Bulletin of Pure and Applied Sciences
Section - E - Mathematics & Statistics

Website : https : //www.bpasjournals.com/

Bull. Pure Appl. Sci. Sect. E Math. Stat.
38E(2), 615–624 (2019)
e-ISSN:2320-3226, Print ISSN:0970-6577
DOI 10.5958/2320-3226.2019.00062.6

The water – level dependent inventory model developed to the reference of Sonbeel lake *

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Abstract Sonbeel is the second largest fresh water lake in Asia. The water-level of Sonbeel is of variable pattern. Throughout the year, the level of water is not constant. During monsoon the water level increases and from the month of October it gradually decreases. Based on this variable level of water, an inventory model is proposed depending on the water level, so that a better water level management of the Sonbeel lake may be developed for the optimum use of water resources.

Key words Inventory model, Optimation technique, Demand, Sonbeel.

2010 Mathematics Subject Classification 90B05, 90B50, 90B99.

1 Introduction

In classical inventory models it is observed that the demand rate is assumed to be constant. Demand in inventory modeling is the most important property which needs analysis. In reality demands are of variable pattern such as time dependent demand, stock dependent demand and price dependent demand.
Here we focus on variable water level management of the Sonbeel lake to meet the seasonal demands such as fishing, cultivation of buro rice, flood protection, hydropower production, navigation, recreation etc. with the water reserve of Sonbeel. An inventory model may be used for optimal utilization of the available water resources of Sonbeel during the different seasons throughout the year. Present study will be based on the water level management of Sonbeel for optimum use of water resources such as agriculture, fish cultivation, tourism, etc. by using the inventory modeling. Like water reservoir system the water level may be maintained by optimization technique of inventory management. In this paper we discuss an inventory model, in which it is considered that in rainy season the demand increases due to the increasing water level and in the summers the demand decreases due to the decreasing water level and reaches a fixed level till the winters.

* Communicated, edited and typeset in Latex by *Lalit Mohan Upadhyaya* (Editor-in-Chief).
Received February 02, 2019 / Revised May 09, 2019 / Accepted June 11, 2019. Online First Published on December 24, 2019 at https://www.bpasjournals.com/.
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A lot of work has been done for determining the demand in inventory modeling by different researchers. In some works it has been observed that seasonal demand over the entire time horizon is three folded. At the beginning of the season inventory is built by meeting the demand. In the mid of the season inventory level decreases due to demand and reaches a fixed level. At the end of the season the inventory level and the demand both become constant.

From the literature it can be seen that after the development of the classical economic Order Quantity (EOQ) model by Wilson [28] under the assumption of constant demand rate, researchers extensively studied several aspects of inventory and inventory systems. Baker and Urban [2] studied a deterministic inventory level dependent rate. Urban [27] investigated an inventory system in which the demand rate during stock-out periods differs from the in-stock period demand by a given amount. A note on an inventory model with inventory level dependent rate was developed by Dutta and Pal [8]. Mandal and Phaujdar [22] developed another model with the same concept. Hwang and Hahn [17] developed an inventory –level dependent demand rate and a fixed life time. Gupta and Vrat [14] constructed an inventory model with stock dependent demand rate. Chao –Ton et al. [5] studied a deteriorating inventory model with inflation and stock dependent demand rate.

Goh [15] developed the only existing inventory model in which the demand is stock dependent and the holding cost is time dependent. Hesham K. Alfares [1] developed an inventory model with a storage–time dependent holding cost and a stock-level dependent demand rate. Dye [11] presented a deteriorating inventory model with partial back logging under conditions of permissible delay in payment and demand depend on stock. Das etal. [9] developed two warehouse production model for deteriorating inventory items with stock dependent demand under inflation over a random planning horizon. Ghosh et al. [16] focused ona multi item inventory model for deteriorating items in limited storage space with stock- dependent demand.

But the application of inventory modeling with reference to the water management has not been done yet. Charnes et al. [6], Chavas et al. [7], Oduol et al. [23] suggested inventory models to be major tools for assessing efficiency, effectively forecasting and allocating resources even under uncertainty. Flides et al. [13] observed that increased specialization of these fields has been narrowing the scope of interest of Operational Research.

To achieve both agriculture and environmental sustainability, inventory models may thus be very helpful in integrating spatially distributed variables of plant water with mathematical description of water availability and farmer's water demand as discussed in Brasington et al. [3] and Efkih et al. [12]. Now days, scientists recognize the need to use water rationally to alleviate food insecurity and proverty facing semi-arid and arid lands. Clarke and King [4], Swarp et al. [26], Sehring [24], Kemper [18], Subramanian [25] advice the use of efficient methods and empirical techniques of water rationalization in agriculture. Furthermore, because of the temporal and spatial variations of the relationships between water demand and supply, the desired schemes for water allocation may also vary dynamically. This may lead to insufficient water for a reservoir to meet the water demand, particularly when the incoming flows are continuously low in a long period. Therefore, effective planning for water resources management under various uncertainties and complexities is desired as highlighted by Li et al. [19]. Luwesi in his works cite20, 21. addressed the techniques of efficient management of water resources in Kenya under fluctuating rainfall conditions due to the climate changes. In the work of by Dutta and Dutta Choudhury [10], a review of 'inventory models for decision making in water control management' is made.

Motivated by these studies,iIn the present work, we develop an inventory model to meet the demand with respect to the increasing water level down to a fixed level for the Sonbeel lake. The water level of the Sonbeel lake may be maintained to store sufficient water in the rainy season so as to spread maximum water area during the winter season.

2 Study area

Fig.1 shows the satellite picture of Sonbeel, which is the largest Oxbow lake (Wetland) of Asia. Sonbeel, the largest fresh water lake in the State of Assam, lies approximately between $24^0 30'$-$24^0\ 45''$ N latitude and $92^0\ 15'$-$92^0\ 30'$ E Longitude. The main inlet of the beel is the Singla River which flows northwards from Mizoram and finishes its course at the southern end of the beel. The outlet is Kachua river which originates from the northern part of the beel and touching Ratakandi- another beel - on the North

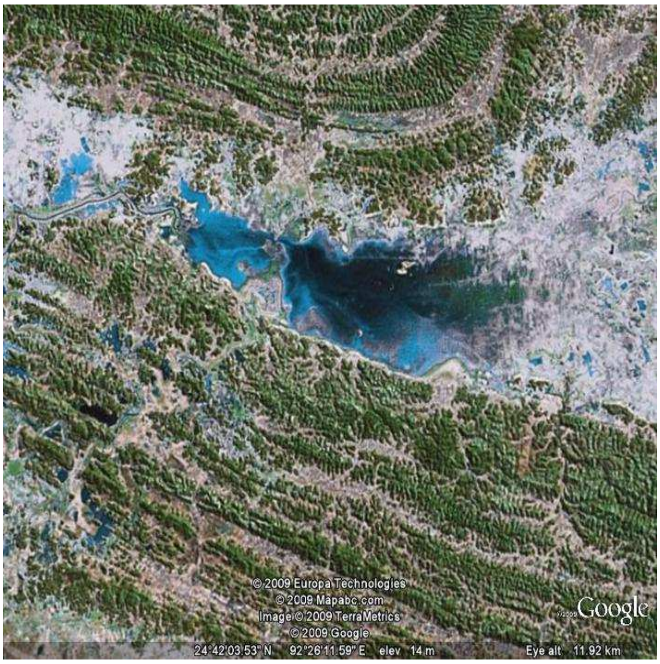

Fig. 1: Satellite picture of Sonbeel. (Courtesy- Google.)

connected with Kushiara river flowing along with the northern boundary of the district .Sonbeel is a natural basin situated between Singla and Kachua River. Sonbeel is said to cover an area of 30,000 bighas of land having an approximate length of 12 km and largest breadth of 3.5 km and about 50,000 peoples' livelihood is directly or indirectly related to the resources generated by the Sonbeel lake. Major production of the beel is the fish which is one of the main sources fishers in Southern Assam. The main resource of the Sonbeel is water and at the same time the main problem of the same is also its uncontrolled water.

Now the present condition of Sonbeel lake is that the whole beel is flooded with water during the rainy season while from the winters to the summers the beel dries almost up to 95% of its area, thus causing a severe scarcity of water in the area. Moreover, due to the shortage of rainfall, water does not remain in the entire beel throughout the year. A model can, therefore, be developed for storage of optimum of water level in the Sonbeel throughout the year and may be used as the reservoir to retain optimum water level and spread area throughout the year.

Assuming that the water level is maintained throughout the year up to a certain level, we develop the following model for this purpose. In rainy season the demand increases due to the increasing water level of Sonbeel and in summers the demand decreases due to the decreasing water level and reaches a fixed level till winter.

3 Sonbeel in pictures

Figs. 2, 3 and 4 show three pictures of Sonbeel lake. In the first of these pictures, the water waves and vegetation can be seen on the surface of the lake while, the next one depicts a rainy season sunset scene of the lake and the last picture shows the view of Sonbeel lake during the winters.

Fig. 2: Waves in Sonbeel. The photo shows the *Baryingtonia acutangularis* vegetation.

4 Development of the innvetory model for Sonbeel lake with the assumptions and notations involved in the process

In this section we state the assumptions and notations used by us for developing the proposed inventory model for Sonbeel lake.

4.1 Assumptions used in the model

Our proposed model is based on the following assumptions:

- The replenishment rate or increasing rate of water level is finite and the lead time is assumed to be zero here.
- The demand rate remains constant throughout the period of increasing water-level.
- During the rainy season the water level is increasing at the rate of ϕ.
- As soon as the rainy season ends, the demand due to water-level is governed by the relation

$$R(Q) = \alpha Q^{\beta} \tag{4.1}$$

  where α and β are constants, $\alpha > 0$ and $0 < \beta < 1$ and Q is the water level.
- (4.1) is maintained till the level of water level decreases and reaches a fixed point Q_0. Thus from Q_0 onwards there is constant demand rate given by the relation:

$$R(Q) = \alpha Q_0^{\beta} = D \tag{4.2}$$

- The water level is maintained at a constant level at times $t = 0$ and at $t = T$ and assumed as $Q = 0$.
- Shortages are not permitted.

4.2 Notations used in the model

ϕ – the increasing rate of water level, i.e., the replenishment rate,
S – flow out rate,

Fig. 3: A sunset scene in Sonbeel during the rainy season.

Q – the water level at any time t,
Q_1 – the maximum level of water,
q – the lot size,
C_1 – the holding cost per unit per unit time,
C_3 – the set up cost per set up,
$0 - t_1$ is the time duration in the beginning of the season when the water level is increasing in the beel,
$t_1 - t_2$ is time duration at the mid of the season when the water level is maintained a particular fixed level and the demand rate follows the relation (4.1),
$t_2 - T$ is the time duration at the end of the season when the water level is decreasing and the demand rate becomes constant.

4.3 The mathematical formulation of the model

Since ϕ is the increasing rate of water level and S is the demand rate, therefore reserving rate of water is $\phi - S$. The reserving period is

$$t_1 = \frac{q}{\phi} \tag{4.3}$$

Now the maximum level of water is

$$Q_1 = (\phi - S)t_1 = (\phi - S)\frac{q}{\phi} \tag{4.4}$$

From (4.4) we have

$$t_1 = \frac{Q_1}{(\phi - s)} \tag{4.5}$$

During the depletion period water level Q at any time t is as follows: $Q = Q_1$, when $t = t_1$ and $Q = Q_0$, when $t = t_2$ and $Q = 0$, when $t = T$, i.e. the water level becomes stagnant at time T, which may be assumed as 0. A schematic representation of the variation of water level with time (season) in the Sonbeel lake is represented in Fig. 5.

Fig. 4: Sonbeel during the winter season.

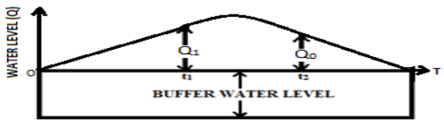

Fig. 5: The schematic representation of variation of the water level with time (season) in Sonbeel lake.

The depletion of the water level is governed by the differential equation

$$\frac{dQ}{dt} = -\alpha Q^{\beta}, \text{when} t_1 \leq t \leq t_2 \tag{4.6}$$

and

$$\frac{dQ}{dt} = -D, \text{when} t_2 \leq t \leq T \tag{4.7}$$

Using the condition $Q = Q_1$ at $t = t_1$, the solution of (4.6) becomes

$$Q^{(1-\beta)} = Q_1^{(1-\beta)} + \alpha(1-\beta)t_1 - \alpha(1-\beta)t \tag{4.8}$$

Since $Q = Q_0$, at $t = t_2$ so we have from (4.8)

$$Q_0{}^{(1-\beta)} = Q_1^{(1-\beta)} + \alpha(1-\beta)t_1 - \alpha(1-\beta)t_2 \tag{4.9}$$

On solving (4.7) using $Q = 0$ at $t = T$, we have

$$Q = D(T - t_2), t_2 \leq t \leq T \tag{4.10}$$

At $t = t_2$, $Q = Q_0$ therefore (4.10) can be written as

$$Q_0 = D(T - t_2) \tag{4.11}$$

$$\Rightarrow t_2 = (DT - Q_0)/D \tag{4.12}$$

Using t_2 from (4.12) in (4.9) we have

$$Q_0{}^{(1-\beta)} = [Q_1^{(1-\beta)} + \alpha\,(1-\beta)\,t_1 - \alpha\,(1-\beta)\,\frac{DT - Q_0}{D}]$$

$$\Rightarrow T=\frac{\{Q_1^{(1-\beta)}-Q_0{}^{(1-\beta)}\}}{\alpha\,(1-\beta)}+t_1\ +\ \frac{Q_0}{D} \tag{4.13}$$

Total inventory I during the complete cycle is calculated as

$$I=\int_0^T Qdt=\int_0^{t_1} Qdt+\int_{t_1}^{t_2} Qdt+\int_{t_2}^{T} Qdt$$

$$=\int_0^{t_1}(\phi-S)dt+\int_{t_1}^{t_2}\{Q_1^{(1-\beta)}+\alpha\,(1-\beta)\,t_1-\alpha\,(1-\beta)\,t\}^{1/(1-\beta)}dt+\int_{t_2}^{T}D(T-t)dt$$

$$=\frac{(\phi-s)\,t_1^2}{2}+\frac{\{Q_1^{1-\beta}\}^{(2-\beta)/(1-\beta)}}{\{\alpha(2-\beta)\}}-\frac{\{\ Q_1^{(1-\beta)}\ +\alpha(1-\beta)t_1-\alpha(1-\beta)t_2\}^{(2-\beta)/(1-\beta)}}{\{\alpha(2-\beta)\}}+\frac{D(T-t_2)^2}{2}$$

Eliminating t_1, t_2, from (4.3), (4.9) and (4.11) we have

$$I=\frac{(\phi-s)q^2}{2\phi^2}+\frac{\{Q_1^{(2-\beta)}-Q_0{}^{(2-\beta)}\}}{\alpha\,(1-\beta)}+\frac{Q_0^2}{2D}$$

$$=\frac{(\phi-s)q^2}{2\phi^2}+\frac{[Q_0^2\,(2-\beta)+2D\{Q_1^{(2-\beta)}-Q_0{}^{(2-\beta)}\}]}{2\alpha D(2-\beta)} \tag{4.14}$$

Eliminating q by using (4.4) in (4.14) we have

$$I=\frac{Q_1^2}{\{2\,(\phi-s)\}}+\frac{[Q_0^2\,(2-\beta)+2D\{Q_1^{(2-\beta)}-Q_0{}^{(2-\beta)}\}]}{2\alpha D(2-\beta)} \tag{4.15}$$

The cost function per unit time $K_1(Q_1)$ is given by the relation

$$K_1(Q_1)=\frac{[C_1 I+C_3]}{T}$$

$$=\frac{C_1\left\{\frac{Q_1^2}{\{2(\phi-s)\}}+\frac{[Q_0^2(2-\beta)+2D\{Q_1^{(2-\beta)}-Q_0{}^{(2-\beta)}\}]}{2\alpha D(2-\beta)}\right\}+C_3}{\frac{\{Q_1^{(1-\beta)}-Q_0{}^{(1-\beta)}\}}{\alpha(1-\beta)}+t_1\ +\ \frac{Q_0}{D}} \tag{4.16}$$

Eliminating t_1 and D from (4.16) using equation (4.3) and the relation $D=\alpha Q_0^{\beta}$

$$K_1(Q_1)=\frac{C_1\left\{\frac{Q_1^2}{\{2(\phi-s)\}}+\frac{[Q_0^2(2-\beta)+2\alpha Q_0^{\beta}\{Q_1^{(2-\beta)}-Q_0{}^{(2-\beta)}\}]}{2\alpha D(2-\beta)}\right\}+C_3}{\frac{\{Q_1^{(1-\beta)}-Q_0{}^{(1-\beta)}\}}{\alpha(1-\beta)}+\frac{Q_1}{(\phi-s)}\ +\ \frac{Q_0}{\alpha Q_0^{\beta}}}$$

To obtain the minimum cost using the condition $\frac{dK_1(Q_1)}{dQ_1}=0$ we have

$$A_1Q_1^{(2-\beta)}+A_2Q_1^{2(1-\beta)}+A_3Q_1^2+A_4Q_1+A_5Q_1^{(1-\beta)}+A_6Q_1^{-\beta}+A_7=0 \tag{4.17}$$

where,

$$A_1=\alpha Q_0^{2\beta}C_1(\phi-s)\{2+(1-\beta\ )(2-\beta)\}$$
$$A_2=2C_1Q_0^{2\beta}(\phi-s)^2$$

$$A_3=\alpha^2Q_0^{2\beta}C_1\,(1-\beta)\,(2-\beta)$$

$$A_4=-2\alpha\beta Q_0^{(1+\beta)}C_1(\phi-s)(2-\beta)$$

$$A_5=-2\alpha\beta Q_0^{(1+\beta)}C_1(\phi-s)^2(2-\beta)$$
$$A_6=(1-\beta)\beta(\phi-s)^2Q_0^{\beta}\{\beta Q_0^2C_1-2\alpha Q_0^{\beta}C_3(2-\beta)\}$$

$$A_7 = \alpha\beta C_1 Q_0^{(2+\beta)}(\phi - s)(1-\beta) - 2C_3\alpha^2 Q_0^{2\beta}(1-\beta)(2-\beta)(\phi - s)$$

Equation (4.17) can be solved by using a suitable numerical method such as the Newton– Raphsons method. As the criterion of optimality is based on minimum cost, the following condition should be satisfied

$$\frac{d^2K_1(Q_1)}{dQ_1^2} > 0$$

then, q can be calculated by the formula $q = \frac{\phi Q_1}{(\phi - s)}$. In case Q_1 obtained from (4.17) is such that $Q_1 < Q_0$ then we proceed as follows:
Since the demand follows a linear relation as soon as the level of water reaches the point Q_0, we have the following formula to calculate cost function $K_2(Q_1)$,

$$K_2(Q_1) = \frac{C_1 Q_1}{2} + \frac{C_3}{T}$$

$$\begin{aligned} T = t_1 + t_2 &= \frac{q}{\phi} + \frac{(\phi - s)q}{(\alpha Q_0^\beta \phi)} \\ &= \frac{q}{\phi} + \frac{(\phi - s)q}{(\alpha Q_0^\beta \phi)} \\ &= \frac{q}{\phi}\frac{\{\alpha Q_0^\beta + (\phi - s)\}}{(\alpha Q_0^\beta)} \end{aligned}$$

which, with the help of (4.4) yields

$$T = \frac{Q_1}{(\phi - s)}\frac{\{\alpha Q_0^\beta + (\phi - s)\}}{(\alpha Q_0^\beta)}$$

Thus,

$$K_2(Q_1) = \frac{C_1 Q_1}{2} + \frac{C_3\alpha Q_0^\beta(\phi - s)\}}{[Q_1\left\{\alpha Q_0^\beta + (\phi - s)\right\}]}.$$

Applying the condition of optimality we have ,

$$Q_1 = [\frac{2\ C_3\alpha Q_0^\beta(\phi - s)\}}{C_1\left\{\alpha Q_0^\beta + (\phi - s)\right\}}]^{1/2}. \tag{4.18}$$

It can be seen that if this $Q_1 < Q_0$ then Q_1 obtained from (4.17) is also less than Q_0.

5 Results and discussion

5.1 Numerical examples

Example 5.1. $\alpha = 0.7$ units, $\beta = 0.6$, $\phi = 12$ units/unit time, $s = 8$ units/unit time, $C_1 = 2$ units/unit time, $C_3 = 55$ units replenishment, $Q_0 = 8$ units.

Solution. Using the values of the parameters in (4.18) we have the optimal lot –size $Q_1 = 9.12698$. Since this $Q_1 > Q_0$, thus (4.17) is applicable, from which Q_1 comes out to be 9.13.

Example 5.2. Assume $\alpha = 0.7$ units, $\beta = 0.6, \phi = 12$ units/unit time, $s = 8$ units/unit time, $C_1 = 2$ units/unit time, $C_3 = 36$ units/replenishment, $Q_0 = 8$ units.

Solution. Using the values of the parameters in (4.18) we have the optimal lot–size $Q_1 = 7.38409$. Since this $Q_1 < Q_0$ so we need not to calculate Q_1 from (4.17).

5.2 Discussion

The tragic aspect of the Sonbeel lake is that it is flooded with water during the rainy season and suffers from water scarcity during the winter for most of its part. The main resource of the Sonbeel is water and at the same time the main problem of the same is also its uncontrolled water. Based on the above model developed by us an inventory model can be further developed for the maintenance of an optimum water level in the Sonbeel so that it may be used as a precious water reservoir to irrigate its surrounding catchment area throughout the year. Our present study is based on the water level management of Sonbeel for optimum use of water resources such as agriculture, fish cultivation, tourism etc. by using the inventory modeling.

Land masses are also there, but due to their higher elevations these areas are not included in the beel area. But these areas also get submerged in the rainy season. 'Buro' crop is cultivated in comparatively higher areas during the winters. If the optimum water level in the Sonbeel lake can be maintained throughout the year then the production of 'Buro' crop can also be maximized.

References

[1] Alfares, H.K. (2007). Inventory model with stock-level dependent demand rate and variable holding cost, *Int. J. Production Economics*, 108, 259–265.

[2] Baker, R.C., Urban, T.L. (1988). A deterministic inventory system with an inventory level dependent rate, *Journal of the Operation Research Society*, 39 (9), 823–831.

[3] Brasington, J., El-Hames, A. and Richards, K. (1998). Hydrological modelling in humid tropical catchments, David Harper & Thomas Brown: Chichester, 313–336.

[4] Clarke, R. and King, J. (2004) The atlas of water, Earthscan Publications, London.

[5] Su, Chao-Ton, Tong, Lee-Ing and Liao, Hung-Chang. (1996). An inventory model under inflation for stock dependent consumption rate and exponential decay, *Opsearch*, 33, 71–82 .

[6] Charnes, A. Cooper, W.W and Rhodes, E. (1978). Measuring efficiency of decision making units, *American Journal of Agricultural Economics*, 87, 160–179.

[7] Chavas J.P., Ragan, P. and Michael, R. (2005). Farm household efficiency: evidence from the Gambia, *Euro. J. Oper. Res.*, 2, 449–458.

[8] Dutta, T.K., Pal, A.K. (1990). A note on an inventory model with inventory-level-dependent demand rate, *J. Oper. Res. Soc.*, 41 (10), 971–975.

[9] Das, et al. (2012). Two-warehouse production model for deteriorating inventory items with stock-dependent demand under inflation over a random planning horizon, *Central European Journal of Operations Research*, 20(2), 251–280.

[10] Dutta, N.and Dutta Choudhury, K. (2017). Review of the inventory models for decision making in water control management, *ISOR Journal of Mathematics*, 13(1), (2017), 16–21.

[11] Dye, C.Y.n(2002). A deteriorating inventory model with stock-dependent demand and partial backlogging under conditions of permissible delay in payment, *Opsearch*, 39, 189–201.

[12] Efkih, S., Feijoo, M.L. and Romero, C. (2009). Agricultural sustainable management: a normative approach based on goal programming, *J. Oper. Res..Soc.*, 60, 534–543.

[13] Flides, R., Nikolopoulos, K., Crone, S.F. and Syntetos, A.A. (2008). Forecasting and operational research: a review. *J. Oper. Res. Soc.*, 59, 1150–1172.

[14] Gupta, R. and Vrat (2009). Inventory model for stock-dependent consumption rate, *Opsearch* , 23 (1), 19–24.

[15] Goh, M. (1994). EOQ models with general demand and holding cost functions. *Eur. J. Oper. Res.*, 73(1), 50–54.

[16] Ghosh, Santanu Kumar, Sarkar, Tania and Choudhury, Kripasindhu (2015). A multi-item inventory model for deteriorating items in limited storage space with stock-dependent demand, *American Journal of Mathematical and Management Sciences*, 34 (2), 147–161.

[17] Hwang, H., Hahn, K.H. (2000). An optimal procurement policy for items with an inventory level dependent demand rate and fixed life time, *European Journal of Operation Research* , 127(3), 537–545.

[18] Kemper, K.E. (2003). Rethinking groundwater management, in Caroline M. Figures, Cecilia Tortajada & Johan Rockström (Eds.), *Rethinking Water Management: Approaches to Contemporary Issues*, Earthscan Publications Ltd., London, UK, 120–143.

[19] Li, Y.P., Huang, G.H., Nie, S.L. and Liu, L. (2008). In exact multistage stochastic integer programming for water resources management under uncertainty, *Journal of Environmental Management*, 88(1), 93–107.

[20] Luwesi, C.N. (2009). An evaluation of efficiency of use of agricultural watershed resources under fluctuating rainfall regimes at Muooni Dam site, Machakos district, Kenya, Master's Thesis, School of Pure and Applied Sciences, Kenyatta University, Kenya.

[21] Luwesi,C.N. (2010). Hydro-Economic inventory in a changing Environment – an assessment of the efficiency of farming water demand under fluctuating rainfall regimes in semi-arid lands of South-East Kenya, Lambert Academic Publishing AG & CO.KG, Saarbrucken, Germany (ISBN 978-3-8433-7607-5).

[22] Mandal, B.N. and Phaujdar (1989). An inventory model for deteriorating items and stock-dependent consumption rate, *J. Oper. Res. Soc.* , 40, 483–488.

[23] Oduol J.B.A., Hotta, K., Shankar, S. and Tsuji, M. (2006). Farm size and productive efficiency: lessons from smallholder farms in Embu District, Kenya, *Kyushu University Journal of the Faculty of Agriculture* , 51 (2), 449–458.

[24] Sehring, J. (2008). The pitfalls of irrigation water pricing in Kyrgyzstan, *Development*, 51, 130–134.

[25] Subramanian, V. (2001). Water quality and quantity, in V. Subramanian and A. Ramanathan (Eds.), *Ecohydrology*, Capital Publishing Company, New Delhi, 241–264.

[26] Swarp, K., Gupta, P.K. and Mohan, M. (2007). Operations Research, Sultan Chand & Sons, New Delhi.

[27] Urban, T.L. (1995). Inventory models with the demand rate dependent on stock and shortage levels, *International Journal of Production Economics*, 40(1), 21–28.

[28] Wilson, R.H. (1934). A scientific routine for stock control, *Harvard Business Review*, 13, 116–128.