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Computation of Working Periods of Two Teams by Linear Equations

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ABSTRACT

The mathematical problem tackled here is a simple application of simultaneous linear equations. It has origin in several puzzles asked by my father to school going children. Yesterday, I recalled the problem and framed it in a different style. I circulated it to my fellow colleagues in the mathematical world. There were not many responses as the academicians' world is shrinking. People do not bother to waste their time unless there are any immediate (monetary) gains. It is heartening to note that above three intellectuals gave attention to the problem and came out with their correct answers. In addition, a junior lady colleague of mine (in the past) Prof. Dr. Mrs. Geetika Shrivastava (currently visiting the I.C.T.P., Trieste, Italy) also took pains in solving it. However, her answers were not correct hence, she is not included as a co-author.

Keywords: Real numbers, Linear equations, Mathematical computation.

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1. Simpler case

Problem: Teams A and B work for different periods of time at the same rate of wages. If the team B spares one year's wages to the other team, their income shall be equal. On contrary, if the team A transfers their one year's income to the other team its income becomes 1/4 th of that of the team B. To compute the periods for which the two teams have worked for.

Solution. Suppose the two teams A and B worked for x and y years respectively on the same wages, say Rs. a per year. Thus, they earned the respective wages: Rs. ax and ay.

As per the first hypothesis, if the team B transfers its one-year's income, i.e. a rupees to the team A, their incomes become equal:

$$ax + a = ay - a \quad \Rightarrow \quad x - y = -2. \tag{1.1}$$

On contrary, if team A transfers its one-year's income to the team B, their respective incomes would be: Team A: $a\,(x - 1)$, and Team B: $a\,(y + 1)$;

which, as per the second hypothesis, would satisfy the relation:

$$4a\,(x - 1) = a\,(y + 1) \quad \Rightarrow \quad 4x - y = 5. \tag{1.2}$$

Solving the simultaneous linear equations (1.1) and (1.2) we, thus, have

$$x = 7/3 \text{ years}, \quad \text{and} \quad \text{consequently}, \quad y = x + 2 = 13/3 \text{ years}.$$

Converting these periods to months, we have the desired answer:

Working periods of: Team A = 28 months; and Team B = 52 months.

2. Generalization of above problem

First condition remains the same, i.e. transferring one year's wages from team B to team A makes their income equal. Now, generalizing the second condition, i.e. transfer of one year's income from the team A to another team raises the income of team B by n times to that of the team A. To compute their periods of work. The general answer is: team A works for years $(n + 3) / (n - 1)$, whereas the team B works for years $(3n + 1) / (n - 1)$, where $n = 2, 3, 4, 5, 6, 7, ...$

Solution. Proceeding as before, following the first hypothesis, their incomes satisfy the relation (1.1). On the other hand, following the second (general) hypothesis, the relation (1.2) amends as

$$na\,(x - 1) = a\,(y + 1) \quad \Rightarrow \quad nx - y = n + 1. \tag{2.1}$$

Solving the simultaneous linear equations (1.1) and (2.1) we, thus, have

$$x = (n + 3) / (n - 1), \quad \text{and} \quad y = (3n + 1) / (n - 1), \tag{2.2}$$

where $n = 2, 3, 4, 5, 6, 7, ...$

2.1. Particular cases

n	**Work Period (in years)**		**Remarks**
	x (Team A)	y (Team B)	
2	5	7	
3	3	5	
4	7/3	13/3	Alternate answers: 28 & 52 months
5	2	4	
6	9/5	19/5	Replacing year by week (of 5 working days) – alternate answers are 9 & 19 days
7	5/3	11/3	Alternate answers: 20 & 44 months

8	11/7	25/7	Replacing year by 7-days' week - alternate answers are 11 & 25 days
9	3/2	7/2	Alternate answers: 18 & 42 months
10	13/9	31/9	Counting year of 9 working months (U.S. pattern) – alternate answers are 13 & 31 years

3. More general cases

The problem can be further generalized by transferring more than one-year's income (say for m number of years) of one team to another team. Accordingly, the answers given by Eqns. (2.2) amend to

$$x = m(n+3)/(n-1), \quad \text{and} \quad y = m(3n+1)/(n-1), \tag{3.1}$$

where n = 2, 3, 4, 5, 6, 7, ... For instance, when m = 2, above answers reduce to

$$x = 2(n+3)/(n-1), \quad \text{and} \quad y = 2(3n+1)/(n-1), \tag{3.2}$$

where n = 2, 3, 4, 5, 6, 7, ...

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REFERENCES

1. Misra, R.B.: Basic Mathematics at a Glance. *Lambert Academic Publishers*, *Saarbrücken* (*Germany*), 2010, ISBN 978-3-8433-8696-8.

2. Misra, R.B.: Shri Harḍwārī Lal - a biography". *Harḍwārī Publications*, *Prayāgrāj* (*Indiā*), Both in English and Hinḍī, 2015, pp. xi + 394 + 90 colour photographs.