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A neighbor balanced design and its optimal certainty contribution *

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Abstract Specially in an agricultural design of experiment, the nature of the treatments may be such that the treatment received by a plot may influence the responses on the neighboring plots or the following plot in the same block. As an example of the second condition, the tall varieties may affect the other crops grown on the neighbor plots by their shades. Bailey ((2003). Designs for one-sided neighbor effect, Jour. Ind. Soc. Ag. Statistics, 56(3), 302–314) has developed such design concerned with the study of one sided neighbor effect, under the above mentioned second condition. This paper gives a new series of Universally Optimal One-Sided Circular Neighbor Balanced Designs.

Key words One Sided Circular Neighbor Balanced design, Universally Optimal, Equivalence class of sequences.

2010 Mathematics Subject Classification Primary: 62K05, 62K10.

1 Introduction and preliminaries

Either in the allied subjects or agricultural design of experiment where the treatment applied to one experimental plot may affect the response on the neighboring plots and response on the plot to which, it is applied, Bailey [2] has proposed particular types of designs which are concerned with the study of one-sided neighbor effect only. An example is the case of sunflower crops as well as in the case of cereal crops where tall varieties may shade the plot on their neighboring and affect the response of the plot. Similarly, as in the case of pesticide or fungicide experiment where some portion of the treatment applied may spread to the neighboring plot immediately by wind and spores, thus from the untreated plots, one-sided neighbor effect of the preceding plot-treatment may occur to the following plot. The linear ridge is the form of the blocks of such design (Welham et al. [8]) where the plots are in 1− dimension and 2− dimension are studied. Azais, Bailey and Monod [1] give a catalogue of circular neighbour balanced designs with $t-1$ blocks of size t or t blocks of size $t-1$, where t is the number of treatments. Many contributions on these and related topics are available in the literatures of Smart [7], Langton [6], David and Kempton [4]. Later on Bailey and Druilhet [3] extended the work of Bailey [2] taking into account the effect of the treatment on the preceding plot and following plot and that on the concerned plot by taking into account the effect of the block. Under the name of Circular Neighbor Balanced design, both of such 1− dimensional and 2− dimensional designs are studied. Kumam and Meitei [5] have contributed a construction-method of such design.
For a better understanding of the concepts developed in this paper we give below the necessary definitions.

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Definition 1.1. One Sided Circular Neighbor Balanced design is an arrangement of v treatments in b linear blocks of size k (not necessarily distinct) such that (i) each treatment is replicated r times, (ii) every pair of distinct treatments has concurrence μ, and, (iii) every treatment is followed by each other treatment λ times assuming that in every block the last plot is followed by the first plot. It is denoted by One Sided Circular Neighbour Balanced design $(v, b, r, k, \mu, \lambda)$.

Such design is neighbour balanced as every treatment is followed by each other treatment λ times and also pairwise balanced in the sense that every pair of distinct treatments has concurrence μ . Clearly, $vr = bk$. These designs become circular, after having recommended to have a border plot before the first plot of each block, assuming that the treatment already applied to the last plot is applied to this border plot. But its response is not measured. It is only to get the neighbor effect of the treatment in the border plot to the last plot. So, from the practical point of view, for conducting an experiment based on such designs of block size k, the planning of the design compels blocks to be of size $k+1$. It becomes Universally Optimal for estimation of the total effect (Bailey and Druilhet [3], proposition 9, p. 1657) if

- there are only s different types of treatments in every block,
- out of the s different types of treatments n_1 repeat m times in the block and each of the remaining n_2 occurs $m+1$ times, where, $n_1 + n_2 = s$, and
- all the occurrences of a treatment in a block must be in a single sequence of adjacent plots (possibly including both the last plot and the first plot).

For the use in the sequel, Universally Optimal One Sided Circular Neighbor Balanced design $(v, b, r, k, s, m, n_1, n_2, \mu, \lambda)$ denotes the design. Clearly, $n_1 = s - n_2, n_2 = k - sm, bs = v\,(v-1)\,\lambda$. The contribution of each block to the sum of the concurrences of all possible pairs of treatments is $\theta/2$ where

$$\begin{aligned}\theta &= n_1\,(n_1-1)\,m^2 + n_2\,(n_2-1)\,(m+1)^2 + 2n_1 n_2 m\,(m+1)\\ &= sm\,(m+1) + k\,(k-2m-1)\,.\end{aligned}$$

Thus, $b\theta = v\,(v-1)\,\mu$. If a classΔ of competing designs contains a design D such that the information matrix C_D is completely symmetric and trace$\,(C_D) \geq$ trace$\,(C_d)$ for all $d \in \Delta$, then the design D is said to be Universally Optimal. Two sequences of treatments on a block are equivalent if one sequence can be obtained from the other one by relabeling the treatments. If we denote by ξ the equivalence class of the sequence 1 on the block u of the design d the trace of C_d is given by (Bailey and Druilhet [3]) as

$$\begin{aligned}C\,(\xi) &= \text{trace}\,(C_{du})\\ &= \tfrac{1}{2}\left(k - \tfrac{2}{k}\textstyle\sum_{i=1}^{v} g_i^2 + \sum_{i=1}^{v} h_i\right)\end{aligned}$$

where, g_i is the number of occurrences of the treatment i in the sequence 1 and h_i is the number of times the treatment$i$ is on the left hand side of itself in sequence 1.

2 Construction

In this paper as we add a new dimension to the sphere of construction of Universally Optimal One Sided Circular Neighbor Balanced Design, therefore, a lemma of Kumam and Meitei [5] , using difference sets will be recalled as below:
Given a set S of size k, i.e., $\{i_1, i_2, \ldots, i_k\}$, the forward and the backward differences arising from this set are defined as follows:

$$F.D. = (i_2 - i_1, i_3 - i_2, \ldots, i_k - i_{k-1}, i_1 - i_k)$$

and

$$B.D. = (i_1 - i_2, i_2 - i_3, \ldots, i_{k-1} - i_k, i_k - i_1)$$

respectively. Clearly, $B_k = -F_k$.

Lemma 2.1. *Let M be a module of v elements. Consider t to be the initial blocks each containing k elements (not necessarily distinct) of M. These t blocks when developed, module v generate a*

Universally Optimal One Sided Circular Neighbor Balanced design with the parameters $v, b = tv, r = kt, k, s, m, n_1, n_2, \mu, \lambda$ if the following conditions are satisfied:
(i) there are only s different types of treatments in every initial block,
(ii) each of the n_1 out of these s treatments occurs m times and each of the remaining $s - n_1 = n_2$ (say) occurs $(m+1)$ times in the block,
(iii) all the occurrences of a treatment in every initial block is in a single sequence of adjacent plots (assuming that the last plot and the first plot are neighbors),
(iv) among the totality of forward (or backward) differences arising from the t initial blocks, every non-zero element of M occurs exactly λ times,
(v) among the totality of differences arising from the t initial blocks, every non-zero element of M occurs exactly μ times.

For, if $v = 4t-1$ be a prime or a prime power and let x be the primitive elements of GF (Galois Field) $(v = 4t-1)$, then $x^{v-1} = x^0 = 1$, i.e., $x^{4t-2} - 1 = 0$, which may be rewritten as $\left(x^{2t-1}+1\right)\left(x^{2t-1}-1\right) = 0$, which gives,$x^{2t-1} = -1$, since x is a primitive element.

$$C_1 = \left\{x^0, x^2, x^4, \ldots, x^{4t-6}, x^{4t-4}\right\} \ldots \tag{2.1}$$

$$C_2 = \left\{x^1, x^3, x^5, \ldots, x^{4t-5}, x^{4t-3}\right\} \ldots \tag{2.2}$$

Consider the initial block as shown in (2.1), then the differences arising from this block can be exhibited as follows :

$$\begin{array}{ll}
1^{\text{st}} \text{ type:} & \pm\left(x^2 - x^0\right), \pm\left(x^4 - x^2\right), \ldots, \pm\left(x^{4t-4} - x^{4t-6}\right), \\
2^{\text{nd}} \text{ type:} & \pm\left(x^4 - x^0\right), \pm\left(x^6 - x^2\right), \ldots, \pm\left(x^{4t-4} - x^{4t-8}\right), \\
\vdots & \vdots \quad \vdots \quad \vdots \quad \ldots \quad \vdots \quad \vdots \\
i^{\text{th}} \text{ type:} & \pm\left(x^{2i} - x^0\right), \pm\left(x^{2i+2} - x^2\right), \ldots, \pm\left(x^{4t-4} - x^{4t-(2i+4)}\right), \\
\vdots & \vdots \quad \vdots \quad \vdots \quad \ldots \quad \vdots \quad \vdots \\
t^{\text{th}} \text{ type:} & \pm\left(x^{2t} - x^0\right), \pm\left(x^{2t+2} - x^2\right), \ldots, \pm\left(x^{4t-4} - x^{4t-(2t+4)}\right), \\
\vdots & \vdots \quad \vdots \quad \vdots \quad \ldots \quad \vdots \quad \vdots \\
[2t-(i+1)]^{\text{th}} \text{ type:} & \pm\left(x^{2[2t-(i+1)]} - x^0\right), \pm\left(x^{2[2t-(i+1)]+2} - x^2\right), \ldots, \\
& \pm\left(x^{4t-4} - x^{4t-(2[2t-(i+1)]+4)}\right), \\
\vdots & \vdots \quad \vdots \quad \vdots \quad \ldots \quad \vdots \quad \vdots \\
[2t-4]^{\text{th}} \text{ type:} & \pm\left(x^{4t-8} - x^0\right), \pm\left(x^{4t-6} - x^2\right), \pm\left(x^{4t-4} - x^4\right), \\
[2t-3]^{\text{th}} \text{ type:} & \pm\left(x^{4t-6} - x^0\right), \pm\left(x^{4t-4} - x^2\right), \\
[2t-2]^{\text{th}} \text{ type:} & \pm\left(x^{4t-4} - x^0\right)
\end{array}$$

Now, the terms of the i^{th} type differences and the $[2t-(i+1)]^{\text{th}}$ type differences may be written as

$$\begin{array}{l}
\pm\left(x^{2i} - x^0\right), \pm\left(x^{2i+2} - x^2\right), \ldots, \pm\left(x^{4t-4} - x^{4t-(2i+4)}\right); \pm\left(x^{2[2t-(i+1)]} - x^0\right), \\
\pm\left(x^{2[2t-(i+1)]+2} - x^2\right), \ldots, \pm\left(x^{4t-4} - x^{4t-(2[2t-(i+1)]+4)}\right)
\end{array}$$

or,

$$\begin{array}{l}
\pm\left(x^{2i} - x^0\right), \pm\left(x^{2i+2} - x^2\right), \ldots, \pm x^{4t-2i-4}\left(x^{2i} - x^0\right); \pm x^{4t-2i-2}\left(x^{2i} - x^0\right), \\
\pm x^2\left(x^{4t-2i-2} - x^0\right), \ldots, \pm x^{4t-4}\left(x^0 - x^{-4t+2i+2}\right)
\end{array}$$

i.e.,

$$\pm x^{q_i}, \pm x^{q_i+2}, \pm x^{q_i+4}, \ldots, \pm x^{q_i+4t-2i-4}, \pm x^{q_i+4t-2i-2}, \pm x^{q_i+4t-2i}, \ldots, \pm x^{q_i+4t-4} \ldots. \tag{2.3}$$

All the positive terms from (2.3) can be written together as

$$x^{q_i}, x^{q_i+2}, x^{q_i+4}, \ldots, x^{q_i+4t-2i-4}, x^{q_i+4t-2i-2}, x^{q_i+4t-2i}, \ldots, x^{q_i+4t-4} \ldots \tag{2.4}$$

and all the negative terms can be exhibited together from (2.3) as

$$-x^{q_i}, -x^{q_i+2}, -x^{q_i+4}, \ldots, -x^{q_i+4t-2i-4}, -x^{q_i+4t-2i-2}, -x^{q_i+4t-2i}, \ldots, -x^{q_i+4t-4}$$

which on using the fact that, $x^{2t-1} = -1$, may be rewritten as

$$x^{q_i+2t-1}, x^{q_i+2t+1}, x^{q_i+2t+3}, \ldots, x^{q_i+6t-2i-5}, x^{q_i+6t-2i-3}, x^{q_i+6t-2i-1}, \ldots, x^{q_i+6t-5} \ldots \tag{2.5}$$

Similarly all the possible differences arising from (2.2) can be written as follows:

$$\pm\, x^{q_i+1}, \pm x^{q_i+3}, \ldots, \pm x^{q_i+4t-2i-3}, \pm x^{q_i+4t-2i-1}, \pm x^{q_i+4t-2i+1}, \ldots, \pm x^{q_i+4t-3} \ldots \tag{2.6}$$

All the positive terms from (2.6) can be written together as

$$x^{q_i+1}, x^{q_i+3}, \ldots, x^{q_i+4t-2i-3}, x^{q_i+4t-2i-1}, x^{q_i+4t-2i+1}, \ldots, x^{q_i+4t-3} \ldots \tag{2.7}$$

and all the negative terms can be exhibited together as follows:

$$-x^{q_i+1}, -x^{q_i+3}, \ldots, -x^{q_i+4t-2i-3}, -x^{q_i+4t-2i-1}, -x^{q_i+4t-2i+1}, \ldots, -x^{q_i+4t-3}$$

which, on using the relation, $x^{2t-1} = -1$, becomes

$$x^{q_i+2t}, x^{q_i+2t+2}, \ldots, x^{q_i+6t-2i-4}, x^{q_i+6t-2i-2}, x^{q_i+6t-2i}, \ldots, x^{q_i+6t-4} \ldots \tag{2.8}$$

From (2.3) and (2.6), it is seen that every non-zero element of GF$(4t-1)$ occurs exactly twice in (2.3) and occurs exactly twice in (2.6) as $i = 1, 2, \ldots, t-1$, among the totality of all possible differences arising from C_1 and C_2 every non-zero elements of GF$(4t-1)$ exactly occur $2\,(t-1)$ times.
Let $x^p = x^2 - x^0$, for some p , the forward differences arising from C_1 and C_2 are given as follows:

$$F.D. = \left(x^2 - x^0\right), \left(x^4 - x^2\right), \ldots, \left(x^{4t-4} - x^{4t-6}\right), \left(x^0 - x^{4t-4}\right)$$

i.e.,

$$x^p, x^{p+2}, \ldots, x^{p+4t-6}, x^{p+4t-4} \ldots \tag{2.9}$$

Similarly for the forward differences arising from C_2 we have,

$$x^{p+1}, x^{p+3}, x^{p+5}, \ldots, x^{p+4t-5}, x^{p+4t-3} \ldots \tag{2.10}$$

The forward differences arising from C_1 and C_2 can be written together in a combined form by using (2.9) and (2.10) as follows,

$$x^p, x^{p+1}, x^{p+2}, x^{p+3}, \ldots, x^{p+4t-6}, x^{p+4t-5}, x^{p+4t-4}, x^{p+4t-3} \ldots \tag{2.11}$$

From (2.11), we clearly see that the forward differences are the powers of the primitive element x, which are increasing from p to $p+4t-3$. All the $2t-2$ differences are nothing but they are the $2t-2$ non-zero elements of GF $(4t-1)$. Thus among the totality of the forward or backward differences arising out of C_1 and C_2 every non-zero elements of GF$(4t-1)$, occurs exactly once. Thus for $\lambda = 1$, developing the initial blocks C_1 and C_2 under the reduction modulo of GF$(4t-1)$, the Universally Optimal One Sided Circular Neighbor Balanced design given in the following theorem can be constructed.

Theorem 2.2. *Developing (2.1) and the (2.2) under* mod(v)*, where x is a primitive element of GF(v), $v = 4t-1$, prime for some t, a construction of Universally Optimal One Sided Circular Neighbor Balanced design with parameters $v = 4t-1, b = 2v, r = 2\,(2t-1)\,, k = 2t-1 = s = n_1, m = 1, n_2 = 0, \mu = 2\,(t-1)\,, \lambda = 1$ is always guaranteed.*

Proof. Obviously, all the $(2t-1)$ elements in C_1 and $(2t-1)$ elements in C_2 are distinct. Hence, $k = s = 2t-1 = n_1, m = 1, n_2 = 0$. □

Example 2.3. An illustrative example for a construction of Universally Optimal One Sided Circular Neighbor Balanced design is given below with the parameters as mentioned in the above theorem, when taking $t = 3$, since the primitive element of 11 is 2, from C_1 and C_2, we get,

$$C_1 = \{x^0, x^2, x^4, \ldots, x^{4t-4}\} = \{2^0, 2^2, 2^4, \ldots, 2^8\} = \{1, 4, 5, 9, 3\}$$

$$C_2 = \{x^1, x^3, x^5, \ldots, x^{4t-3}\} = \{2^1, 2^3, 2^5, \ldots, 2^9\} = \{2, 8, 10, 7, 6\}$$

as $i = 1, 2, \ldots, t-1$.
Developing C_1 and C_2 under the reduction modulo of 11, a solution of a Universally Optimal One Sided Circular Neighbor Balanced design with the parameters becomes, $v = 11, b = 22, r = 10, k = 5 = s = n_1, n_2 = 0, m = 1, \mu = 4, \lambda = 1$.

$$\begin{aligned}
&(1,4,5,9,3), (2,5,6,10,4), (3,6,7,0,5), (4,7,8,1,6), \\
&(5,8,9,2,7), (6,9,10,3,8), (7,10,0,4,9), (8,0,1,5,10), \\
&(9,1,2,6,0), (10,2,3,7,1), (0,3,4,8,2), (2,8,10,7,6), \\
&(3,9,0,8,7), (4,10,1,9,8), (5,0,2,10,9), (6,1,3,0,10), \\
&(7,2,4,1,0), (8,3,5,2,1), (9,4,6,3,2), (10,5,7,4,3), \\
&(0,6,8,5,4), (1,7,9,6,5).
\end{aligned}$$

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