Bulletin of Pure and Applied Sciences
Section - E - Mathematics & Statistics
Website : https : //www.bpasjournals.com/

Bull. Pure Appl. Sci. Sect. E Math. Stat.
38E(2), 661–667 (2019)
e-ISSN:2320-3226, Print ISSN:0970-6577
DOI 10.5958/2320-3226.2019.00067.5

Common fixed point theorems satisfying implicit relation conditions in quasi metric spaces and their applications *

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Abstract The aim of this paper is to establish and prove several results on common fixed point theorems for self operator satisfying implicit relation conditions in quasi metric space.

Key words Common fixed point, Quasi valued metric spaces, Implicit relation, Integral type contraction.

2010 Mathematics Subject Classification 30G35, 46E15, 47H10, 47H09.

1 Introduction

Fixed point theory has proved itself to be a useful branch of functional analysis. In 1922, Banach introduced the most useful principle (Banach contraction principle) which has been used in and generalized to many other branches of mathematics. In 1997, Popa [16] defined an implicit relation which covers several well known contractions of the existing literature in one go besides admitting several new contractions. An implicit relation is general enough to a multitude of new contractions. Following this Waszkiewick [1] introduced the concept of quasi metric spaces which is more general than the metric space. The study of fixed point and common fixed points of mappings for a certain metrical contractive condition has attracted the attention of many researchers (see [7]). In a series of recent papers Berinde [8], [10] has focused on such (common) fixed point theorems, which are called constructive (common) fixed point theorems, (see [9]). All these results are obtained by considering operators that satisfy an implicit contractive type condition. The aim of this paper is to define the notion of fixed point theorems in quasi metric spaces satisfying an implicit relation and in this work we prove common fixed point theorems for self operator satisfying implicit relation conditions in a quasi metric space .

* Communicated, edited and typeset in Latex by *Lalit Mohan Upadhyaya* (Editor-in-Chief).
Received April 11, 2019 / Revised October 23, 2019 / Accepted November 20, 2019. Online First Published on December 24, 2019 at https://www.bpasjournals.com/.
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2 Preliminaries

Let X be a set. Following Waszkiewick (see [1]), a distance on X is a map $d: X \times X \to [0, \infty)$. A pair (X, d) is called a distance space if d satisfies the following conditions, for every $x, y, z \in X$ one of the following holds:
(1) $d(x, x) = 0$;
(2) $d(x, y) = d(y, x) = 0 \Rightarrow x = y$;
(3) $d(x, y) \leqslant d(x, z) + d(z, y)$,
then it is called a quasi metric (or simply q-metric) on X. If d satisfies (2) and (3), then d is said to be a dislocated quasi-metric (or simply dq-metric) on X. It is clear that if d satisfies (1),(2), (3) and
(4) $d(x, y) = d(y, x) \forall x, y \in X$,
then d is a metric on X.

Example 2.1. Let $X = \mathbb{R}$ ($\mathbb{R}$ is the set of all real numbers) be endowed with the metric d defined by $d(x, y) = |x - y| \ \forall x, y \in X$. We find that

$$d(rx, ry) = |rx - ry| = |r(x - y)| = |r||x - y| = |r|d(x, y),$$

for all $x, y \in X$ and for each $r \in R$.

Definition 2.2. (Petryshyn and Williamson [3]) Let (X, d) be a metric space. A mapping $T : X \to X$ is called quasi-nonexpansive if for each $x \in X$ and for every $p \in F(T), d(T(x), p) \leq d(x, p)$.

Definition 2.3. (Aubin [2]) Let (X, d) be a metric space. The map $T : D \to X$ is said to be quasi-nonexpansive w.r.t. $(x_n) \subseteq D$ if for all $n \in \mathbb{N} \bigcup \{0\}$ and for every $p \in F(T), d(x_{n+1}, p) \leq d(x_n, p)$. Lemma 2.1 in Aubin [2] states that quasi-nonexpansiveness $\Rightarrow$ quasi-nonexpansiveness w.r.t. $(T^n(x_0))$ (respectively, $(T^n_\lambda(x_0)), (T^n_{\lambda,\mu}(x_0))$) for each $x_0 \in D$. The reverse implications are not true (see, Example 2.1 in (see [2])). Also, Aubin [2]) showed that the continuity of $T : D \to X$ leads to the closedness of $F(T)$ and the converse is not true (see, Example 2.2 in Aubin [2]).

Definition 2.4. (Kirk [4]) Let (X, d) be a metric space. The mapping $T : X \to X$ is called as asymptotically regular at a point $x_0 \in X$ if $lim_{n\to\infty} d(T^n(x_0), T^{n+1}(x_0)) = 0$.

Definition 2.5. (Zeyada et al. [5]) A sequence (x_n) in a dq-metric space (X, d) is said to be convergent to an $x \in X$ if $lim_{n\to\infty} d(x_n, x) = 0$ or $lim_{n\to\infty} d(x, x_n) = 0$. In this case x is called a limit of (x_n) and we write $lim_{n\to\infty} x_n = x$.

Definition 2.6. (Zeyada et al. [5]) A sequence (x_n) in a dq-metric space (X, d) is said to be a Cauchy sequence if for every $\epsilon > 0$ there is an $n_0 = n_0(\epsilon) \in N$ such that $\forall m, n > n_0, d(x_m, x_n) < \epsilon$ or, $d(x_n, x_m) < \epsilon$.

Definition 2.7. Let S and T be self mappings of a metric space (X, d). Seaas [14] defines S and T to be weakly commuting if (see, Jungck [15])

$$d(STx; TSx) \leqslant d(Tx; Sx) \text{ for all } x \in X.$$

and he also defines S and T to be compatible if

$$\lim d(STx_n, TSx_n) = 0$$

whenever x_n is a sequence in X such that $\lim Sx_n = \lim Tx_n = t$ for some $t \in X$.
Clearly, commuting mappings are also weakly commuting.

3 Main result

Popa (see [6]) utilized the idea of implicit function to unify the fixed point theorems. Now, we define the following class of implicit functions as below:
Let Ψ be the family of all continuous real functions $F : [0, \infty)^6 \to [0, \infty)$, satisfying the following conditions:

(ψ_1): F is non-increasing in 5^{th}, 6^{th} coordinate variables,
(ψ_{21}): there exists $h \in (0,1)$ such that for every $u, v > 0$ with $F(u,v,v,u,u+v,0) \leq 0$, or,
(ψ_{22}): $F(u,v,u,v,0,u+v) \leq 0$, we have $u \leq hv$,
(ψ_3): $F(u,u,0,0,u,u) > 0$ for all $u > 0$.

Example 3.1. The function F given by $F(t_1,t_2,t_3,t_4,t_5,t_6) = t_1 - ct_2$ where $c \in [0,1)$ satisfying $\psi_1, \psi_2, \psi_{22}$.

Example 3.2. Let $a \in [0,3)$, then the function F given by $F(t_1,t_2,t_3,t_4,t_5,t_6) = t_1 - a(t_5+t_6)$ where $c \in [0,1)$ satisfying $\psi_1, \psi_2, \psi_{22}$ with $h = \frac{a}{1-a} \leq 1$.

Theorem 3.3. *Let I and J be two self mappings of a quasi metric space (X,d) into itself and let S and T be two mappings from X into X such that Sx, Tx are nonempty closed subset of X for all $x \in X$ then*

1) $SX \subset J(X), TX \subset I(X)$,
2) the pairs (S,J) and (T,I) are D-compatible mappings,
3) $I(X)$ is x_0 joint orbitally complete for some $x_0 \in X$.
If there is a $F \in \Psi$ such that for all $x, y \in X$,

$$F(H(Sx,Ty), d(Ix,Jy), d(Ix,Sx), d(Jy,Ty), d(Ix,Ty), d(Jy,Sx)) \leq 0$$

then S and T have a unique common fixed point such that there exists $z \in X$ such that $z = Iz = Jz$ and $z \in Sz \cap Tz$.

Proof. Let $x_0 \in X$ there exists $y_1 = Jx_1 \in SX_0$, but $SX_0 \in CP(X)$ and $TX_1 \in CP(X)$ then there exists $y_2 = Ix_2 \in STx_1$ such that $d(y_1,y_2) \leqslant H(SX_0,Tx_1)$ and

$$F(d(y_1,y_2), d(y_0,y_1), d(y_0,y_1), d(y_1,y_2), d(y_0,y_1)+d(y_1,y_2), 0) \leq 0$$

$$F(d(Tx_0,Sx_1), d(y_0,Tx_0), d(y_0,Tx_0), d(Tx_0,Sx_1), d(y_0,Tx_0)+d(Tx_0,Sx_1), 0) \leq 0$$

from the property ψ_{21} $\exists$ $h \in (0,1)$ such that

$$F(d(y_1,y_2), d(y_0,y_1), d(y_0,y_1), d(y_1,y_2), d(y_0,y_1)+d(y_1,y_2), 0) \leq 0$$

and

$$F(H(Sx_0,Tx_1), d(Ix_0,Jx_1), d(Ix_0,Sx_0), d(Jx_1,Tx_1), d(Ix_0,Tx_1), d(Jx_1,Sx_0)) \leq 0$$

since H is a Hausdorff metric space and from the property ψ_{21} $\exists$ $h \in (0,1)$ such that

$$d(y_1,y_2) \leq 0hd(y_0,y_1).$$

Similarly one can deduce from the property ψ_{22} $\exists$ $h \in (0,1)$ such that

$$d(y_2,y_3) \leq 0hd(y_1,y_2)$$

then we have an orbit $o(S,T,I,J,x_0)$

$$y_{2n+1} = Jx_{2n+1} \in Sx_{2n}, y_{2n+2} = Ix_{2n+2} \in Tx_{2n+1}.$$

By induction we obtain

$$d(y_n,y_{n+1}) \leq 0h^n d(y_0,y_1).$$

Since

$$d(y_n,y_m) \leq d(y_n,y_{n+1}) + d(y_{n+1},y_{n+2}) + ... + d(y_{m-1},y_m)$$

$$d(y_n,y_m) \leq h^n d(y_0,y_1) + h^{n+1} d(y_0,y_1) + ... + h^{m-1} d(y_0,y_1) = \frac{h^n}{1-h} d(y_0,y_1).$$

therefore $\lim d(y_n, y_m) = 0$ hence y_n is Cauchy sequence. As y_{2n+2} is a Cauchy sequence in $I(X)$ and $I(X)$ is joint orbitally complete, therefore there exists $z \in X$ such that $y_{2n+2} \to z = Iu$ for some $u \in X$. Next we show that $z \in Su$ since

$$F(d(y_{2n+2}, Su), d(z, y_{2n+1}), d(z, Su), d(y_{2n+1}, y_{2n+2}), d(z, y_{2n+2}), d(y_{2n+1}, Su)) \leq 0$$

$$F(d(z, Su), 0, d(z, Su), 0, 0, d(z, Su)) \leq 0$$

By the property ψ_{22} we have $d(z, Tu)) \leq h.0 = 0$ $z \in Su$ therefore there exists $v \in X$ such that $z = Jv$. Similarly we can show that $Jv \in Tv$. Since the pair (S, I) are D-compatible and $z = Iu \in Su$ therefore, $Iz = IIu \in ISu \in Sz$.
Also, $Jz = JJu \in JTu \in Tz$. Next we show that $z = Iz$ if not, we suppose $d(z, Iz) \succ \theta$, then

$$F(d(z, Iz), d(z, Iz), d(z, Iz), d(z, Iz), d(z, Iz), d(z, Iz)) \leq$$
$$F(H(Sz, Tv), d(Iz, Jv), d(Iz, Sz), d(Jv, Tv, d(Iz, Tv), d(Jv, Sz)) \leq 0$$

$$F(d(z, Iz), d(z, Iz), 0, 0, d(z, Iz), d(z, Iz)) \leq 0.$$

It contradicts ψ_3. Thus $d(z, Iz) = 0$ therefore $Iz \in Sz$ similarly, $z = Jz \in Tz$. Hence $z \in Sz \cap Tz$. □

Corollary 3.4. *Let I be a self mapping from a quasi metric space (X, d) into itself and S, T be two mappings from X into X such that SX, TX are nonempty closed subsets of X for all $x \in X, TX \subset IX, SX \subset IX$, the pairs $(S, I), (T, I)$ are D-compatible mappings, and $I(X)$ is x_0 joint orbitally complete for some $x_0 \in X$.*
If there is a $F \in \psi$ such that for all $x, y \in X$

$$F(d(Sx, Ty), d(Ix, Iy), d(Ix, Sx), d(Iy, Ty), d(Ix, Ty), d(Iy, Sx)) \leq 0$$

then there exists $z \in X$ such that $z = Iz \in Tz, z = Iz \in Sz \cap Tz$.

Remark 3.5. If we put $J = I$ and $S = T = W$ in the Theorem 3.3, we have an orbit (x_0, I, W) of x_0 for I and T, in this case we have the following result.

Corollary 3.6. *Let I be a self mapping from a quasi metric space (X, d) into itself and T be a quasi metric space from X into X, $TX \subset I(X)$ the pair (T, I) is D-compatible mappings and $I(X)$ is x_0 jointly orbitally complete for some $x_0 \in x$ If there is a $F \in \psi$ such that for all $x, y \in X$*

$$F(H(Tx, Ty), d(Ix, Iy), d(Ix, Tx), d(Iy, Ty), d(Ix, Ty), d(Iy, Tx)) \leq 0$$

then there exists $z \in X$ such that $z = Iz \in Tz$.

Corollary 3.7. *Let S and T be two mappings from complete quasi metric space (X, d) into X if there $F \in \psi$ such that for all $x, y \in X$*

$$F(H(Sx, Ty), d(x, y), d(x, Sx), d(y, Ty), d(x, Ty), d(y, Sx)) \leq 0.$$

then there exists $z \in X$,$z \in Sz \cap Tz$.

Theorem 3.8. *Let I and J be two self mappings from a quasi metric space (X, d) into itself and $T_n, S_n, n \in N$ be two mappings from X into X such that*
(1) $S_i(X) \subset J(X)$, $T_j(X) \subset I(X), i = 2n, j = 2n + 1$
(2) The pairs (Si, I) and (Tj, J) are D-compatible mappings,
(3) $I(X)$ is x_0 joint orbit all y complete for some $x_0 \in X$.
If there is a $F \in \Psi$ such that for all $x, y \in X$,

$$F(H(S_ix, T_jy), d(Ix, Iy), d(Ix, S_ix), d(Iy, T_jy), d(Ix, T_jy), d(Iy, S_ix)) \leq 0$$

then there exists $z \in X$ such that $z = Iz = Jz and ,z \in \sqcap_{n=0}^{\infty} T_nz$.

4 Result with integral type contraction

In 2002 Branciari [11] defined an integral type contraction and obtained a generalization of Banach contraction principle. Some results on fixed point theorems of integral type contraction have appeared in the literature (see, [12], [13]). In this section, we prove a fixed point result for integral type contractive condition with implicit relation for two pairs and non self mappings in a quasi metric space. Let Ψ be the family of all continuous mappings $F : [0,\infty)^6 \to [0,\infty)$ satisfying the following properties:

(ψ_1): F is non-increasing in the 1^{st} variable and non-increasing in the $3^{\text{rd}}, 4^{\text{th}}, 5^{\text{th}}, 6^{\text{th}}$ coordinate variables,

(ψ_{21}): there exists $h \in (0,1)$ such that for every $u, v \geq 0$ with $\int_0^{F(u,v,v,u,u+v,0)} \varphi(s)ds \leq 0$, or,

(ψ_{22}): $\int_0^{F(u,v,u,v,0,u+v)} \varphi(s)ds \leq 0$, we have $u \geq hv$,

(ψ_3): $\int_0^{F(u,u,0,0,u,u)} \varphi(s)ds > 0$ for all $u > 0$,

where $[0,\infty)^6 \to [0,\infty)$ is a summable nonnegative Lebesgue integral function such that for each $\varepsilon \in [0,1]$, $\int_0^{\varepsilon} \varphi(s)ds \leq 0$.

Theorem 4.1. *Let I and J be two self mappings of a quasi metric space (X,d) into itself and S, T be two mappings from X into X such that Sx, Tx are nonempty closed subsets of X for all $x \in X$ and*
1) $SX \subset J(X), TX \subset I(X)$,
2) the pairs (S,J) and (T,I) are D-compatible mappings,
3) $I(X)$ is x_0 joint orbitally complete for some $x_0 \in X$.
If there is a $F \in \Psi$ such that for all $x, y \in X$,

$$\int_{\theta}^{F(H(Sx,Ty),d(Ix,Jy),d(Ix,Sx),d(Jy,Ty),d(Ix,Ty),d(Jy,Sx))} \varphi(s)ds \leq 0$$

then S and T have a unique common fixed point such that there exists $z \in X$ such that $z = Iz = Jz$ and $z \in Sz \cap Tz$.

Proof. Let $x_0 \in X$ there exists $y_1 = Jx_1 \in SX_0$, but as $SX_0 \in CP(X)$ and $TX_1 \in CP(X)$ thus there exists $y_2 = Ix_2 \in STx_1$ such that $d(y_1,y_2) \leqslant H(SX_0,Tx_1)$ and

$$\int_0^{F(d(y_1,y_2),d(y_0,y_1),d(y_0,y_1),d(y_1,y_2),d(y_0,y_1)+d(y_1,y_2),0)} \varphi(s)ds \leq 0$$

$$\int_0^{F(d(Tx_0,Sx_1),d(y_0,Tx_0),d(y_0,Tx_0),d(Tx_0,Sx_1),d(y_0,Tx_0)+d(Tx_0,Sx_1),0)} \varphi(s)ds \leq 0$$

from the property ψ_{21} $\exists$ $h \in (0,1)$ such that

$$\int_0^{F(d(y_1,y_2),d(y_0,y_1),d(y_0,y_1),d(y_1,y_2),d(y_0,y_1)+d(y_1,y_2),0)} \varphi(s)ds \leq 0$$

$$\int_0^{F(H(Sx_0,Tx_1),d(Ix_0,Jx_1),d(Ix_0,Sx_0),d(Jx_1,Tx_1),d(Ix_0,Tx_1),d(Jx_1,Sx_0))} \varphi(s)ds \leq 0$$

from the property ψ_{21} $\exists$ $h \in (0,1)$ such that

$$d(y_1,y_2) \leq hd(y_0,y_1).$$

Similarly one can deduce from the property ψ_{22} $\exists$ $h \in (0,1)$ such that

$$d(y_2,y_3) \leq hd(y_1,y_2)$$

then we have an orbit $o(S, T, I, J, x_0)$

$$y_{2n+1} = Jx_{2n+1} \in Sx_{2n}, y_{2n+2} = Ix_{2n+2} \in Tx_{2n+1}$$

By induction we obtain

$$d(y_n, y_{n+1}) \leq h^n d(y_0, y_1).$$

Since

$$d(y_n, y_m) \leq d(y_n, y_{n+1}) + d(y_{n+1}, y_{n+2}) + ... + d(y_{m-1}, y_m)$$

$$d(y_n, y_m) \leq h^n d(y_0, y_1) + h^{n+1} d(y_0, y_1) + ... + h^{m-1} d(y_0, y_1) = \frac{h^n}{1-h} d(y_0, y_1)$$

therefore, $\lim d(y_n, y_m) = 0$ hence, y_n is a Cauchy sequence. As y_{2n+2} is a Cauchy sequence in $I(X)$ and $I(X)$ is joint orbitally complete, therefore there exists $z \in X$ such that $y_{2n+2} \to z = Iu$ for some $u \in X$. Next we show that $z \in Su$. Since

$$\int_0^{F(d(y_{2n+2},Su),d(z,y_{2n+1}),d(z,Su),d(y_{2n+1},y_{2n+2}),d(z,y_{2n+2}),d(y_{2n+1},Su))} \varphi(s)ds \leq 0$$

$$\int_0^{F(d(z,Su),\theta,d(z,Su),0,0,d(z,Su))} \varphi(s)ds \leq 0$$

by the property ψ_{22} we have $d(z, Tu) \leq h.\theta = 0$, $z \in Su$, therefore, there exists $v \in X$ such that $z = Jv$. Similarly we can show that $Jv \in Tv$. Since the pair (S, I) is D-compatible and $z = Iu \in Su$ therefore $Iz = IIu \in ISu \in Sz$.
Also, $Jz = JJu \in JTu \in Tz$. Next we show that $z = Iz$. If not, suppose that $d(z, Iz) > 0$, then

$$\int_0^{F(d(z,Iz),d(z,Iz),d(z,Iz),d(z,Iz),d(z,Iz),d(z,Iz))} \varphi(s)ds$$
$$\leq \int_0^{F(H(Sz,Tv),d(Iz,Jv),d(Iz,Sz),d(Jv,Tv),d(Iz,Tv),d(Jv,Sz))} \varphi(s)ds \leq 0$$

$$\int_0^{F(d(z,Iz),d(z,Iz),0,\theta,d(z,Iz),d(z,Iz))} \varphi(s)ds \leq 0$$

which contradicts ψ_3. Thus $d(z, Iz) = 0$, therefore, $Iz \in Sz$. Similarly $z = Jz \in Tz$. Hence $z \in Sz \cap Tz$. □

Example 4.2. The function F given by

$$\int_0^{F(t_1,t_2,t_3,t_4,t_5,t_6)} \varphi(s)ds = \int_0^{t_1 - ct_2} \varphi(s)ds$$

where $c \in [0, 1)$ satisfies $\psi_1, \psi_2, \psi_{22}$.

5 Conclusion

The results derived above in this paper generalize and extend some results of Zeyada et al. [5] in a quasi metric space by using an implicit condition.

Acknowledgments The authors acknowledge with thanks the valuable help received from the Editor-in-Chief for his numerous useful comments and for pointing out certain errors in the proofs of some of the results derived in the original version of this paper. The useful advice and suggestions of the Editor-in-Chief have been utilized beneficially by them in the revised version of this manuscript.

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