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Odd Graceful Labeling of Super Subdivision of Few Graphs

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ABSTRACT

Odd graceful labeling was originally introduced by Gnanajothi [3] in 1991. The graph Gdemonstrates *Odd graceful labeling* as injecting β from $V(G) \rightarrow \{0,1,2, \ldots (2q-1)\}$ in such a way that whenever the label $|\beta(x)-\beta(y)|$ is allocated to every edgexy, the derived edge labels are $\{1,3,5, \ldots, (2q-1)$. This paper demonstrates that the Super subdivision of the star and super subdivision of path permits odd graceful labeling.

KEYWORDS: Odd graceful labeling, Star graph, Path, super subdivision.

2010 Mathematics Subject Classification: 05C78

1. Introduction

Graph theory, the branch of mathematics dealing with the structure and behavior of graphs of points connected by lines has developed into a significant area of mathematical research. Rosa [7] in 1967 introduced Graph labeling, which involves assigning labels to vertex or edges or both, under some conditions. Gnanajothi [3] introduced odd graceful classification as one of the nuances of graceful labeling in 1991.The graph GdemonstratesOdd graceful labelingas injecting β from $V(G) \rightarrow \{0,1,2, \ldots (2q-1)\}$in such a way that whenever the label $|\beta(x)-\beta(y)|$ is allocated to every edgexy,the derived edge labels are $\{1,3,5, \ldots,(2q-1)\}$. She demonstrated that any graph with an odd cycle is not odd-graceful and she demonstrated that the pathP_n is odd graceful. Eldergill [1] enhanced Gnanajothi's result on stars by implying that the graphs formed by merging one end point from each of any odd number of equal-length paths are odd-graceful. Crown graphs are odd graceful, as demonstrated by Mahmoud I Moussa and El-Sayed Badr [5]. Neela and Selvaraj [6] remedied finite union of stars as odd graceful. The dynamic survey by Gallian [2] contains a wide range of results on odd graceful labeling. Graph Labeling is a key component

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of graph theory and is used in many fields including coding theory, X-ray diffraction, missile guidance codes and radio astronomy problems, circuit design, network addressing, and database administration.

We demonstrated in this paper that the Super subdivision of the star graph and super subdivision of path permits for odd graceful labeling.

2. Main results

Under this section, we state few definitions and few theorems on super subdivision of few graphs on odd graceful labeling.

Definition 2.1:
A Star graph denoted as $k_{1,n}$is a tree with one vertex adjacent to every other vertex as $n$number of pendant vertices are connected to one vertex.

Definition 2.2:
Super subdivision of graph [4]of a path is the graph derived from$G$ by replacing every edge $uv$with a complete bipartite graph$K_{2,m}$.

Definition 2.3:
The super subdivision graphof a stardenoted by $D_m^*(k_{1,n})$is obtained from$k_{1,n}$by replacing every edge uv of $k_{1,n}$witha complete bipartite graph $K_{2,m}$.

Theorem 2.4:
Super subdivision of the star graph is Odd graceful.

Proof:
Let $k_{1,n}$ be the star graph with the central vertex as z and the pendant vertices as $x_1, x_2, x_3, \dots, x_n$ as shown in Figure 1. The supersubdivision of the star graph denoted as $D_m^*(k_{1,n})$ is obtained by replacing each edge zx_i, for $1 \leq i \leq n$ of the star $k_{1,n}$ by a complete bipartite graph$K_{2,m}$ with m fixed. The vertices of the first partite set of $K_{2,m}$ in $D_m^*(k_{1,n})$ are z and x_i respectively and the vertices of the second partite set of $K_{2,m}$ in $D_m^*(k_{1,n})$ are y_i^k for $1 \leq i \leq n$ and $1 \leq k \leq m$. The graph $D_m^*(k_{1,n})$ is given in Figure 2.

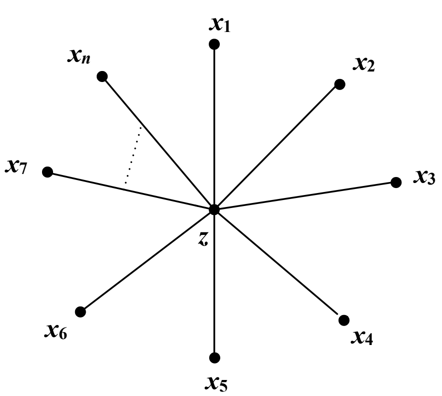

Figure 1: Star graph

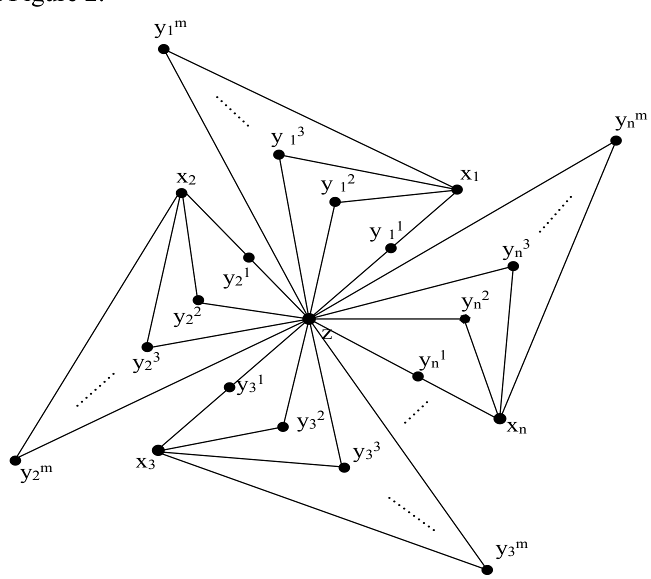

Figure 2: Super subdivision of star graph $D_m^*(k_{1,n})$

The number of vertices in $D_m^*(k_{1,n})$ is$p = |V(D_m^*(k_{1,n}))| = nm + (n+1)$
The number if edges in $D_m^*(k_{1,n})$ is$q = |E(D_m^*(k_{1,n}))| = 2mn$

The vertex labels for the graph $D_m^*(k_{1,n})$is defined as follows:
$f(z) = 0$
$f(x_i) = 2mn + 2n + 2 - 4i\ , \quad 1 \le i \le n$
$f(y_i^k) = 2q - (2i - 1) - 2n - 2n(k - 2)\ , \quad 1 \le i \le n\ ,\ 1 \le k \le m$

The Edge labels for the graph $D_m^*(k_{1,n})$are obtained as follows:

$$|f(y_i^k) - f(z)| = |2q - (2i - 1) - 2n - 2n(k - 2)|,\quad 1 \le i \le n\ ,\ 1 \le k \le m$$

which yields the edges label set$E_1 = \{2q - 1, 2q - 3, \dots , q + 1\}$

$$|f(y_i^k) - f(x_i)| = |2q - 4n - 2n(k - 2) - 2mn + 2i - 1|, 1 \le i \le n,\ 1 \le k \le m$$

which yields the set in$E_2$with edge labels given as $E_2 = \{1, 3, 5, \dots , (q - 1)\}$

Therefore, $E_1 \cup E_2 = \{(2q - 1), (2q - 3), \dots , (q + 1)\} \cup \{1, 3, 5, \dots , (q - 1)\}$
$= \{1, 3, 5, \dots , (2q - 1)\}$

From the above computed edge labels, it is observed that the labels for the edges are distinct and odd. Hence the super subdivision of star graph $D_m^*(k_{1,n})$is odd graceful. The theorem is illustrated in Figure 3.

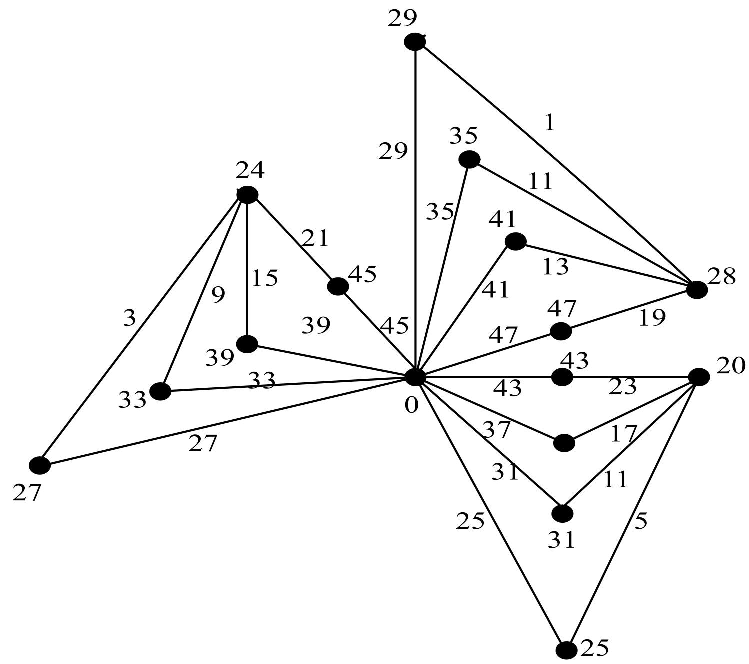

Figure 3: The graph $D_4^*(k_{2,m})$

Theorem 2.5:
Super subdivision of the path is Odd graceful.

Proof:
Let $p_1, p_2, \dots , p_n$ be the vertices of the path shown in figure 4. The super subdivision of the path denoted as $D_m^*(P_n)$ is obtained by replace each edge $p_i p_{i+1}$ for $1 \le i \le n - 1$ of the path p_n by a complete bipartite graph $K_{2,m}$ in $D_m^*(P_n)$ are $p_i$and $p_{i+1}$for $1 \le i \le n - 1$ respectively and the vertices of the second partite set of $K_{2,m}$ in $D_m^*(P_n)$ are $b_i^k$for $1 \le i \le n$ and for $1 \le k \le m$. The graph $D_m^*(P_n)$ is shown in Figure 5.
The number of vertices in $D_m^*(P_n)$ is$p = |V(D_m^*(P_n))| = nm + (n + 1)$
The number of edges in $D_m^*(P_n)$ is$q = |E(D_m^*(P_n))| = 2mn$

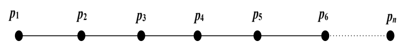

Figure 4: Path

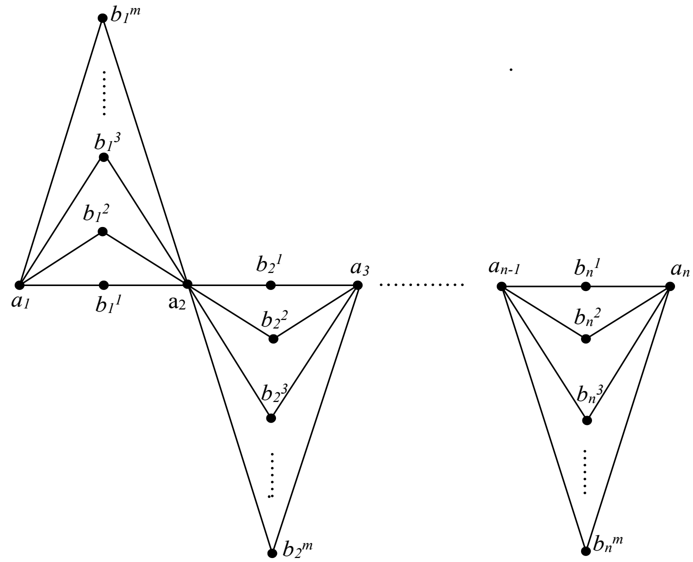

Figure 5: Super subdivision of a path $D_m^*(P_n)$

The vertex labels for the graph $D_m^*(P_n)$ is defined as follows:
$f(a_i) = 2(i-1)$, $\quad 1 \leq i \leq n$
$f(b_j^k) = 2q - 4k - 4mi + 4m + 2i + 1$, $\quad 1 \leq i \leq n$, $1 \leq k \leq m$
The edge labels for the graph $D_m^*(P_n)$are obtained as follows:
$E_1 = |f(b_i^k) - f(a_i)| = |2q - 4k - 4mi + 4m + 3|$,$1 \leq i \leq n$
$E_2 = |f(b_i^k) - f(a_{i+1})| = |2q - 4k - 4mi + 4m + 1|$, $1 \leq i \leq n, 1 \leq k \leq m$
Therefore, $E = E_1 \cup E_2$
From the above computed edge labels, it is observed that the edge labels are distinct and odd. Hence the super subdivision of path graph $D_m^*(P_n)$ is odd graceful. Illustration for the theorem is shown in Figure 6.

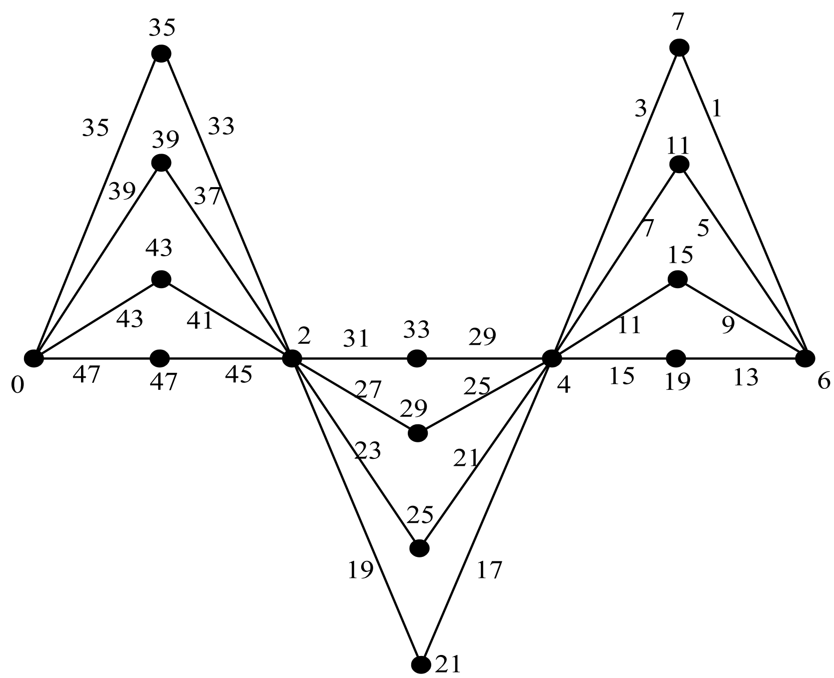

Figure 6: The graph $D_4^*(P_3)$

3. Conclusion

We have shown in this paper that the super subdivision of star and super subdivision of path permits odd graceful labeling.

References

[1] Eldergill P. (1997). *Decomposition of the Complete Graph with an Even Number of Vertices*, M. Sc. Thesis, McMaster University.

[2] Gallian J.A. (2020). *A dynamic survey of Graph labeling*, The Electronic Journal of Combinatorics, (2020), 77-81.

[3] Gnanajothi R.B. (191). *Topics in Graph Theory,* Ph.D. Thesis, Madurai Kamaraj University.

[4] Kaladevi V. and Backiyalakshmi P (2011). *Maximum Distance Matrix of Super Subdivision of Star Graph*, J. Comp. & Math. Sci. 2(6), 828-835.

[5] Mahmoud I Moussa, El-Sayed Badr (2009). *Odd Graceful labelings of Crown Graphs*, 1st Int Conference on Computer Science from Algorithms to Applications (CSAA-2009), Cairo, Egypt, 8-10.

[6] Neela N and Selvaraj C. (2016). *Conjecture on odd graceful graphs*, J. Combin. Math. Combin. Comput., 97, 65-82.

[7] Rosa A (1967). *On certain valuations of the vertices of a graph*, Theory of Graphs (International Symposium, Rome, July 1966), Gordon and Breach, N.Y. and Dunod Paris, (1967), 349-355.

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