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An Alternative Approach to Solve Multi-objective Fuzzy Non-linear Programming Problems

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ABSTRACT
An alternative way of solving multi-objective fuzzy non-linear programming problems (MOFNLPP) is presented. This method reduces the MOFNLPP to crisp without using ranking function. The resulting multi-objective non-linear programming problem (MONLPP) is reduced to a single objective non-linear programming problem using Zimmerman's technique. At last the single objective NLPP is solved by convex separable programming method.

Keywords: Multi-objective fuzzy non-linear programming problem (MOFNLPP), Multi-objective non-linear programming problems (MONLPP), Convex separable programming, Fuzzy numbers.

AMS 2000 Subject Classification: 90C29, 90C30

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1. Introduction

To deal with the real life problems having imprecise data, we take the help of fuzzy-linear programming models. Tanaka et.al [1] first developed the theory of fuzzy programming. Zimmerman [2] provided a solution of fuzzy linear programming Problems (FLPP). Maleki [4] solved FLPP using ranking functions. Zhang et.al. [5] evolved a technique to reduce a FLPP into a Multi- objective linear programming problem having four objective functions. Thakre P.A.et.al. [6] solved similar type of problem using weights. R.B. Dash and P.D.P. Dash [7] solved fuzzy integer programming problem using Thakre's Technique.

In more complex situation we accept non-linear models rather than linear ones. Therefore, not much work has been done in the area of non-linear programming. In 1951 HW Kuhn and A.W. Tucker [8] first presented a paper on non-linear programming. C.A. Hildreth (1957) [9], P.Wolfe (1959)[10], R. Fletcher (1971)[11] work on quadratic programming algorithm.

F.M. Ali [12] first attempted to solve fuzzy non-linear programming problems. Then Nasseri S.H. [13], P.Raj and Ranjana[14], C. Loganathan and M. Laliltha [15] suggested different methods for solving fuzzy non-linear programming problems.

PDP Dash and R.B. Dash [16] used ranking function approach to solve MOFNLPP. Recently many authors, K.P. Gadle and T.S. Pawar [17], D.Rani and T.R. Gulati [18], S.K. Sing and S.P. Yadav [19], Priyadarshini et.al [20] produced papers based on solution of multi-objective non-linear optimization problems in Fuzzy environment.

Recently Sk.S.Ali et. al. [21] solved a multi objective fuzzy linear programming problem without using ranking function.

In this paper, a novel method of solving MOFNLPP is presented. The MOFNLPP is converted to a set of crisp MOFLPPS without taking the help of ranking function. Each member of the set is solved without using weighted combination method. Then utilizing the results a non-linear programming problem is derived using Zimmerman's fuzzy programming technique [3]. At last the resulting NLPP is solved using separable programming technique [22].

2. Preliminaries:

Fuzzy number:

A fuzzy number is a representation of a quantity with arrange of possible values, rather than a single value. Fuzzy numbers have been introduced by Zadeh [23] and Sakawa [24] in order to deal with imprecise numerical quantities in a practical way. We define this as follows.

Definition 1 A number ā whose membership functions in general is defined as follows is called a fuzzy number.

$$\mu_{\tilde{a}}(x) = \begin{cases} \mu_{\tilde{a}}^{L}(x) & a_1 \le x \le a_2 \\ 1 & a_2 \le x \le a_3 \\ \mu_{\tilde{a}}^{R}(x) & a_3 \le x \le a_4 \\ 0 & otherwise \end{cases}$$

$$where \ \ \mu_{\tilde{a}}^{L}: [a_1, a_2] \to [0,1]$$

$$\mu_{\tilde{a}}^{R}: [a_3, a_4] \to [0,1]$$

are strictly monotonic and continuous functions.

Definition 2 (Trapezoidal Fuzzy Number)

A fuzzy number $\tilde{a} = (a_1,\ a_2,\ a_3,\ a_4)$ having following membership function is called Trapezoidal fuzzy number.

$$\mu_{\tilde{a}}(x) = \begin{cases} 0 & for \quad x < a_1 \\ \frac{x-a_1}{a_2-a_1} & for \quad a_1 \le x < a_2 \\ 1 & for \quad a_2 \le x \le a_3 \\ \frac{a_4-x}{a_4-a_3} & for \quad a_3 \le x \le a_4 \\ 0 & for \quad x > a_4 \end{cases}$$

Definition 3 (Triangular Fuzzy Number)

A fuzzy number $\tilde{a} = (a_1,\ a_2,\ \ a_3)$ associated with following membership function is called Triangular fuzzy number.

$$\mu_{\tilde{a}}(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1} & for \quad a_1 \le x \le a_2 \\ 1 & for \quad x = a_2 \\ \dfrac{x-a_1}{a_2-a_1} & for \quad a_2 \le x \le a_3 \\ 0 & Otherwise \end{cases}$$

Definition 4 (Optimal Solution)
X* is called a complete optimal solution for (2.1) of $\exists\ x^* \in X$ such that

$$f_i(x^*) \ge f_i(x),\ i = 1,\ 2,\ \ldots.,\ K,\ \forall\ x \in X$$

Definition 5 (Pareto-optimal Solution)
An optimal solution X* of a multi-objective decision making is said to be Pareto-optimal solution of any change in X* cannot improve optimality of some objectives at the cost of other objectives.

It is a compromise optimal solution for the set of objective functions of Multi-objective optimization problem.

2.1 General Multi-objective Non-linear Programming

The problem to optimize multiple conflicting non-linear objective functions simultaneously under given constraints is called multi-objective non-linear programming problem (MONLPP). The formulation of this problem is to

$$maximize\ \ f(x) = (f_1(x), f_2(x), \ldots f_k(x))^T$$

Subject to

$$x \in X = \{x \in R^n | g_j(x) \le 0,\ j = 1,\ 2,\ \ldots,\ m\} \tag{2.1}$$

where $f_1(x), f_2(x), \ldots, f_k(x)$ are k distinct non-linear objective functions of decision variables and x is the set of constrained decision.

3. Fuzzy Non-linear Programming Problem

We consider the following fuzzy non-linear programming problem (FNLPP) in which the cost of the decision variables as well as the co-efficient matrix of the constraints are fuzzy in nature.

$$f(x) = \langle \tilde{c}, x \rangle = Max\tilde{Z} = \sum_{j=1}^{n} \tilde{C}_j x_j^{\alpha_j} \tag{3.1}$$

subject to the constraints

$$\sum_{j=1}^{n} \tilde{a}_{ij}\ x_j \le \tilde{b}_i, \quad 1 \le i \le m, \quad x_j \ge 0$$

Where $\tilde{C}_j = (\alpha_j,\ \beta_j,\ \gamma_j,\ \ \delta_j)$, $\quad j = 1,\ \ 2, \ldots, n$.
and $\tilde{a}_{ij} = (a_{ij}^1,\ \ a_{ij}^2,\ \ a_{ij}^3)$, $\tilde{b}_{ij} = (b_i^1,\ \ b_i^2,\ \ b_i^3)$

The above problem can be rewritten as

$$f(x) = max \sum_{j=1}^{n} \tilde{C}_j\ x_j^{\alpha j}$$

such that

$$\sum_{j=1}^{n} (a_{ij}^1,\ a_{ij}^2,\ a_{ij}^3)\ x_j \le (b_i^1,\ b_i^2,\ b_i^3), \quad 1 \le i \le m, \quad 1 \le j \le n$$

Using the transformation adopted in the paper Thakre et. al. [6], the problem is further reduced to

$$f(x) = max \sum \tilde{C}_j \ x_j^{\alpha j} = max \sum (\alpha_j, \ \beta_j, \ \gamma_{j,} \delta_j) \ x_j^{\alpha j}$$

Such that

$$\sum_{j=1}^{n} \ a_{ij}^1 \ x_j \leq b_i$$
$$\sum_{j=1}^{n} \left(a_{ij}^1 - a_{ij}^2\right) x_j \leq b_i^1 - b_i^2$$
$$\sum_{j=1}^{n} \left(a_{ij}^1 + a_{ij}^3\right) \ x_j \leq b_i^1 + b_i^3, \quad x_j \geq 0 \tag{3.2}$$

Definition 6 A point $x^* \in X$ is said to be an optimal solution to a FLPP/FNLPP if $\langle \tilde{C}, x^* \rangle \geq \langle \tilde{C}, x \rangle$ for all $x \in X$.

4. Reduction to Multi-objective Non-linear Programming Problem

The fuzzy objective function of (3.2) breads four crisp objective functions such as

$$f_1(x) = \sum \alpha_j x_j^{\alpha j}$$
$$f_2(x) = \sum \beta_j x_j^{\alpha j}$$
$$f_3(x) = \sum \gamma_j x_j^{\alpha j}$$
$$and \quad f_4(x) = \sum \delta_j x_j^{\alpha j}$$

Ultimately the FNLPP (3.2) reduces to a crisp multi-objective linear programming problem as follows:

$$\operatorname*{Max}_{x \in X} \{f_1(x), \ f_2(x), \ f_3(x), \ f_4(x)\} \tag{4.1}$$

subject to the constraints of (3.2),
where $f_i: R^n \to R$.

This crsip multi-objective function could have been solved reducing to weighted single objective function such as

$$Max \ \{w_1 f_1 + w_2 f_2 + w_3 f_3 + w_4 f_4\}$$

subject to the constraints of (3.2).
But in this paper, we solve (4.1) using Zimmerman's fuzzy technique.

5. Solving Multi-objective Fuzzy Non-linear Programming Problem(MOFNLPP)

A fuzzy multi-objective non-linear programming problem is defined as follows.

$$Max \ \widetilde{Z_r} = \sum_j \widetilde{C_{rj}} x_j^{rj} \quad r = 1,2, \ldots q \tag{5.1}$$

such that

$$\sum_j \widetilde{a_{ij}} x_j \leq \widetilde{b_i}, \ j = 1,2, \ldots, m$$

$$x_j \geq 0$$

$$Where \quad \widetilde{a_{ij}} = (a_{ij}^1, a_{ij}^2, a_{ij}^3, a_{ij}^4)$$

$$\widetilde{C_{rj}} = (c_{rj}^1, c_{rj}^2, c_{rj}^3, c_{rj}^4)$$

The method of solution:

Step-1 Reduce for each r, $\tilde{Z}_r$ to a set of four crisp objective functions.
$r = 1,2, \ldots, q$

Step-2 Reduce the constraints of (5.1) to the form as described in section-3.

Step-3 Ultimately we get a crisp multi-objective function having 4q objectives subject to the constraints of the step-2.

Step-4 The crisp MONLPP is reduced to single objective non-linear programming problem by Zimmerman's technique and convex separable programming technique.
Step-5 The resulting NLPP is solved using convex separable programming method.

6 Numerical Example

$$Max: \widetilde{z_1}(x) = \tilde{2}x_1 + \tilde{3}x_2 - \tilde{2}x_1^2$$
$$Max: \widetilde{z_2}(x) = \tilde{3}x_1 + \tilde{4}x_2 - \tilde{5}x_1^2 \qquad (6.1)$$

such that

$$\tilde{1}x_1 + \tilde{4}x_2 \leq \tilde{4}$$

$$\tilde{1}x_1 + \tilde{1}x_2 \leq \tilde{2}$$

$$x_1, x_2 \geq 0$$

where

$$\tilde{2} = (2,2.1,2.2,2.5)$$

$$\tilde{3} = (2.1,2.3,3.3,3.7)$$

$$\tilde{2} = (1.1,1.3,2.3,2.7)$$

$$\tilde{3} = (2.2,2.5,3.3,3.4)$$

$$\tilde{4} = (3.1,3.4,4.2,4.7)$$

$$\tilde{5} = (4.2,4.4,5.2,5.6)$$

$$\tilde{1} = (0.7,0.9,1)$$

$$\tilde{4} = (3.1,4.0,4.3)$$

$$\tilde{1} = (0.6,0.9,1)$$

$$\tilde{1} = (0.5,0.8,1.2)$$

$$\tilde{4} = (3.2,3.4,4)$$

$$\tilde{2} = (1.7,1.9,2.1)$$

Now the problem is converted into a crisp multi-objective non-linear programming problem as follows

$$Max: z_{11} = 3x_1 + 3.3x_2 - 1.8x_1^2$$

$$Max: z_{12} = 3.2x_1 + 3.4x_2 - 1.9x_1^2$$

$$Max: z_{13} = 3.3x_1 + 3.8x_2 - 2.4x_1^2$$

$$Max: z_{14} = 3.7x_1 + 3.9x_2 - 2.7x_1^2$$

$$Max: z_{21} = 3.4x_1 + 3.3x_2 - 4.4x_1^2$$

$$Max: z_{22} = 3.6x_1 + 3.5x_2 - 4.6x_1^2$$

$$Max: z_{23} = 3.9x_1 + 4.3x_2 - 5.3x_1^2$$

$$Max: z_{24} = 4.6x_1 + 4.9x_2 - 5.8x_1^2$$

subject to

$$1.9x_1 + 3.8x_2 \leq 3.8$$

$$0.9x_1 + 0.8x_2 \leq 0.8$$

$$4.1x_1 + 8.6x_2 \leq 8$$

$$1.8x_1 + 1.7x_2 \leq 1.8 \quad (6.2)$$

$$0.8x_1 + 0.7x_2 \leq 0.8$$

$$4x_1 + 4.1x_2 \leq 4.2$$

Solving $Max: z_{11} = 3x_1 + 3.3x_2 - 1.8x_1^2$,subject to (6.2) by using convex separable method,we get
$x_1 = 0.982, x_2 = 0.994, z_{11} = 4.490$

Solving $Max: z_{12} = 3.2x_1 + 3.4x_2 - 1.9x_1^2$,subject to (6.2) by using convex separable method,we get
$x_1 = 0, x_2 = 0.9302, z_{12} = 3.163$

Solving $Max: z_{13} = 3.3x_1 + 3.8x_2 - 2.4x_1^2$,subject to (6.2) by using convex separable method,we get
$x_1 = 0.987, x_2 = 0.9302, z_{13} = 4.454$

Solving $Max: z_{14} = 3.7x_1 + 3.9x_2 - 2.7x_1^2$,subject to (6.2) by using convex separable method,we get
$x_1 = 0.966, x_2 = 0.9301, z_{14} = 4.682$

Solving $Max: z_{21} = 3.4x_1 + 3.3x_2 - 4.4x_1^2$,subject to (6.2) by using convex separable method,we get
$x_1 = 0.0123, x_2 = 0.9302, z_{21} = 3.11$

Solving $Max: z_{22} = 3.6x_1 + 3.5x_2 - 4.6x_1^2$,subject to (6.2) by using convex separable method,we get
$x_1 = 0.0021, x_2 = 0.9302, z_{22} = 3.263$

Solving $Max: z_{23} = 3.9x_1 + 4.3x_2 - 5.3x_1^2$,subject to (6.2) by using convex separable method,we get
$x_1 = 0.999, x_2 = 0.93, z_{23} = 2.605$

Solving $Max: z_{24} = 4.6x_1 + 4.9x_2 - 5.8x_1^2$,subject to (6.2) by using convex separable method,we get
$x_1 = 0.987, x_2 = 0.9433, z_{24} = 1.011$

	1	2	3	4	5	6	7	8
z_{11}	4.490	4.689	4.703	4.906 U_1	2.375 L_1	2.578	2.993	3.794
z_{12}	3.534	3.163 L_2	3.535	3.628	3.069	3.256	3.999 U_2	4.558
z_{13}	4.277	4.47	4.454	4.649 U_3	2.139 L_3	2.328	2.686	3.448
z_{14}	4.288	4.480	4.483	4.682 U_4	2.248 L_4	2.440	2.821	3.588
z_{21}	3.106 L_5	3.201	3.575	3.673	3.11	3.299	4.047	4.614 U_5
z_{22}	3.076 L_6	3.169	3.541	3.636	3.077	3.263	4.008	4.567 U_6
z_{23}	4.269	4.463	4.435	4.628 U_7	2.074 L_7	2.260	2.605	3.363
z_{24}	4.320	4.514	4.503	4.700 U_8	2.182	2.373	2.742	1.011 L_8

Utilising $U_i, L_i, i = 1,2, \ldots,8$ from the above table and adopting Zimmerman's procedure we have,

$$min\lambda:$$

such that

$$3x_1 + 3.3x_2 - 0.1125a_{11} - 0.45a_{12} - 1.0125a_{13} - 1.8a_{14} + 2.531\lambda \geq 4.906$$

$$3.2x_1 + 3.4x_2 - 0.1187a_{11} - 0.475a_{12} - 1.0687a_{13} - 1.9a_{14} + 0.836\lambda \geq 3.999$$

$$3.3x_1 + 3.8x_2 - 0.15a_{11} - 0.6a_{12} - 1.35a_{13} - 2.4a_{14} + 2.51\lambda \geq 4.649$$

$$3.7x_1 + 3.9x_2 - 0.168a_{11} - 0.675a_{12} - 1.5187a_{13} - 2.7a_{14} + 2.434\lambda \geq 4.682$$

$$3.4x_1 + 3.3x_2 - 0.275a_{11} - 1.15a_{12} - 2.475a_{13} - 4.4a_{14} + 1.508\lambda \geq 4.614$$

$$3.6x_1 + 3.5x_2 - 0.2875a_{11} - 1.15a_{12} - 2.5875a_{13} - 4.6a_{14} + 1.491\lambda \geq 4.567$$

$$3.9x_1 + 44.3x_2 - 0.3312a_{11} - 0.1325a_{12} - 2.9812a_{13} - 5.3a_{14} + 2.55\lambda \geq 4.628$$

$$4.6x_1 + 4.9x_2 - 0.3625a_{11} - 1.45a_{12} - 3.2625a_{13} - 5.8a_{14} + 3.689\lambda \geq 4.7$$

$$1.9x_1 + 3.8x_2 \leq 3.8$$

$$0.9x_1 + 0.8x_2 \leq 0.8$$

$$4.1x_1 + 8.6x_2 \leq 8$$

$$1.8x_1 + 1.7x_2 \leq 1.8$$

$$0.8x_1 + 0.7x_2 \leq 0.8$$

$$4x_1 + 4.1x_2 \leq 4.2$$

$$a_{10} + a_{11} + a_{12} + a_{13} + a_{14} = 1$$

$$x_1 + x_2 \geq 0$$

Now solving by two-phase method the optimal solution of the above problem is

$$x_1^* = 0.1076$$

$$x_2^* = 0.8789$$

Now the optimal values of the objective function of MOFNLPP (??) we get

$$z_1^* = \tilde{2}x_1^* + \tilde{3}x_2^* - \tilde{2}x_1^*2$$

$$= (2,2.1,2.2,2.5)(0.1076) + (2.1,2.3,3.3,3.7)(0.8789) - (1.1,1.3,2.3,2.7)(0.1076)^2$$

$$= (0.2152,0.2259,0.2367,0.269) + (1.8456,2.0214,2.9003,3.2519) - (0.0127,0.0150,0.0266,0.0312)$$

$$= (2.0481,2.2323,3.1104,3.4897)$$

$$z_2^* = \tilde{3}x_1^* + \tilde{4}x_2^* - \tilde{5}x_1^*2$$

$$= (2.2,2.5,3.3,3.4)(0.1076) + (3.1,3.4,4.2,4.7)(0.8789) - (4.2,4.4,5.2,5.6)(0.1076)^2$$

$$= (0.2367,0.269,0.3550,0.3658) + (2.7245,2.9882,3.6913,4.1308) - (0.0486,0.0509,0.0602,0.0648)$$

$$= (2.9126,3.2063,3.9861,4.4318)$$

$$\mu_{\widetilde{z_1}}(x) = \begin{cases} 0 & x < a \\ \frac{x-a}{b-a} & a < x \leq b \\ 1 & b < x \leq c \\ \frac{d-x}{d-c} & c < x \leq d \\ 0 & x > d \end{cases}$$

$$\mu_{\widetilde{z_1}}(x) = \begin{cases} 0 & x < 2.0481 \\ \frac{x-2.0481}{0.1842} & 2.0481 < x \leq 2.2323 \\ 1 & 2.2323 < x \leq 3.1104 \\ \frac{3.4897-x}{0.3793} & 3.1104 < x \leq 3.4897 \\ 0 & x > 3.48497 \end{cases}$$

$$\mu_{\widetilde{z_2}}(x) = \begin{cases} 0 & x < 2.9126 \\ \frac{x-2.9126}{0.2937} & 2.9126 < x \leq 3.2063 \\ 1 & 3.2063 < x \leq 3.9861 \\ \frac{4.4318-x}{0.4457} & 3.9861 < x \leq 4.4318 \\ 0 & x > 4.4318 \end{cases}$$

7. Conclusion

An alternative way of solving MOFNLPP is presented in this paper. A numerical verification shows the results are very close to those obtained by other methods using ranking function [16].This method may be adopted for MOFNLPP in intuitionistic fuzzy environment.

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