 1, \text{ for } 1 \leq i \leq n.$$

The following two cases arise for our discusion:

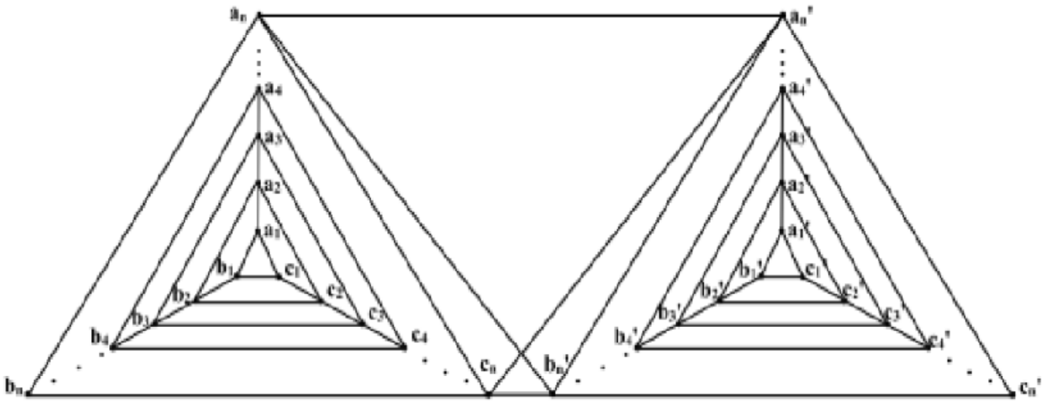

Fig. 2: The cordial labeling for $D_2(NT_n)$.

Case 1: When n is even,

$$h(c_i') = \begin{cases} 0, & i \equiv 1(\text{mod } 2) \\ 1, & i \equiv 0(\text{mod } 2) \end{cases} \quad \text{for } 1 \leq i \leq n.$$

Case 2: When n is odd,

$$h(c_i') = 1,$$

$$h(c_i') = \begin{cases} 0, & i \equiv 1(\text{mod } 2) \\ 1, & i \equiv 0(\text{mod } 2) \end{cases} \quad \text{for } \ 2 \leq i \leq n.$$

Thus,

$$v_h\,(0) = 3n \ = v_h\,(1)\,.$$

From the above labeling pattern, we obtain $e_h\,(0) = 6n - 1 = e_h(1)$. Thus, $|e_h\,(0) - e_h(1)| \leq 1$. Therefore, the shadow of the Nested Triangle graph, $D_2(NT_n)$ is cordial. □

Theorem 3.3. *The double graph of the Nested Triangle, $D(NT_n)$ is a cordial graph.*

Proof. Let G' and G'' be the two copies of the Nested Triangular graph NT_n. Let $\{a_i, b_i, c_i : 1 \leq i \leq n\}$ and $\{a_i', b_i', c_i' : 1 \leq i \leq n\}$ be their corresponding vertex sets. The double graph of the Nested Triangular graph denoted by $D(NT_n)$ is obtained from G' and G'' by joining b_n of G' with b_n' of G'', a_n of G' with a_n' of G'', c_n of G' with c_n' of G''. Then $|V(D(NT_n))| = 6n$, and $|E(D(NT_n))| = 12n - 3$. The graph for the $D(NT_n)$ is shown in Fig. 3.
Thus, the vertex labeling in this case is defined as below:

$$h\,(a_i) = 0, \text{ for } 1 \leq i \leq n,$$

$$h\,(b_i) = 1, \text{ for } 1 \leq i \leq n,$$

$$h\,(c_i) = \begin{cases} 0, \ i \equiv 1(\text{mod } 2) \\ 1, \ i \equiv 0(\text{mod } 2) \end{cases} \quad \text{for } 1 \leq i \leq n,$$

$$h(a_i') = 0, \text{ for } 1 \leq i \leq n,$$

$$h(b_i') = 1, \text{ for } 1 \leq i \leq n,$$

$$h(c_i') = \begin{cases} 0, & i \equiv 1(\text{mod } 2) \\ 1, & i \equiv 0(\text{mod } 2) \end{cases} \quad \text{for } \ 1 \leq i \leq n.$$

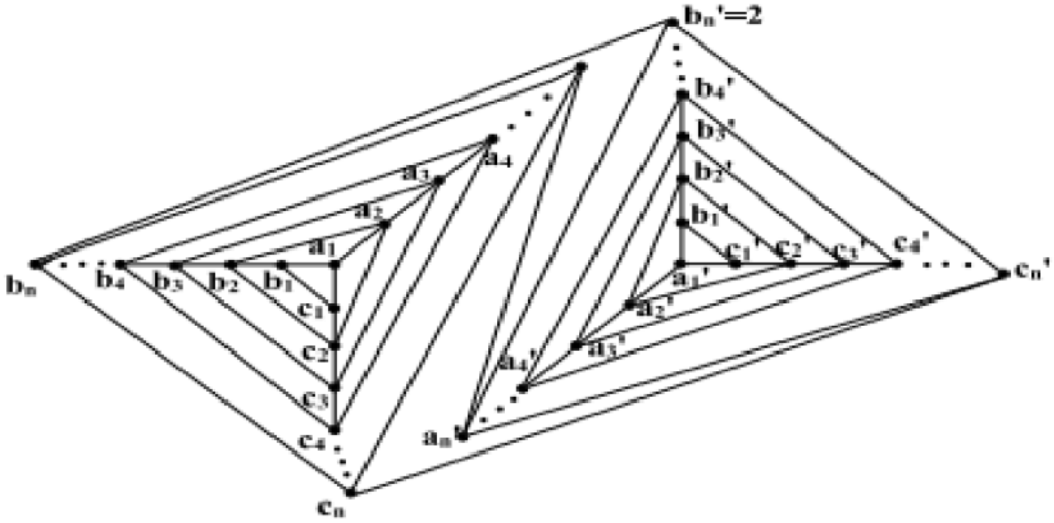

Fig. 3: The cordial labeling for $D(NT_n)$.

Thus,

$$v_h(0) = 3n = v_h(1).$$

In the above labeling pattern, we obtain $e_h(0) = 6n - 1$ and $e_h(1) = 6n - 2$. Thus,$|e_h(0) - e_h(1)| \le 1$. Therefore, the double graph of the Nested Triangle $D(NT_n)$ is cordial. □

4 Conclusion

In this paper the cordial labeling for the Nested Triangle graph NT_n, the shadow of the Nested Triangle graph $D_2(NT_n)$ and the double graph of the Nested Triangle graph $D(NT_n)$ is established.

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