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Possible regression models for the municipal finances of the municipal corporations of various Indian states *

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Abstract As an extension of the ongoing long term research work of the first author on the detailed mathematical and statistical analysis of the budget of the Bangalore Municipal Corporation (see, A. Sathyavathi https://ssrn.com/abstract=4403863 and A. Sathyavathi https://ssrn.com/abstract=4461494), in the present paper we propose two possible regression models - the first one a polynomial regression model and the other one a nonlinear regression model to explain the observed trends of the revenue receipts and the revenue expenditures of the municipal corporations of the various Indian States including the Union Territory of Chandigarh for the 2017-18 Accounts Category, which pertains to the data of finances of municipal corporations of the various Indian States during the financial year 2017-2018. We also compare these two models of ours. The entire data used by us in this study is gratefully taken by us from the Report on Municipal Finances which was published by the Reserve Bank of India recently on 10 November, 2022 (see, https://m.rbi.org.in/scripts/AnnualPublications.aspx?head=Report%20on%20Municipal%20Finances,https://rbidocs.rbi.org.in/rdocs/Publications/PDFs/RMF101120223A34C4F7023A4A9E99CB7F7FEF6881D0.PDF).

Key words Municipal finances, Urban Local Bodies, municipal corporations, revenue receipts, revenue expenditures, nonlinear regression, Exponential Association 2 model, Akaike Information Criterion, F-test.

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1 Introduction

The three-tier system of governments in India, viz., the Union (Central) Government followed by the respective State Governments and then the local governments at the lowest level of governance, which consist of the Urban Local Bodies (ULBs) comprising of the Municipal Corporations (Nagar Mahapalikas for larger metropolitan cities) or the Municipal Councils (Nagar Palikas for smaller cities) or the Nagar Panchayats (for the smallest towns) and the Gram Panchayats (for rural areas) are well known in the context of the system of governance of the country. The Urban Local Bodies were given the Constitutional Status by the enactment of the Seventy Forth Constitution Amendment Act in the year 1992 [1]. As the growth of urbanization increases in India the urban population will automatically increase which means that the Indian Cities will have to cope up with the ever increasing pressure on the limited resources of life in the urban habitat to cater to the basic necessities of life, like the safe drinking water supply, sanitation, housing, medical facilities, education and employment opportunities, etc. Due to this rapid phenomenon of urbanization of the country, it has become a challenging task to provide the basic amenities of life to the dwellers of towns and cities of the country, where people continuously migrate from the rural areas of the country in search for employment and with the dreams of living a more comfortable life full of all facilities including a good education, advanced medical care facilities and the like. This process has led to the scarcity of necessary funds with the Urban Local Bodies of the country, viz., the municipal corporations and the municipal councils who have the primary responsibility under the relevant provisions of the Constitution of India to provide the minimum basic amenities of life to the urban populace of the country. In particular by the provisions of the Article 243W of the Constitution the Union Government empowered the State Legislatures to devolve by law such authority and powers to the Municipalities so that they may perform and implement schemes entrusted to them with respect to the matters described in the Twelfth Schedule of the Constitution of India. In order to be able to efficiently discharge their duties and functions as are mandated to them under the Constitution Amendment Act [1], the ULBs of India need proper and sufficient financial resources so that they may significantly improve the quality of life of the very large urban population of the country.

It has been well three decades after the historic Constitution Amendment Act [1] was passed by the Union Government, but still the condition of the financial resources of the ULBs of the country is far from satisfactory. We briefly mention just a few recent reports concerning this major burning issue being faced by the country today. In this direction we only mention here the works [2–19] among many others for the sake of the interested reader.

As the issue of financial resources of municipalities of India is one of the prime concerns of the Union Government and much research work is already done in this direction by many scholars as mentioned in [2–19], our present attempt is one among these in that direction. Our present work is primarily *an extension* of the ongoing long term study of the first author (see, A. Sathyavathi [20–22]) about the mathematical and statistical analysis of the budgets and finances of the Bangalore Municipal Corporation (also known as the Bruhat Bengaluru Mahanagara Palike (BBMP)) during the financial years 2015-2016 (Actuals) to 2021-2022 (Budget Estimates), wherein the first author has proclaimed that "we hereby announce to the academic research community that in a series of our forthcoming research papers we propose to make a detailed study of the statistical and mathematical investigations into the Budget of the Bangalore Municipal Corporation during the financial years 2015-2016 (Actuals) to 2021-2022 (Budget Estimates) and we also propose to give a thorough statistical analysis of the various aspects of the Revenue Account (Revenue Receipts and Revenue Expenditure), the Capital Account (Capital Receipts and Capital Expenditure) and the Extraordinary Account (Extraordinary Receipt and Extraordinary Expenditure) of the Bangalore Municipal Corporation during the financial years 2015-2016 (Actuals) to 2021-2022 (Budget Estimates)" and further that "we announce here that in the forthcoming series of a multitude of our research papers on the analysis of the financial and budgetary aspects of not only the BBMP but also possibly of a number of other Municipal Corporations of our country, we will also present a number of our mathematical regression models besides forecasting the trends of the various budgetary aspects of the BBMP and most probably also for the other Municipal Corporations that we intend to study on the basis of the various regression models that we would try to formulate for these Municipal Corporations" (see, Sathyavathi [20]). In the present study we propose to present two regression models which include a polynomial regression model and a nonlinear regression

model which mathematically describe the trends of the revenue receipts and the revenue expenditures of the municipal corporations of the various states of India including the Union Territory of Chandigarh for the 2017-18 Accounts Category which corresponds to the data of the Revenue Receipts and the Revenue Expenditures of the municipal corporations of these States during the financial year 2017-18. We mention here that we have made use of the secondary data for the purpose of this study which we have obtained from the most recently published "Report on Municipal Finances" by the Reserve Bank of India (see [6]), which is the first ever Report of its kind published by the Reserve Bank of India. This report tries to throw light on the recent trends of the various aspects of the current state of municipal finances in India. Most sincerely and humbly we take this opportunity to thank the Reserve Bank of India and the Authorities of this organization which is one of the most prestigious institutions of our country and we also most gratefully acknowledge the Reserve Bank of India for the Report [6] which is the original source of the entire data that we present in this paper in Table 1 and Table 2 in the Section 2 below and we use a portion of the same in this paper of ours for proposing our regression models for the 2017-18 Accounts Category of the same. We also compare our fitted polynomial model with our fitted nonlinear model for the 2017-18 Accounts Category of the budget and further we present our representative tables of forecasts for the revenue expenditures (our response variable) for given values of the revenue receipts (our predictor variable) based on our fitted models of regression.
Since the method of regression analysis is a very powerful research tool for the determination of a mathematical relation as accurate as possible, within the limitations of the techniques used, between the predictor variables and the response variable, in order to determine the behavior of the various measurable quantities involved in any experiment, we naturally employ this technique of regression analysis in finding the best possible models which can quantify the relation of the revenue expenditures of the municipal corporations as functions of their revenue receipts during a given financial year. As regression analysis is a very wide ranging topic that is very extensively used in almost all disciplines of study like the engineering, physical, chemical, biological, medical, technological, computer, actuarial, social and other sciences including econometrics and financial mathematics and many more wide and diverse topics than those we can mention here, and since there is no dearth of research material available on the introductory topics as well as the advanced topics on regression analysis and their applications to various diverse fields of human study, it is only for the sake of completeness and appropriateness to the context of our present study we briefly mention only a few representative references in this field [23–36] with our presumption that the interested readers would easily be able to look for more research monographs and research papers in this field.

2 Description of the data

In this section we give the data of the revenue receipts in Table 1 and the revenue expenditures in Table 2 of the municipal corporations of the various States and Union Territories of India which we have taken respectively from the Appendix I and the Appendix II of the Reserve Bank of India's Report on Municipal Finances [6]. At present we only use the data of the third columns of these two tables of this section for our entire regression analysis in the succeeding sections of this paper, while we would communicate our related papers on this topic for the remaining portions of the data of the fourth, fifth and sixth columns respectively of these two tables (viz., Table 1 and Table 2 of this section) in near future.

Table 1: Revenue Receipts of Municipal Corporations (in Rs. Lakhs).
Source: Report on Municipal Finances, The Reserve Bank of India, dated 10 November, 2022 [6, Appendix-I].

S. No.	State/ U.T.	2017-18 Acc.	2018-19 B.E.	2018-19 R.E.	2019-20 B.E.
1.	Andhra Pradesh	207,678.1	310,267.4	330,663.3	367,270.3
2.	Assam	13,229.5	22,717.2	22,882.4	25,628.3
3.	Bihar	40,805.9	76,853.6	68,109.6	128,265.8

Continued on the next page

Table 1 – Continued from the previous page

S. No.	State / U.T.	2017-18 Acc.	2018-19 B.E.	2018-19 R.E.	2019-20 B.E.
4.	Chhattisgarh	155,878.5	140,923.1	117,838.6	174,391.7
5.	Goa	4,882.8	4,769.9	4,101.8	6,146.7
6.	Gujarat	852,938.9	1,057,062.3	987,043.9	1,167,004.8
7.	Haryana	200,250.4	290,213.6	290,147.8	357,421.3
8.	Himachal Pradesh	10,157.4	19,718.8	9,470.5	13,950.7
9.	Jammu and Kashmir	30,646.2	38,844.5	38,844.5	42,008.8
10.	Jharkhand	13,885.1	51,493.4	50,049.3	47,752.4
11.	Karnataka	107,143.1	165,249.9	140,777.7	167,416.6
12.	Kerala	135,129.2	220,371.3	161,664.9	210,570.2
13.	Madhya Pradesh	286,775.2	309,051.2	309,051.2	340,266.3
14.	Maharashtra	4,304,296.1	5,245,861.0	4,686,343.2	5,786,325.1
15.	Mizoram	5,336.8	2,527.1	2,527.1	3,069.0
16.	Odisha	47,514.2	49,598.9	45,050.0	57,195.9
17.	Punjab	169,396.6	235,775.4	214,886.4	251,363.3
18.	Rajasthan	142,097.5	266,363.4	216,671.7	275,667.8
19.	Sikkim	1,574.2	1,750.8	1,953.5	2,289.9
20.	Tamil Nadu	403,992.0	596,614.8	578,100.9	679,189.5
21.	Telangana	306,337.3	424,327.3	382,657.4	420,324.2
22.	Tripura	10,057.4	21,775.7	21,775.7	20,706.5
23.	Uttar Pradesh	588,159.5	952,227.2	892,135.4	818,754.9
24.	Uttarakhand	26,754.2	29,419.2	30,794.5	37,295.1
25.	West Bengal	399,624.2	538,361.8	501,110.8	543,568.4
26.	Chandigarh	18,816.6	17,093.0	21,270.0	27,686.0
27.	Delhi	1,405,074.2	1,981,183.3	1,959,108.6	2,180,172.5

Abbreviations used in the table above: U.T. = Union Territory, Acc. = Accounts, B.E. = Budget Estimates, R.E. = Revised Estimates.

Table 2: Revenue Expenditures of Municipal Corporations (in Rs. Lakhs). *Source: Report on Municipal Finances, The Reserve Bank of India, dated 10 November, 2022 [6, Appendix-II].*

S. No.	State/ U.T.	2017-18 Acc.	2018-19 B.E.	2018-19 R.E.	2019-20 B.E.
1.	Andhra Pradesh	169,327.7	264,788.2	269,252.3	295,408.5
2.	Assam	10,022.2	34,588.7	36,877.5	41,302.8
3.	Bihar	57,889.3	144,940.1	114,048.3	178,718.0
4.	Chhattisgarh	106,101.9	174,513.5	131,923.2	199,207.2
5.	Goa	2,902.5	4,958.5	4,259.0	5,626.8
6.	Gujarat	758,442.0	916,796.7	889,215.1	993,408.0
7.	Haryana	213,389.7	388,350.7	386,773.7	470,735.2
8.	Himachal Pradesh	13,380.0	22,629.7	11,223.5	12,438.7

Continued on the next page

Table 2 – Continued from the previous page

S. No.	State / U.T.	2017-18 Acc.	2018-19 B.E.	2018-19 R.E.	2019-20 B.E.
9.	Jammu and Kashmir	12,686.9	14,738.2	14,738.2	12,929.2
10.	Jharkhand	11,501.6	9,198.0	8,801.5	10,392.4
11.	Karnataka	96,404.9	143,549.8	121,141.4	141,929.1
12.	Kerala	215,116.7	179,834.6	136,188.7	198,039.8
13.	Madhya Pradesh	191,823.4	571,316.9	571,316.9	582,536.1
14.	Maharashtra	2,774,922.2	3,649,951.8	3,182,861.2	3,931,060.7
15.	Mizoram	2,632.5	2,991.0	2,597.5	3,399.2
16.	Odisha	56,410.3	66,004.4	68,081.5	77,283.1
17.	Punjab	134,698.8	160,259.8	150,027.0	177,364.1
18.	Rajasthan	109,059.2	186,264.1	185,050.5	220,279.6
19.	Sikkim	522.5	532.2	693.8	689.5
20.	Tamil Nadu	643,129.0	660,257.1	695,438.7	767,200.1
21.	Telangana	219,540.8	295,967.7	279,620.9	315,543.8
22.	Tripura	9,026.8	9,394.8	9,394.8	7,851.7
23.	Uttar Pradesh	349,387.9	681,307.5	658,777.1	539,027.8
24.	Uttarakhand	26,844.6	41,927.4	37,107.0	35,491.6
25.	West Bengal	320,126.5	515,960.9	492,488.8	506,576.3
26.	Chandigarh	53,145.1	65,351.0	64,734.0	79,441.0
27.	Delhi	1,269,803.9	1,988,092.3	1,992,980.9	2,146,466.1

Abbreviations used in the table above: U.T. = Union Territory, Acc. = Accounts, B.E. = Budget Estimates, R.E. = Revised Estimates.

3 Regression analysis of the municipal finances for the category 2017-18 Accounts

In this section we fit a polynomial regression model for the revenue receipts and the revenue expenditures of the municipal corporations of the twenty seven States and Union Territories of India for the category 2017-18 Accounts, the financial data pertaining to which is displayed in the third columns of Table 1 and Table 2. We present our results of investigations as below:

3.1 Polynomial regression model for the 2017-18 Accounts category

By scaling the data of both the variables, viz., the revenue receipts (our predictor variable) and the revenue expenditures (our response variable) by a factor of 10^{-6} we find that the data given in the third columns of Table 1 and Table 2 corresponding to the revenue receipts and revenue expenditures of the municipal corporations of the Indian States can be modeled very accurately by a *eighth degree polynomial* given below in (3.1) with the predictor variable x representing the revenue receipts and the response variable y representing the revenue expenditures for this category:

$$y = a + bx + cx^2 + dx^3 + \ldots + ix^8, \tag{3.1}$$

in which model the coefficients $a, b, \ldots, i$ (regression parameters) are given by

$$a = 18156479779494\text{E} - 021;\ b = -5.55183539764263\text{E} - 01;\ c = 4.86112035953556\text{E} + 01;$$
$$d = -4.77003520261670\text{E} + 02;\ e = 1.95442223030907\text{E} + 03;\ f = -3.80177002039624\text{E} + 03;$$
$$g = 3.57057224338027\text{E} + 03;\ h = -1.47723014562316\text{E} + 03;\ i = 1.92771126545275\text{E} + 02$$

where the number $a = 18156479779494\text{E} - 021$ means the number $a = 18156479779494 \times 10^{-21}$, etc. The standard errors of the values of the regression parameters and their 95% confidence intervals are displayed as below:

Parameter	Value	Std Err	Range (95% Confidence)
a	0.011816	0.030039	− 0.051294 to 0.074925
b	− 0.555184	1.737451	− 4.205433 to 3.095066
c	48.611204	21.079339	4.325157 to 92.897251
d	− 477.003520	107.728038	− 703.331730 to − 250.675310
e	1954.422230	298.263832	1327.793173 to 2581.051288
f	− 3801.770020	476.345927	− 4802.535677 to − 2801.004364
g	3570.572243	411.245813	2706.576851 to 4434.567636
h	− 1477.230146	164.726566	− 1823.307818 to − 1131.152474
i	192.771127	21.272102	148.080098 to 237.462155

The standard deviations of the values of the parameters of regression in the Polynomial Degree 8 model of (3.1) and their uncertainties at 95% confidence level are as displayed underneath:

Parameter Standard Deviations	Parameter Uncertainties at 95% Confidence Level
a_stddev = 3.00390110291200E − 02	a_unc = 6.31096203381194E − 02
b_stddev = 1.73745134218857E + 00	b_unc = 3.65024981865021E + 00
c_stddev = 2.10793385297138E + 01	c_unc = 4.42860469107763E + 01
d_stddev = 1.07728038291156E + 02	d_unc = 2.26328209997812E + 02
e_stddev = 2.98263831654109E + 02	e_unc = 6.26629057728837E + 02
f_stddev = 4.76345926829494E + 02	f_unc = 1.00076565645509E + 03
g_stddev = 4.11245812978597E + 02	g_unc = 8.63995392443547E + 02
h_stddev = 1.64726565503897E + 02	h_unc = 3.46077672080334E + 02
i_stddev = 2.12721021158189E + 01	i_unc = 4.46910281773802E + 01

In the box display below we present the other relevant information about the polynomial regression model of (3.1):

Name	Polynomial Regression (degree = 8)
Kind	Regression
Family	Linear Regressions
Equation	$y = a + b * x + c * x^2 + \ldots$
# of Indep. Vars	1
Standard Error	0.060311
Correlation Coeff. (r)	0.996152
Coeff. of Determination(r^2)	0.992318
DOF	18
AICc	− 138.592821.

The plot of the above polynomial fit (3.1) to the data set is shown below in Fig. 1 and its Residual Plot is shown in Fig. 2 for the P-Value of 72.42% at 95% confidence level (it being understood that we have scaled the values of both the variables by a factor of 10^{-6} for convenience as already mentioned earlier). It may be remarked here that the Residuals Plot of Fig. 2 graphically plots the difference between the values of the revenue expenditures as predicted by the Polynomial Degree 8 Model as a function of the revenue receipts and their observed values against the values of the revenue receipts. A light red colored line is also superimposed on the Residual Plot, which is a straight-line fit to the residual points. In Fig. 2 this light red colored line merges with the black colored zero-line, which corresponds to the most ideal situation (theoretically speaking) when the fitted model exactly fits the given data, in which case all the residuals would be exactly be equal to zero each! From the Residual Plot of the Polynomial Degree 8 Fit it can be seen that the observed number of runs is sixteen and the P-Value of the Wald-Wolfowitz runs test performed on the residuals is 0.7242 which being greater than the critical value of 0.05, the pattern is not unlikely, which is indicative of the fact that the Polynomial Degree 8 Model used to fit the data is correct, since we can also check this fact visually by observing that in Fig. 2 the residuals are randomly distributed around the curve.

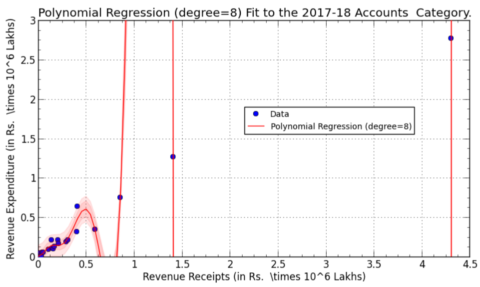

Fig. 1: Polynomial Degree 8 Fit to the 2017-18 Accounts Category with Values of the Variables in Rs. $\times 10^6$ Lakhs.

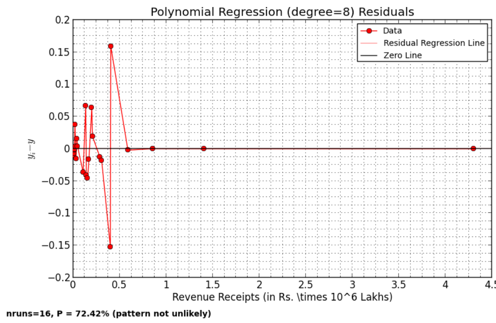

Fig. 2: Residual Plot of Polynomial Degree 8 Fit to the 2017-18 Accounts Category with P-Value = 0.7242 at 95% Confidence Level.

In Table 3 we present a sample prediction table for predicting (forecasting) ten values of the response variable revenue expenditure (the y variable in (3.1)) for given values of the predictor variable revenue receipt (the x variable in (3.1)). Since it is well known that the family of polynomials oscillates wildly at many points within and outside the range of the data set, therefore, the presence of negative response values in Table 3 should not come as a surprise to us because this is a limitation of the polynomial regression model owing to the intrinsic algebraic oscillating behavior of the polynomials.
In order to justify that among a number of possible polynomial fits to the financial data of the 2017-18 Accounts category, our polynomial model of (3.1) gives a best fit to this data, we present as a representative result, the predicted value of the revenue expenditures for the minimum and maximum values of the data set in Table 4. Owing to the inherent oscillating character of the polynomials we get a negative predicted value at the lower end (minimum) of the data set and the agreement between the predicted and observed values at the upper end (maximum) of the data set is excellent. Fig. 3 shows the graphical representation of the evaluation for the maximum value of the predictor by the model of (3.1), whereas we can also observe that the slope of the curve of (3.1) at $x = 2.15293515$ is $\frac{dy}{dx} = -44988.1292752$ which is shown in Fig. 4. The area under the entire curve of (3.1) ranging from the minimum value of the data set to the maximum value, i.e., from $x = 0.0015742$ to $x = 4.3042961$ is -422960.638792 (Rs. $\times\, 10^6$Lakhs)^2, meaning thereby that $\int_{0.0015742}^{4.3042961} f(x)\, dx = -422960.638792$(Rs. $\times\, 10^6$Lakhs)^2. The negative sign of the area under the curve is not surprising since the curve of Fig. 5 shows a distinct minimum near $x = 3.875$ approximately. Further we remark that the entire arc length of the curve from $x = 0.0015742$ (minimum value) to $x = 4.3042961$ (maximum value) is 802747.842246 (in Rs. $\times 10^6$ Lakhs) as shown in Fig. 6. The polynomial model of (3.1) has a 9×9 covariance matrix of parameters which we give in Table 5 for the interested researches who may utilize the same for further researches on this model fit.

Table 3: A representative prediction table for the Polynomial Degree 8 Fit of (3.1) to the 2017-18 Accounts Category.

S. No.	Revenue Receipt x (in Rs. $\times 10^6$Lakhs)	Revenue Expenditure y (in Rs. $\times 10^6$Lakhs)
1.	0.0000000000000000E+00	1.1815647977949373E-02
2.	5.0000000000000000E-01	6.1000400436322399E-01
3.	1.0000000000000000E+00	9.8297496571031182E+00
4.	1.5000000000000000E+00	-1.0506053544371244E+02
5.	2.0000000000000000E+00	-5.2279935347153114E+03
6.	2.5000000000000000E+00	-3.7835314024895328E+04
7.	3.0000000000000000E+00	-1.4094896404101924E+05
8.	3.5000000000000000E+00	-3.2310249568424886E+05
9.	4.0000000000000000E+00	-3.6685931261750567E+05
10.	4.5000000000000000E+00	6.0793891113703209E+05

Table 4: Prediction table for the Polynomial Degree 8 Fit of (3.1) to the minimum and the maximum values of the 2017-18 Accounts Category.

	Revenue Receipt x (in Rs. $\times 10^6$Lakhs)	Observed Revenue Expenditure y_{obs} (in Rs. $\times 10^6$Lakhs)	Predicted Revenue Expenditure y_{pred} (in Rs. $\times 10^6$Lakhs) by (3.1)	Difference $= y_{\text{obs}} - y_{\text{pred}}$ (in Rs. $\times 10^6$Lakhs)
Minimum	0.0015742	0.0005225	0.011060292906	-0.010537792906
Maximum	4.3042961	2.7749222	2.77492220028	-0.00000000028

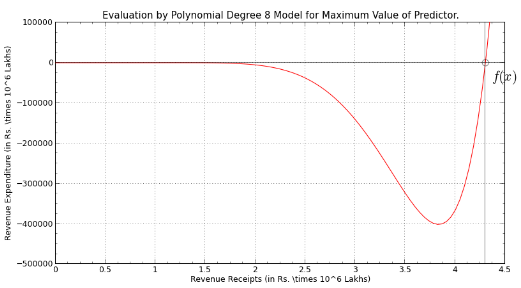

Fig. 3: Evaluation by Polynomial Degree 8 Model for the Maximum Value of the Predictor.

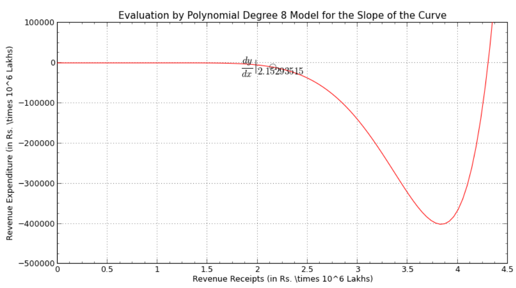

Fig. 4: Evaluation by Polynomial Degree 8 Model for the Slope of the Curve.

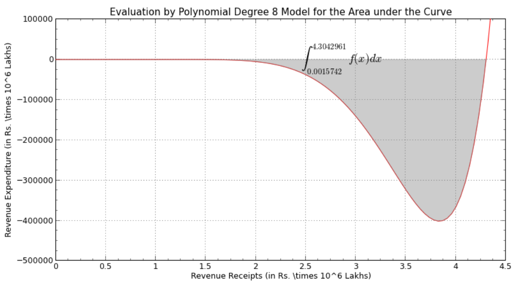

Fig. 5: Evaluation by Polynomial Degree 8 Model for the Area under the Curve.

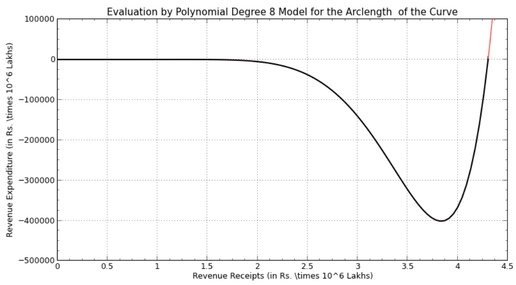

Fig. 6: Evaluation by Polynomial Degree 8 Model for the Arclength of the Curve.

Table 5: (9×9) Covariance Matrix of the Polynomial Degree 8 Fit given by (3.1) to the Financial Data of 2017-18 Accounts Category.

Parameters	Entries of Covariance Matrix	Description
a	2.4807466518135982E-01 −1.1431421426653623E+01 1.1999574736418928E+02 −5.0707363340462371E+02 1.0264668862306924E+03 −1.0447327030562683E+03 5.1863676263410719E+02 −1.1231351662990645E+02 8.5348773866943350E+00	First row of the (9×9) covariance matrix
b	−1.1433914335349757E+01 8.2992042868845283E+02 −9.6660454511146436E+03 4.3408833755640655E+04 −9.3077340616134199E+04 1.0199092764258938E+05 −5.6651059490854444E+04 1.4685682796277470E+04 −1.3897835346364989E+03	Second row of the (9×9) covariance matrix
c	1.2011366070370298E+02 −9.6734206068018157E+03 1.2215889914601677E+05 −5.9541884567916382E+05 1.4064255256027298E+06 −1.7439401798998883E+06 1.1364887057060024E+06 −3.5649490634598676E+05 3.9636194543468504E+04	Third row of the (9×9) covariance matrix

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Table 5 – Continued from the previous page

Parameters	Entries of Covariance Matrix	Description
d	−5.0831161681929160E+02 4.3502245878908616E+04 −5.9617469617775700E+05 3.1905727831801865E+06 −8.3752390874473164E+06 1.1662023080362689E+07 −8.5653656725259293E+06 2.9949985058068498E+06 −3.5740507950989000E+05	Fourth row of the (9 × 9) covariance matrix
e	1.0316093602816934E+03 −9.3491665726452702E+04 1.4108091194657243E+06 −8.3882982266397420E+06 2.4457515569158647E+07 −3.7551839582251824E+07 2.9995557998712886E+07 −1.1181801971524103E+07 1.3839037231377596E+06	Fifth row of the (9 × 9) covariance matrix
f	−1.0547240222288442E+03 1.0282156094252951E+05 −1.7536698976755533E+06 1.1700129382323826E+07 −3.7604734743978120E+07 6.2381536158134915E+07 −5.2793261912401997E+07 2.0471764365398172E+07 −2.5869362236106163E+06	Sixth row of the (9 × 9) covariance matrix
g	5.2797894110464756E+02 −5.7441062062438068E+04 1.1462168704941010E+06 −8.6076164280152675E+06 3.0072515994836025E+07 −5.2846689949490480E+07 4.6495843601496466E+07 −1.8479932761400823E+07 2.3644662228948469E+06	Seventh row of the (9 × 9) covariance matrix
h	−1.1616340273123440E+02 1.5014335507513653E+04 −3.6065035945103469E+05 3.0139483444761094E+06 −1.1219907103760907E+07 2.0506199327606391E+07 −1.8491543715237726E+07 7.4599933518041233E+06 −9.6150425418923213E+05	Eighth row of the (9×9) covariance matrix

Continued on the next page

Table 5 – Continued from the previous page

Parameters	Entries of Covariance Matrix	Description
i	9.0361809594139864E+00 −1.4327684594636812E+03 4.0186283002758493E+04 −3.5996703793291136E+05 1.3892578343045970E+06 −2.5921823721353142E+06 2.3666908768436913E+06 −9.6179797799391276E+05 1.2440332022296246E+05	Ninth row of the (9×9) covariance matrix

3.2 Nonlinear regression model for the category 2017-18 Accounts

We now move ahead to describe a nonlinear regression model to fit the data of the category 2017-18 Accounts, which according to our investigations gives the best fit to this data set. Our explorations have led us to find that the *Exponential Association 2 Model* which belongs to the general class of *Growth Models* best describes the behavior of the predictor (revenue receipts) and the response (revenue expenditures) variables of the 2017-18 Accounts category. The Exponential Association 2 Model which we fit to the 2017-18 Accounts data set (of the finances of the municipal corporations of the various States of India as described above) is defined by

$$y = a(1 - e^{-bx}), \tag{3.2}$$

in which a and b are the *two* regression parameters. For the financial data concerning the category 2017-18 Accounts, the model of (3.2) when fitted to this data yields the following values of the parameters a and b:

$$a = 4.73426194146244E + 00, b = 2.05940539417275E - 01.$$

The other details of the model of (3.2) are tabulated in Table 6 below while the 95% Confidence intervals are given in Table 7. In Fig. 7 we give the plot of the Exponential Association 2 Model fit to the data, while Fig. 8 represents the residual plot for this model with a P-Value of 0.7472 which is the outcome of a Wald-Wolfowitz runs test performed on the residuals reporting fifteen number of runs. Further it can be seen from Fig. 8 that the residuals are randomly scattered around the Residual Regression Line (which in this case again coincides with the Zero Line) showing that the Exponential Association 2 is a good fit to the data. It may be noted that in Fig. 7 the Confidence Band and the Prediction Band at 95% confidence level are also clearly visible, with the narrower Confidence Band in darker red color and the wider Prediction Band in lighter red color. The Confidence Band represents the region that has a 95% chance that the true curve fitting the data in question will lie in this region, while the Prediction Band denotes the region in which there is a 95% probability that the values predicted by the fitted model will lie.

We present the graphs of the Residual History and the Parameter Histories for this nonlinear (Exponential Association 2) Model in Fig. 9 and Fig. 10 respectively. The Residual History (or Convergence History) of the fitted nonlinear model is an important indication and test of the fact whether the process of iteration (computation) has converged, which in turn indicates that the proposed model is a good and reliable fit to the data under consideration and also of the fact that the results predicted by the model fitted to the data are reliable. In the plot of the Residual History (Fig. 9) two graphs are shown: one (the red colored line) is the norm of the residual and the other (the blue colored line) is the change in the residual. Both these quantities are plotted against the iteration number. In our machine computations we have set the tolerance limit of 10^{-8} for both the quantities, viz., the change in the residual as well as the change in the parameters and we have set the maximum number of iterations as 100. We can see from Fig. 9 that the norm of the residual (the red colored line) has attained constancy just after the second iteration and the change in residual has fallen below the preset tolerance limit of 10^{-8} after the fifth iteration, which show that our process of computation for this model has converged.

Table 6: Description of the Nonlinear Fit of (3.2) to the 2017-18 Accounts Category.

Name: Exponential Association 2
Kind: Regression
Family: Growth Models
Equation: $a * (1 - \exp(-b * x))$
No. of Independent variables: 1
Parameters:
a = 4.73426194146244E+00
b = 2.05940539417275E-01
Standard Error: 7.51989808349557E-02
Coefficient of Determination (r^2): 9.83413120933933E-01
Correlation Coefficient (r): 9.91671881689671E-01
DOF 25
AICc -139.649299
Covariance matrix:
6.5133914286833289E+01 −3.9265401718344446E+00
−3.9265401718344450E+00 2.4461026340308270E-01
Parameter Standard Deviations:
a_stddev = 6.06897772966822E-01
b_stddev = 3.71919793739895E-02
Parameter Uncertainties, 95%:
a_unc = 1.24992936100529E+00
b_unc = 7.65983153739367E-02

Table 7: 95% Confidence Intervals for the Parameters of the Nonlinear Fit of (3.2) to the 2017-18 Accounts Category.

Parameter	Value	Standard Error	Range (95% Confidence)
a	4.734262	0.606898	3.484333 to 5.984191
b	0.205941	0.037192	0.129342 to 0.282539

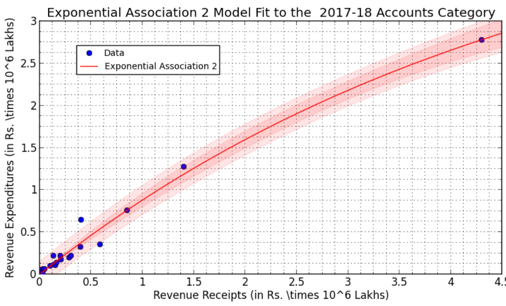

Fig. 7: Exponential Association 2 Model Fit to the 2017-18 Accounts Category with the values of the Variables in Rs. $\times 10^6$ Lakhs.

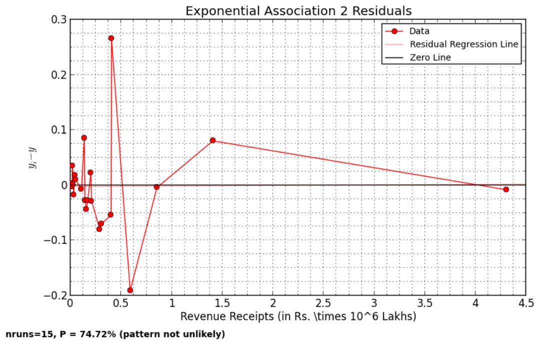

Fig. 8: Residual Plot of the Exponential Association 2 Model Fit to the 2017-18 Accounts Category with P-Value =0.7472 at 95% Confidence Level.

In the Parameter Histories plot of a nonlinear regression model we plot the computed values of each parameter of the model against the number of iterations, which helps us in testing the fact whether the values of the parameters in our chosen model have attained a constancy before the process of computation is completed, i.e., the iteration has terminated. In case the parameter values have attained constancy before the completion of the computation process (which in our case is a maximum run of preset 100 iterations, after which the process of computation automatically terminates), the values of all the parameters of the model should attain constancy, this is indicated by the fact that in the right side of the Parameter Histories plot the values of all the regression parameters should be flat. We can observe from the Parameter Histories plot of Fig. 10 that the value of the parameter a of our proposed Exponential Association 2 Model attains constancy after the fourth iteration, while that of the parameter b practically settles down to a constant value just after the second iteration and very well after the third iteration. Thus our proposed nonlinear regression model Exponential Association 2 satisfies all the desired characteristics of a good robust fit to the 2017-18 Accounts Category data.

We present in Table 8 a representative prediction table for predicting the values of the revenue expenditures for given values of the response variable (the revenue receipts) based on the nonlinear regression model of (3.2). As in the case of the Polynomial Degree 8 Fit, we also give in Table 9 the predicted values of the revenue expenditures for the given values of the minimum and the maximum revenue receipts based on the Exponential Association 2 Model. It can be observed that, as in the case of the Polynomial Degree 8 Fit the agreement in this model is also very unsatisfactory at the lower end (minimum) of the data set, while the agreement at the upper end is excellent. We remark that this anomaly in our models may be due to the presence of outliers in the samples of the the revenue receipts and the revenue expenditures, which requires another detailed investigation, which at present we are not inclined to undertake, but we can address the same in one of our future communications. Here we only remark that the box-and-whisker plot of the sample of Revenue Receipts given in the third column of Table 1 (which corresponds to the values of Revenue Receipts for the 2017-18 Accounts category) as shown in Fig. 11 shows one *outside point* (the entry of Rs. 852939 Lakhs for Gujarat) and two *far outside points* (the entries of Rs. 1.40507×10^6 Lakhs and Rs. 4.3043×10^6 Lakhs corresponding to Delhi and Maharashtra respectively) which need to be tested for possible *outliers* in this sample. Similarly the box-and-whisker plot of the sample of the Revenue Expenditures given in the third column of Table 2 (which corresponds to the values of Revenue Expenditures for the 2017-18 Accounts category) as shown in Fig. 12 reveals the presence of two *outside* points (viz., Rs. 643129 Lakhs (for Tamil Nadu) and Rs. 758442 Lakhs (for Gujarat)) and two far outside points (viz., Rs. 1.2698×10^6

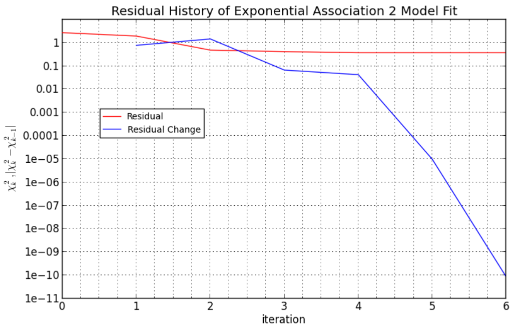

Fig. 9: The Residual History of the Exponential Association 2 Model Fit to the 2017-18 Accounts Category.

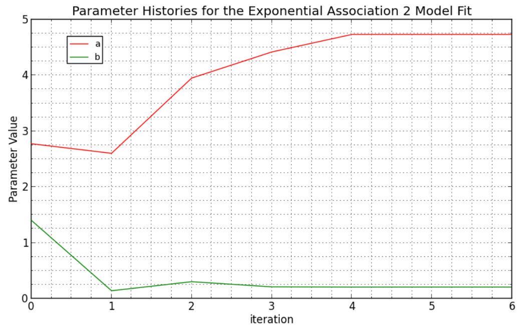

Fig. 10: The Parameter Histories of the Exponential Association 2 Model Fit to the 2017-18 Accounts Category.

(for Delhi) and Rs. 2.77492×10^6 (for Maharashtra)) which are also required to be tested for possible *outliers* in the present sample, which shall form parts of our later investigations in this study.
As in the case of the Polynomial Degree 8 Fit, we show in Fig. 13 the evaluation for the maximum value of the revenue receipts (predictor), while in Fig. 14 we find the slope of the Exponential Association 2 curve at $x = 2.15293515$, which yields a value of $\frac{dy}{dx} = 0.625803664445$. The entire area under the Exponential Association 2 curve from the lower limit of revenue receipts $x =$ Rs 0.0015742×10^6 Lakhs to its upper limit of $x =$ Rs. 4.3042961×10^6 Lakhs is 6.86330112338 $(\text{Rs. } 10^6 \text{ Lakhs})^2$, i.e., $\int_{0.0015742}^{4.3042961} f(x)\,dx = 6.86330112338 (\text{Rs.} \times 10^6 \text{Lakhs})^2$ as shown in Fig. 15. Similarly, the entire arclength of the Exponential Association 2 curve from the lower limit of revenue receipts $x =$ Rs 0.0015742×10^6 Lakhs to its upper limit of $x =$ Rs. 4.3042961×10^6 Lakhs is Rs. $5.15732008288 \times 10^6$ Lakhs as shown in Fig. 16.

3.3 Comparison of the Polynomial Degree 8 and the Exponential Association 2 models

Now we proceed to make a comparison of the above two models fitted by us to the data set of the 2017-18 Accounts Category. In Table 10 we present the details of the results of the comparison made by us, while in Fig. 17 we depict the results of this comparison graphically. Our comparison of the two models involves the Akaike Information Criterion (AIC) test (more precisely the AICc, which is the AIC corrected for small samples) (for example, see [37]) and the usual F-test. It must be borne in mind that since the results of the F-test are valid only when one model (the simpler one) can be derived from the more complex one by suitably choosing some of its parameter values and, as this may not be true or easier to decide in all the cases, yet, without going into the details of the same, we mention the results of the F-test also in Table 10. Since the AIC test is valid in any case therefore, we prefer to rely on the declaration made by the result of the AIC test in case of a contradiction between the results of these two tests employed by us. From the discussion given in Table 10 *we find that the Exponential Association 2 is the better fit to the data set of the 2017-18 Category.* It may also be noted here that since the result of the F-test performed declares that there is only a 2.917% chance (which being less than the critical value of 5% at 95% confidence level) that the Exponential Association 2 model is better than the Polynomial Degree 8 model for the data set in hand, so we discard the same.

Table 8: A representative prediction table for the Exponential Association 2 Model Fit of (3.2) to the 2017-18 Accounts Category.

S. No.	Revenue Receipt x (in Rs. $\times 10^6$Lakhs)	Revenue Expenditure y (in Rs. $\times 10^6$Lakhs)
1.	0.0000000000000000E+00	0.0000000000000000E+00
2.	5.0000000000000000E-01	4.6322956739284421E-01
3.	1.0000000000000000E+00	8.8113387928436770E-01
4.	1.5000000000000000E+00	1.2581478399442239E+00
5.	2.0000000000000000E+00	1.5982724150869831E+00
6.	2.5000000000000000E+00	1.9051170911132211E+00
7.	3.0000000000000000E+00	2.1819381798749737E+00
8.	3.5000000000000000E+00	2.4316733754652318E+00
9.	4.0000000000000000E+00	2.6569729297567095E+00
10.	4.5000000000000000E+00	2.8602277775324336E+00

Table 9: Prediction table for the Exponential Association 2 Model Fit of (3.2) for the minimum and the maximum values of the 2017-18 Accounts Category.

	Revenue Receipt x (in Rs. $\times 10^6$Lakhs)	Observed Revenue Expenditure y_{obs} (in Rs. $\times 10^6$Lakhs)	Predicted Revenue Expenditure y_{pred} (in Rs. $\times 10^6$Lakhs) by (3.2)	Difference= $y_{\text{obs}} - y_{\text{pred}}$ (in Rs. $\times 10^6$Lakhs)
Minimum	0.0015742	0.0005225	0.0015345591811	-0.0010120591811
Maximum	4.3042961	2.7749222	2.78315518133	-0.00823298133

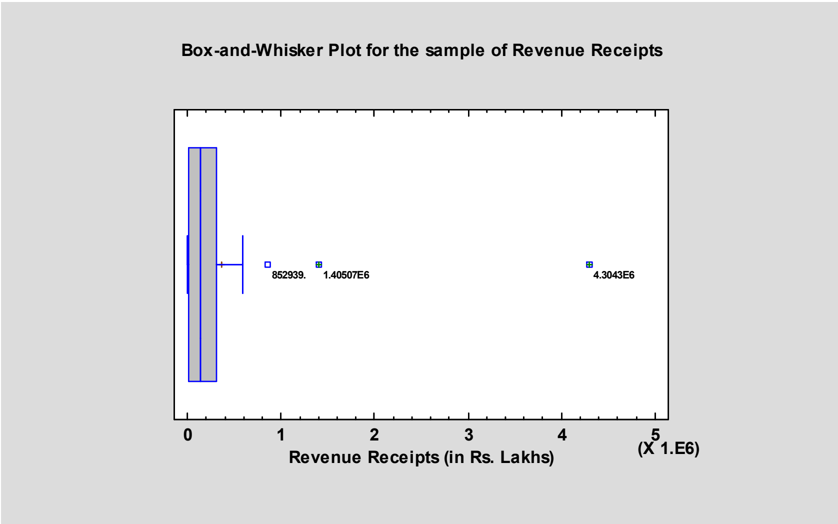

Fig. 11: The box-and-whisker plot for the Revenue Receipts sample corresponding to the 2017-18 Accounts Category.

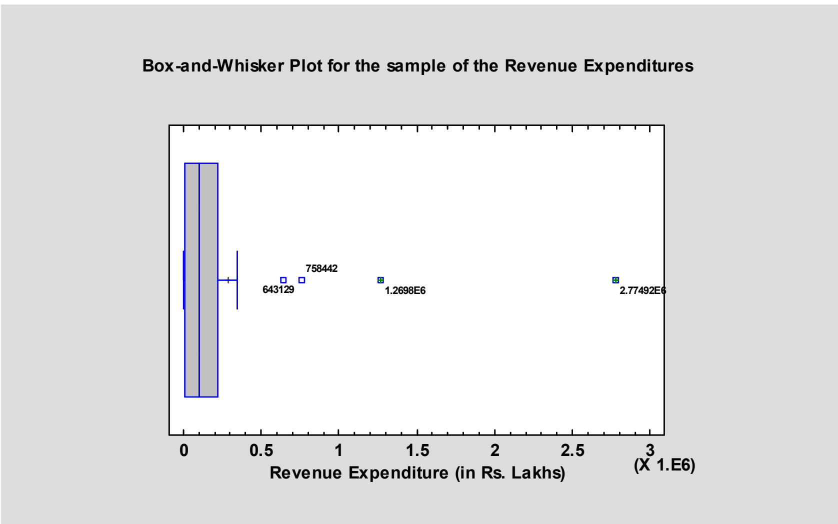

Fig. 12: The box-and-whisker plot for the Revenue Expenditures sample corresponding to the 2017-18 Accounts Category.

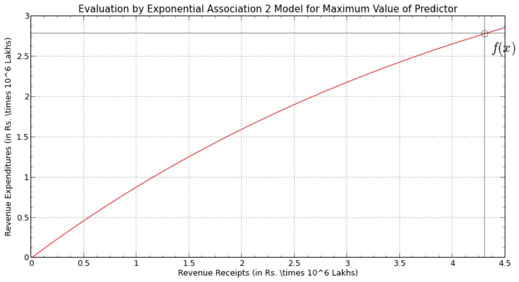

Fig. 13: Evaluation by Exponential Association 2 Model for the Maximum Value of the Predictor.

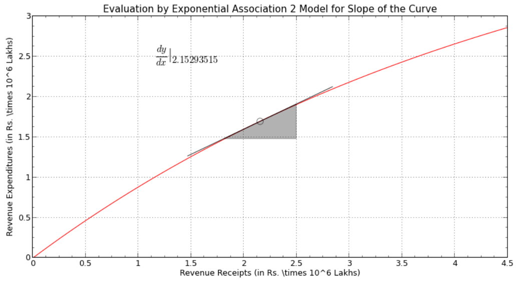

Fig. 14: Evaluation by Exponential Association 2 Model for the Slope of the Curve.

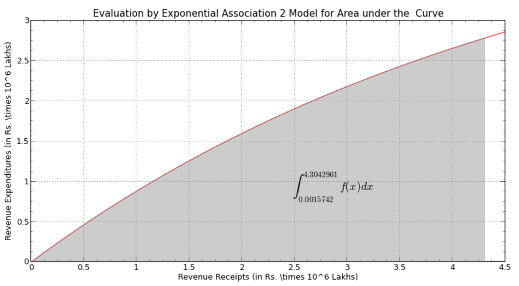

Fig. 15: Evaluation by Exponential Association 2 Model for the Area under the Predictor.

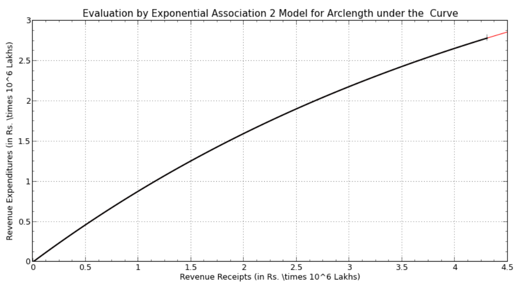

Fig. 16: Evaluation by Exponential Association 2 Model for the Arclength of the Curve.

Table 10: Comparison of the Polynomial Degree 8 and the Exponential Association 2 Models for the data set of the 2017-18 Accounts Category.

Overview

Model	**Sum of Squares**	**DOF**	**AICc**
Exponential Association 2	0.141372	25	−139.649
Polynomial Regression (degree=8)	0.0654729	18	−138.593

Comparison

The best regression is (chosen via the AIC test): **Exponential Association 2**
The best regression is (chosen via the F-Test): **Polynomial Regression (degree=8)**

Justification

Akaike's Information Criterion

Model	**AICc**
Exponential Association 2	**−139.649**
Polynomial Regression (degree=8)	**−138.593**

Delta = 1.05648
Probability = 0.370928
The likelihood that Exponential Association 2 is the better model is 62.9072%.

F-Test

The more complicated model (Polynomial Regression (degree=8)) has a better sum of squares than the simpler model (Exponential Association 2). Performing an F test. Since the result of an F test is strictly not valid unless the simpler model is a subset of the more complex model; i.e., the simpler model can be obtained by setting some parameters in the more complex model to certain constants. As the software cannot currently tell if this is the case, so it presents F-Test results regardless of this fact.
F = 2.98092
P = 0.02917
If the simpler model were correct, the sum of squares would increase by approximately the gain in degrees of freedom when moving from the complex model to the simple one (i.e., F = 1). There is a 2.917% probability that the the simpler regression (Exponential Association 2) is the better fit to the data.

4 Conclusion

We proposed two possible regression models, one a Polynomial Degree 8 Model and the other an Exponential Association 2 Model for the financial data belonging to the 2017-18 Accounts Category as detailed in the third columns of Table 1 and Table 2 of this paper which precisely correspond to the revenue receipts and the revenue expenditures of the municipal corporations of the various States of India including the Union Territory of Chandigarh for the financial year 2017-18. On the basis of the arguments and analysis presented in the preceding sections of this paper, both the models provide a good fit to the data set under consideration. On comparison of the two proposed models we found that the Exponential Association 2 is a better model compared to the Polynomial Degree 8 model for the data set in hand. Though at present we have not analyzed both these samples of the revenue receipts and the revenue expenditures of the Indian States pertaining the financial year 2017-18 for possible outliers, we have only tried here to fit a model that explains in the best possible manner, as far as

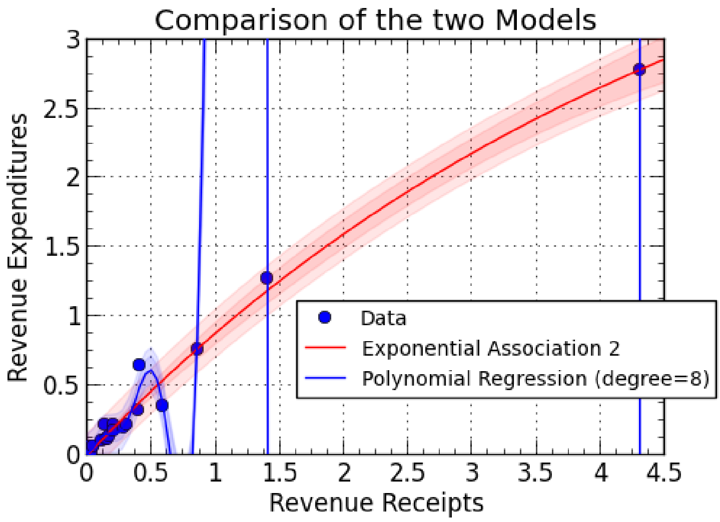

Fig. 17: The comparison of the Polynomial Degree 8 Model and the Exponential Association 2 Model (where both the variables are in Rs. $\times 10^6$ Lakhs) for the data of 2017-18 Accounts Category.

possible, the behavior of the financial data concerning the municipalities of India during the financial year 2017-18.

In our future communications we propose to formulate suitable regression models - both linear and nonlinear - for the financial data of the municipalities of our country belonging to the 2018-19 Budget Estimates, 2018-19 Revised Estimates and the 2019-20 Budget Estimates Categories as mentioned in the fourth, fifth and sixth columns respectively of Table 1 and Table 2 of this paper.

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