*Original Article*

*Available online at*
*www.bpasjournals.com*

*Bulletin of Pure and Applied Sciences.*
*Vol. 38E (Math & Stat.), No.1, 2019. P.450-465*
*Print version ISSN 0970 6577*
*Online version ISSN 2320 3226*
*DOI: 10.5958/2320-3226.2019.00049.3*

# EFFECT OF CHEMICAL REACTION ON MHD FLOW WITH HEAT AND MASS TRANSFER PAST A POROUS PLATE IN THE PRESENCE OF VISCOUS DISSIPATION

**Satyabrat Kar[1,*], Nityananda Senapati[2], Bharatkeshari Swain[3]**

**Authors Affiliation:**
[1,2]Ravenshaw University, Cuttack, Odisha, India, 753003.
[3]IGIT, Sarang, Dhenkanal, Odisha, India, 759146.

***Corresponding Author:**
**Satyabrat Kar,** Ravenshaw University, Cuttack, Odisha, India, 753003.

**E-mail:** satyabratkr@gmail.com

**Received on 18.12.2018, Revised on 03.04.2019, Accepted on 20.04.2019**

**Abstract:**
An attempt is made to study an unsteady MHD free convective flow with heat and mass transfer past a semi-infinite vertical porous plate immersed in a porous medium. The presence of viscous dissipation and chemical reaction are taken into account. It is assumed that the plate is moved with uniform velocity in the direction of fluid flow. Viscous dissipation term lends nonlinearity in the governing equations. Applying perturbation technique, the solutions for velocity, temperature and concentration are obtained. The effect of various parameters such as Reynold number(Rc), Thermal Grashof number(Gr), Mass Grashof number (Gc), Schmidt number (Sc) etc. on velocity, temperature and concentration are shown through graphs. Applications of the present study are shown in material processing systems and different chemical industries. In the physical realm, many irreversible processes are present. Some examples are heat flow through a thermal resistance, fluid flow through a flow resistance, chemical reactions etc. The irreversible process by means of which the work done by a fluid on adjacent layers due to the action of shear forces is transformed into heat is defined as viscous dissipation. It can be seen in many places such as in hydraulic engineering, waves or oscillations etc. viscous dissipation has different applications in various industries.

**Keywords:** MHD, chemical reaction, viscous dissipation, Soret number, Nusselt number, Eckert number.

**2010 Mathematics Subject Classification:** 76W05, 76W99.

**Nomenclature**

$C^*$ Species concentration $\left(kgm^{-3}\right)$

$C_p$ Specific heat at constant pressure $\left(Jkg^{-1}K\right)$

$C_\infty^*$ Species concentration in the free stream $\left(kgm^{-3}\right)$

$C_w^*$ Species concentration at the surface $\left(kgm^{-3}\right)$

$D_M$ Chemical molecular diffusivity $\left(m^2s^{-1}\right)$

$g$ Acceleration due to gravity $\left(ms^{-2}\right)$

$Gr$ Thermal Grashof number
$Gc$ Mass Grashof number
$K$ Permeability parameter
$M$ Hartmann number
$Nu$ Nusselt number
$\Pr$ Prandtl number
$q_r$ Radiative heat flux
$Sh$ Sherwood number
$Sc$ Schmidt number
$T^*$ Temperature $(K)$
$T_w^*$ Fluid temperature at the surface $(K)$
$T_\infty^*$ Fluid temperature in the free stream $(K)$
$u$ Dimensionless velocity component $(ms^{-1})$

*Greek symbols*

$\beta$ Coefficient of volume expansion for heat transfer $(K^{-1})$
$\beta'$ Coefficient of volume expansion for mass transfer $(K^{-1})$
$\theta$ Dimensionless fluid temperature $(K)$
$\kappa$ Thermal conductivity $(Wm^{-1}K^{-1})$
$\nu$ Kinematic viscosity $(m^2s^{-1})$
$\rho$ Density $(kgm^{-3})$
$\sigma$ Electrical conductivity
$C$ Dimensionless species concentration $(kgm^{-3})$
$\tau$ Shearing stress $(Nm^{-2})$

*Subscripts*

$w$ Conditions on the wall
$\infty$ Free stream condition

## 1. INTRODUCTION

In recent years, the subject of magnetohydrodynamics has attracted the attention of many authors in view of its application to the problems in geophysics, astrophysics and many engineering and industrial applications, like cooling of metal in nuclear reactors and magnetic control of iron flow in steel industry, etc. [1-4]. The investigation of unsteady natural convective flow of viscous incompressible fluid past vertical bodies has also several engineering and technological applications. Gupta [5] studied transient free convection of an electrically conducting fluid from a vertical plate in the presence of magnetic field. Free convection effects on flow past an exponentially accelerated vertical plate was studied by Singh and Kumar [6]. Jha et al. [7] analyzed mass transfer effects on exponentially accelerated infinite vertical plate with constant heat flux and uniform mass diffusion. Recently Ahmmed et al.[8] studied the unsteady MHD free convection and mass transfer flow past a vertical porous plate. Chemical reactions happen at a characteristic reaction rate at a given temperature and chemical concentration. Different chemical reactions are used in combinations during chemical synthesis in order to find a desired product. In biochemistry, a consecutive series of chemical reactions form metabolic path ways. The order of chemical reaction is defined as the sum of the powers of the concentration of the reactants in the rate equation of that particular chemical reaction. A first order reaction is the one in which the rate is proportional to the concentration of a single reactant. In the present paper, first order reaction is taken into account. Moreover, coupled heat and mass transfer problems in the presence of chemical reaction are of importance in many processes such as drying, distribution of temperature and moisture over agricultural fields and groves of fruit trees, damage of crops due to freezing evaporation at the surface of a water body and energy transfer in a wet cooling tower, and flow in a desert cooler. Chamkha and Aly [9] obtained numerical solution of steady boundary-layer stagnation point flow of a polar fluid toward a stretching surface embedded in porous media in the presence of the effects of Soret and Dufour numbers and first-order homogeneous chemical reaction. Aurangzaib et al. [10] investigated the effect of thermal stratification and chemical reaction on free convection boundary layer MHD flow with heat and mass transfer of an electrically conducting fluid over time dependent stretching sheet. Abd El-Aziz [11] obtained numerical results to study the effect of time-dependent chemical reaction on stagnation point flow and heat transfer of nanofluid over a stretching sheet. Pal and Mandal [12,13] investigated the mixed convection boundary layer flow of nanofluids at a stagnation point over a permeable stretching/shrinking sheet subject to thermal radiation, heat source / sink, viscous dissipation and chemical reaction using numerical method.

In the physical realm, many irreversible processes are present. Some examples are heat flow through a thermal resistance, fluid flow through a flow resistance, chemical reactions etc. The irreversible process by means of which the work done by a fluid on adjacent layers due to the action of shear forces is transformed into heat is defined as viscous dissipation. It can be seen in many places such as in hydraulic engineering, waves or oscillations, etc. that the viscous dissipation has different applications in various industries. Significant temperature rises are observed in polymer processing flows such as injection molding or extrusion at high rates. Aerodynamic heating in the thin boundary layer around high speed aircraft raises the temperature of the skin. A number of authors have considered viscous heating effects on Newtonian flows. Mahajan et al. [14] reported the influence of viscous heating dissipation effects in natural convective flows, showing that the heat transfer rates are reduced by an increase in the dissipation parameter. Isreal Cookey et al. [15] investigated the influence of viscous dissipation and radiation on unsteady MHD free convection flow past an infinite heated vertical plate in a porous medium with time dependent suction. Zueco [16] used network simulation method (NSM) to study the effects of viscous dissipation and radiation on an unsteady MHD free convection flow past a vertical porous plate. Suneetha et al. [17] have analyzed the thermal radiation effects on hydromagnetic free convection flow past an impulsively started vertical plate with variable surface temperature and concentration by taking into account of the heat due to viscous dissipation. Recently Suneetha et al. [18] studied the effects of thermal radiation on the natural conductive heat and mass transfer of a viscous incompressible gray absorbing-emitting fluid flowing past an impulsively started moving vertical plate with viscous dissipation. Very recently Hiteesh [19] studied the boundary layer steady flow and heat transfer of a viscous incompressible fluid due to a stretching plate with viscous dissipation effect in the presence of a transverse magnetic field. Due to above significant role, it is the purpose of this paper to examine the effect of viscous dissipation and chemical reaction on the MHD flow of a viscous incompressible fluid past a semi-infinite vertical plate. This flow problem was previously studied by Ahmed et al. [8] in the absence of these two parameters.

## 2. MATHEMATICAL FORMULATION

A two dimensional unsteady flow of a laminar and incompressible fluid past a semi-infinite vertical moving plate embedded in a uniform porous medium is considered. Fluid is assumed to be viscous and electrically conducting. A uniform transverse magnetic field $B_0$ is applied in the presence of pressure gradient. Thermal diffusion and thermal radiation are also considered to be present. In the given system $x'$ axis is taken along the plate and $y'$ axis normal to it. No voltage is applied to the system and induced magnetic field is negligible. Since we assume a semi-infinite plate surface, the flow variables are the functions of $y'$ and $t'$ only. Then under the above assumption the unsteady flow with usual Boussinesq's approximation is governed by the following equations

$$\frac{\partial v'}{\partial y'}=0 \tag{2.1}$$

$$\rho\left(\frac{\partial u'}{\partial t'}+V'\frac{\partial u'}{\partial y'}\right)=\frac{\partial p'}{\partial x'}+\mu\frac{\partial^2 u'}{\partial y'^2}-\rho g-\frac{\mu}{k'}u'-\sigma B_0^2 u' \tag{2.2}$$

$$\frac{\partial T'}{\partial t'}+V'\frac{\partial T'}{\partial y'}=\frac{\kappa}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2}-\frac{1}{\rho C_p}\frac{\partial q_r'}{\partial y'}-\frac{Q_0}{\rho C_p}\left(T'-T_\infty'\right)+\frac{\mu}{\rho C_p}\left(\frac{\partial u'}{\partial y'}\right)^2 \tag{2.3}$$

$$\frac{\partial \varphi'}{\partial t'}+V'\frac{\partial \varphi'}{\partial y'}=D_M\frac{\partial^2\varphi'}{\partial y'^2}+D_T\frac{\partial^2 T'}{\partial y'^2}-R_C'\left(C'-C_\infty'\right) \tag{2.4}$$

Boundary conditions for the velocity, temperature and concentration fields are given as follows:

$$u'=u_p',T'=T_w'+\varepsilon\left(T_w'-T_\infty'\right)e^{n't'},C'=C_w'+\varepsilon\left(C_w'-C_\infty'\right) \quad \text{at} \quad y'=0 \tag{2.5}$$

$$u'=U_\infty',U_\infty'=U_0\left(1+\varepsilon e^{n't'}\right),T'\to T_\infty',C'\to C_\infty' \quad \text{at} \quad y'\to\infty \tag{2.6}$$

Now,

$$v'=-v_0\left(1+\varepsilon A e^{n't'}\right) \tag{2.7}$$

In free stream, we have

$$\rho\frac{dU_\infty'}{dt'}=\frac{\partial p'}{\partial x'}-\rho_\infty g-\frac{\mu}{\kappa'}U_\infty'-\sigma B_0^2U_\infty'$$

$$\frac{\partial p'}{\partial x'}=\rho\frac{dU_\infty'}{dt'}+\rho_\infty g+\frac{\mu}{\kappa'}U_\infty'+\sigma B_0^2U_\infty' \tag{2.8}$$

Eliminating $\frac{\partial p'}{\partial x'}$ using (2.2) and (2.8), we obtain

$$\rho\left(\frac{\partial u'}{\partial t'}+v'\frac{\partial u'}{\partial y'}\right)=\rho\frac{dU_\infty'}{dt'}+\rho_\infty g+\frac{\mu}{\kappa'}U_\infty'+\sigma B_0^2U_\infty'+\mu\frac{\partial^2u'}{\partial y'^2}-\rho g-\frac{\mu}{\kappa'}u'-\sigma B_0^2u'$$

$$=\rho\frac{dU_\infty'}{dt'}+\left(\rho_\infty-\rho\right)g+\mu\frac{\partial^2u'}{\partial y'^2}+\frac{\mu}{\kappa'}\left(U_\infty'-u'\right)+\sigma B_0^2\left(U_\infty'-u'\right) \tag{2.9}$$

By using the equation of state, we obtain

$$\left(\rho_\infty-\rho\right)=\beta\left(T'-T_\infty'\right)+\beta'\left(C'-C_\infty'\right) \tag{2.10}$$

Then (2.9) becomes

$$\rho\left(\frac{\partial u'}{\partial t'}+v'\frac{\partial u'}{\partial y'}\right)=\left(\rho_\infty-\rho\right)g+\rho\frac{dU_\infty'}{dt'}+\mu\frac{\partial^2u'}{\partial y'^2}+\frac{\mu}{\kappa'}\left(U_\infty'-u'\right)+\sigma B_0^2\left(U_\infty'-u'\right)$$

$$\frac{\partial u'}{\partial t'}+v'\frac{\partial u'}{\partial y'}=\frac{dU_\infty'}{dt'}+\nu\frac{\partial^2u'}{\partial y'^2}+g\beta(T'-T_\infty')+g\beta'(C'-C_\infty')+\frac{\nu}{k'}(U_\infty'-u')+\frac{\sigma B_0^2}{\rho}(U_\infty'-u') \tag{2.11}$$

The radioactive heat flux term by using the Roseland approximation is given by

$$q_r'=-\frac{4\sigma'}{3\kappa_1'}\frac{\partial T'^4}{\partial y'} \tag{2.12}$$

$T'^4$ may be expressed as a linear combination of the temperature. This is accomplished by expanding in a Taylor series about $T_\infty'$ and neglecting higher order terms to obtain

$$T'^4=4T'T_\infty'^4-3T_\infty'^4 \tag{2.13}$$

By using (2.12) and (2.13) into (2.3) we have

$$\frac{\partial T'}{\partial t'}+V'\frac{\partial T'}{\partial y'}=\frac{\kappa}{\rho C_p}\frac{\partial^2T'}{\partial y'^2}+\frac{16\sigma' T_\infty'^3}{3\rho C_p\kappa_1'}\frac{\partial^2T'}{\partial y'^2}-\frac{Q_0}{\rho C_p}\left(T'-T_\infty'\right)+\frac{\mu}{\rho C_p}\left(\frac{\partial u'}{\partial y'}\right)^2 \tag{2.14}$$

To get the solution of (2.1) to (2.4) with boundary conditions (2.5) and (2.6), the following non dimensional parameters are used.

$$u = uU_0, v' = vV_0, T' = T'_\infty + \theta\left(T'_w - T'_\infty\right), C' = C'_\infty + \varphi\left(C'_w - C'_\infty\right), U'_\infty = U_\infty U_0,$$

$$u'_p = U_p U_0, K' = \frac{Kv^2}{V_0^2}, y' = \frac{yv}{V_0}, Gc = \frac{vg\beta'\left(C'_w - C'_\infty\right)}{V_0^2 U_0}, Gr = \frac{vg\beta\left(T'_w - T'_\infty\right)}{V_0^2 U_0},$$

$$Rc = \frac{Rc'v}{V_0^2}, \Pr = \frac{v\rho C_p}{K}, M = \frac{\sigma B_0^2}{\rho V_0^2}, Q = \frac{Q_0 v}{\rho V_0^2 C_p}, R = \frac{4\sigma' T_\infty'^3}{K_1' K}, Sc = \frac{v}{D_M}, \tag{2.15}$$

$$t' = \frac{tv}{V_0^2}, n' = \frac{nV_0^2}{v}, So = \frac{D_T\left(T'_w - T'_\infty\right)}{v\left(C'_w - C'_\infty\right)}, Ec = \frac{U_0^2}{c_p\left(T'_w - T'_\infty\right)}$$

Using the dimensionless parameters in (2.11), (2.14) and (2.4), we get

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + Gr\theta + Gc\varphi + N\left(U_\infty - U_0\right), \tag{2.16}$$

$$\frac{\partial \theta}{\partial t} + v\frac{\partial \theta}{\partial y} = \frac{1}{\Pr}\left(1 + \frac{4}{3}R\right)\frac{\partial^2 \theta}{\partial y^2} - Q\theta + Ec\left(\frac{\partial u}{\partial y}\right)^2, \tag{2.17}$$

$$\frac{\partial \varphi}{\partial t} + v\frac{\partial \varphi}{\partial y} = \frac{1}{Sc}\frac{\partial^2 \varphi}{\partial y^2} + So\frac{\partial^2 \theta}{\partial y^2} - Rc\varphi. \tag{2.18}$$

The reduced initial and boundary conditions are

$$u = U_p, \theta = 1 + \varepsilon e^{nt}, \varphi = 1 + \varepsilon e^{nt}, \ \text{at} \ y = 0$$

$$u \to U_\infty = 1 + \varepsilon e^{nt}, \theta \to 0, \varphi \to 0 \ \text{as} \ y \to \infty \tag{2.19}$$

Perturbation method is used to solve (2.16), (2.17) and (2.18). The following forms are considered:

$$u(y,t) = u_0(y) + \varepsilon e^{nt} u_1(y) + O\left(\varepsilon^2\right)$$

$$\theta(y,t) = \theta_0(y) + \varepsilon e^{nt} \theta_1(y) + O\left(\varepsilon^2\right)$$

$$\varphi(y,t) = \varphi_0(y) + \varepsilon e^{nt} \varphi_1(y) + O\left(\varepsilon^2\right)$$

From (2.16), (2.17) and (2.18), we obtain

$$u_0'' + u_0' - Nu_0 = -N - Gr\theta_0 - Gc\varphi_0, \tag{2.20}$$

$$u_1'' + u_1' - (N+n)u_1 = -N - Au_0' - Gr\theta_1 - Gc\varphi_1, \tag{2.21}$$

$$(3+4R)\theta_0'' + 3\Pr\theta_0' - 3\Pr Q\theta_0 = -3\Pr Ecu_0'^2, \tag{2.22}$$

$$(3+4R)\theta_1'' + 3\Pr\theta_1' - 3(n+Q)\Pr\theta_1 = -3A\Pr\theta_0' - 6Ec\Pr u_0'u_1', \tag{2.23}$$

$$\varphi_0'' + Sc\varphi_0' - RcSc\varphi_0 = -SoSc\theta_0'', \tag{2.24}$$

$$\varphi_1'' + Sc\varphi_1' - (Rc+n)Sc\varphi_1 = -SoSc\theta_1'' - ASc\varphi_0', \tag{2.25}$$

Corresponding boundary conditions are

$$u_{00} = U_p, u_{01} = 0, \theta_{00} = 1, \theta_{01} = 1, \varphi_{00} = 1, \varphi_{01} = 1 \quad \text{as} \ y = 0$$

$$u_{10} = U_p, u_{11} = 0, \theta_{10} = 1, \theta_{11} = 1, \varphi_{10} = 1, \varphi_{11} = 1$$

$$u_{00} \to 1, u_{01} \to 1, \theta_{00} \to 0, \theta_{01} \to 0, \varphi_{00} \to 0, \varphi_{01} \to 0 \ \text{as} \ y \to \infty \tag{2.26}$$

$$u_{10} \to 1, u_{11} \to 1, \theta_{10} \to 0, \theta_{11} \to 0, \varphi_{10} \to 0, \varphi_{11} \to 0$$

Second Perturbation technique is used to solve equations (2.20), (2.21), (2.22), (2.23), (2.24) and (2.25). The following forms are considered

$$u_0(y,t) = u_{00}(y) + Ecu_{01}(y) + O\left(Ec^2\right)$$

$$u_1(y,t) = u_{10}(y) + Ecu_{11}(y) + O\left(Ec^2\right)$$

$$\theta_0(y,t)=\theta_{00}(y)+Ec\theta_{01}(y)+O\left(Ec^2\right)$$
$$\theta_1(y,t)=\theta_{10}(y)+Ec\theta_{11}(y)+O\left(Ec^2\right)$$
$$\varphi_0(y,t)=\varphi_{00}(y)+Ec\varphi_{01}(y)+O\left(Ec^2\right)$$
$$\varphi_1(y,t)=\varphi_{10}(y)+Ec\varphi_{11}(y)+O\left(Ec^2\right).$$

From equations (2.20), (2.21), (2.22), (2.23), (2.24) and (2.25), we obtain

$$u_{00}''+u_{00}'-Nu_{00}=-N-Gr\theta_{00}-Gc\varphi_{00} \tag{2.27}$$
$$u_{01}''+u_{01}'-Nu_{01}=-N-Gr\theta_{01}-Gc\varphi_{01}, \tag{2.28}$$
$$u_{10}''+u_{10}'-(N+n)u_{10}=-N-Au_{00}'-Gr\theta_{10}-Gc\varphi_{10}, \tag{2.29}$$
$$u_{11}''+u_{11}'-(N+n)u_{11}=-N-Au_{01}'-Gr\theta_{11}-Gc\varphi_{11} \tag{2.30}$$
$$(3+4R)\theta_{00}''+3\Pr\theta_{00}'-3\Pr Q\theta_{00}=0 \tag{2.31}$$
$$(3+4R)\theta_{01}''+3\Pr\theta_{01}'-3\Pr Q\theta_{01}=-3\Pr u_{00}'^{2} \tag{2.32}$$
$$(3+4R)\theta_{10}''+3\Pr\theta_{10}'-3\Pr(Q+n)\theta_{10}=-3A\Pr\theta_{00}' \tag{2.33}$$
$$(3+4R)\theta_{11}''+3\Pr\theta_{11}'-3\Pr(Q+n)\theta_{11}=-3A\Pr\theta_{01}'-6\Pr u_{00}'u_{10}' \tag{2.34}$$
$$\varphi_{00}''+Sc\varphi_{00}'-RcSc\varphi_{00}=-SoSc\theta_{00}'' \tag{2.35}$$
$$\varphi_{01}''+Sc\varphi_{01}'-RcSc\varphi_{01}=-SoSc\theta_{01}'' \tag{2.36}$$
$$\varphi_{10}''+Sc\varphi_{10}'-(Rc+n)Sc\varphi_{10}=-SoSc\theta_{10}''-ASc\varphi_{00} \tag{2.37}$$
$$\varphi_{11}''+Sc\varphi_{11}'-(Rc+n)Sc\varphi_{11}=-SoSc\theta_{11}''-ASc\varphi_{01} \tag{2.38}$$

Solving (2.31) and (2.33), we obtain

$$\theta_{00}=e^{m_2y}, \tag{2.39}$$
$$\theta_{10}=C_1e^{m_2y}+C_2e^{m_4y}, \tag{2.40}$$

Solving (2.35) and (2.37), we obtain

$$\varphi_{00}=B_1e^{m_2y}+B_2e^{m_6y}, \tag{2.41}$$
$$\varphi_{10}=B_3e^{m_6y}+B_4e^{m_2y}+B_5e^{m_8y}+C_3e^{m_2y}+C_4e^{m_4y} \tag{2.42}$$

Solving equations (2.27) and (2.32), we obtain

$$u_{00}=1+A_1e^{m_2y}+A_2e^{m_6y}+A_3e^{m_2y}+A_4e^{m_{10}y} \tag{2.43}$$
$$\begin{aligned}\theta_{01}=&E_1e^{2m_2y}+E_2e^{2m_6y}+E_3e^{2m_2y}+E_4e^{2m_{10}y}+E_5e^{(m_2+m_6)y}+E_6e^{(m_2+m_{10})y}+\\&E_7e^{2m_2y}+E_8e^{(m_2+m_{10})y}+E_9e^{(m_2+m_6)y}+E_{10}e^{(m_6+m_{10})y}+E_{11}e^{m_{12}y}\end{aligned} \tag{2.44}$$

Solving (2.36) and (2.28), we obtain

$$\begin{aligned}\varphi_{01}=&C_5e^{2m_2y}+C_6e^{2m_6y}+C_7e^{(m_2+m_6)y}+C_8e^{2m_2y}+C_9e^{2m_{10}y}+C_{10}e^{(m_2+m_6)y}+\\&C_{11}e^{2m_2y}+C_{12}e^{(m_2+m_{10})y}+C_{13}e^{(m_2+m_6)y}+C_{14}e^{(m_6+m_{10})y}+C_{15}e^{m_{12}y}+C_{16}e^{m_{12}y}\end{aligned} \tag{2.45}$$

$$\begin{aligned}u_{01}=&1+A_{18}e^{2m_2y}+A_{19}e^{2m_6y}+A_{20}e^{(m_2+m_6)y}+A_{21}e^{2m_2y}+A_{22}e^{2m_{10}y}+A_{23}e^{(m_2+m_6)y}+\\&A_{24}e^{2m_2y}+A_{25}e^{(m_2+m_{10})y}+A_{26}e^{(m_2+m_6)y}+A_{27}e^{(m_6+m_{10})y}+A_{28}e^{m_{12}y}+A_{29}e^{2m_2y}\\&+A_{30}e^{2m_6y}+A_{31}e^{(m_2+m_6)y}+A_{32}e^{2m_2y}+A_{33}e^{2m_2y}+A_{33}e^{2m_2y}+A_{34}e^{(m_2+m_6)y}+\\&A_{35}e^{2m_2y}+A_{36}e^{(m_2+m_6)y}+A_{37}e^{(m_2+m_6)y}+A_{38}e^{(m_6+m_{10})y}+A_{39}e^{m_{12}y}+A_{40}e^{m_{14}y}\end{aligned}$$

$$+A_{41}e^{m_{16}y} \tag{2.46}$$

Solving (2.29) and (2.34), we obtain

$$u_{10}=1+A_{42}e^{m_2y}+A_{43}e^{m_8y}+A_{44}e^{m_2y}+A_{45}e^{m_{10}y}+A_{46}e^{m_2y}+A_{47}e^{m_4y}+ A_{48}e^{m_8y}+A_{49}e^{m_2y}+A_{50}e^{m_2y}+A_{51}e^{m_6y}+A_{52}e^{m_{18}y} \tag{2.47}$$

$$\begin{aligned}\theta_{11}=&A_{52}e^{2m_2y}+A_{53}e^{2m_6y}+A_{54}e^{(m_2+m_6)y}+A_{55}e^{2m_4y}+A_{56}e^{(m_2+m_6)y}+A_{57}e^{2m_2y}+\\&A_{58}e^{(m_2+m_6)y}+A_{59}e^{(m_2+m_6)y}+A_{60}e^{(m_6+m_{10})y}+A_{61}e^{(m_6+m_{10})y}+A_{62}e^{m_{20}y}+E_{11}e^{2m_2y}\\&+E_{12}e^{(m_2+m_8)y}+E_{13}e^{2m_2y}+E_{14}e^{(m_2+m_{10})y}+E_{15}e^{2m_2y}+E_{16}e^{(m_2+m_4)y}+E_{17}e^{(m_2+m_8)y}\\&+E_{18}e^{2m_2y}+E_{19}e^{2m_2y}+E_{20}e^{(m_2+m_6)y}+E_{21}e^{(m_2+m_8)y}+E_{22}e^{(m_2+m_6)y}+E_{23}e^{(m_2+m_6)y}\\&+E_{24}e^{(m_2+m_6)y}+E_{25}e^{(m_6+m_{10})y}+E_{26}e^{(m_2+m_6)y}+E_{27}e^{(m_4+m_6)y}+E_{28}e^{(m_6+m_8)y}+\\&E_{29}e^{(m_2+m_6)y}+E_{30}e^{(m_2+m_6)y}+E_{31}e^{2m_2y}+E_{32}e^{(m_2+m_8)y}+E_{33}e^{2m_2y}+E_{34}e^{(m_2+m_8)y}\\&+E_{35}e^{2m_2y}+E_{36}e^{(m_2+m_{10})y}+E_{37}e^{2m_2y}+E_{38}e^{(m_2+m_4)y}+E_{39}e^{(m_2+m_8)y}+E_{40}e^{2m_2y}\\&+E_{41}e^{2m_2y}+E_{42}e^{(m_2+m_6)y}+E_{43}e^{(m_2+m_{18})y}+E_{44}e^{(m_2+m_10)y}+E_{45}e^{(m_8+m_{10})y}+\\&E_{46}e^{(m_2+m_{10})y}+E_{47}e^{2m_{10}y}+E_{48}e^{(m_2+m_{10})y}+E_{49}e^{(m_4+m_{10})y}+E_{50}e^{(m_8+m_{10})y}\\&+E_{51}e^{(m_2+m_{10})y}+E_{52}e^{(m_2+m_{10})y}+E_{53}e^{(m_6+m_{10})y}+E_{54}e^{(m_{10}+m_{18})y}\end{aligned} \tag{2.48}$$

Solving (2.30) and (2.38), we obtain

$$\begin{aligned}\varphi_{11}=&C_{17}e^{2m_2y}+C_{18}e^{2m_6y}+C_{19}e^{(m_2+m_6)y}+C_{20}e^{2m_4y}+C_{21}e^{(m_2+m_6)y}+C_{22}e^{2m_2y}+\\&C_{23}e^{(m_2+m_{10})y}+C_{24}e^{(m_2+m_6)y}+C_{25}e^{(m_6+m_{10})y}+C_{26}e^{m_{12}y}+C_{27}e^{m_{20}y}+C_{28}e^{2m_2y}+\\&C_{29}e^{(m_2+m_6)y}+C_{30}e^{2m_2y}+C_{31}e^{(m_2+m_{10})y}+C_{32}e^{2m_2y}+C_{33}e^{(m_2+m_4)y}+C_{34}e^{(m_2+m_8)y}+\\&C_{35}e^{2m_2y}+C_{36}e^{2m_2y}+C_{37}e^{(m_2+m_6)y}+C_{38}e^{(m_2+m_8)y}+C_{39}e^{(m_2+m_8)y}+C_{40}e^{(m_2+m_6)y}+\\&C_{41}e^{(m_2+m_6)y}+C_{42}e^{(m_6+m_{10})y}+C_{43}e^{(m_2+m_6)y}+C_{44}e^{(m_4+m_6)y}+C_{45}e^{(m_6+m_8)y}+C_{46}e^{(m_2+m_6)y}+\\&C_{47}e^{(m_2+m_6)y}+C_{48}e^{2m_6y}+C_{49}e^{(m_2+m_8)y}+C_{50}e^{2m_2y}+C_{51}e^{(m_2+m_8)y}+C_{52}e^{2m_2y}+\\&C_{53}e^{(m_2+m_{10})y}+C_{54}e^{2m_2y}+C_{55}e^{(m_2+m_4)y}+C_{56}e^{(m_2+m_8)y}+C_{57}e^{2m_2y}+C_{58}e^{2m_2y}+\\&C_{59}e^{(m_2+m_6)y}+C_{60}e^{(m_2+m_{18})y}+C_{61}e^{(m_2+m_{10})y}+C_{62}e^{(m_8+m_{10})y}+C_{63}e^{(m_2+m_{10})y}+C_{64}e^{2m_{10}y}+\\&C_{65}e^{(m_2+m_{10})y}+C_{66}e^{(m_4+m_{10})y}+C_{67}e^{(m_8+m_{10})y}+C_{68}e^{(m_2+m_{10})y}+C_{69}e^{(m_2+m_{10})y}+C_{70}e^{(m_6+m_{10})y}+\\&C_{71}e^{(m_{10}+m_{18})y}+C_{72}e^{2m_2y}+C_{73}e^{2m_6y}+C_{74}e^{(m_2+m_6)y}+C_{75}e^{2m_2y}+C_{76}e^{2m_{10}y}+\\&C_{77}e^{(m_2+m_6)y}+C_{78}e^{2m_2y}+C_{79}e^{(m_2+m_{10})y}+C_{80}e^{(m_2+m_6)y}+C_{81}e^{(m_6+m_{10})y}+C_{82}e^{m_2y}+\\&+C_{83}e^{m_{14}y}+C_{84}e^{m_{22}y},\end{aligned} \tag{2.49}$$

$$\begin{aligned}u_{11}=&1+C_{85}e^{2m_2y}+C_{86}e^{2m_6y}+C_{87}e^{(m_2+m_6)y}+C_{88}e^{2m_2y}+C_{89}e^{2m_{10}y}+C_{90}e^{(m_2+m_{10})y}+\\&C_{91}e^{2m_2y}+C_{92}e^{(m_2+m_{10})y}+C_{93}e^{(m_2+m_6)y}+C_{94}e^{(m_6+m_{10})y}+C_{95}e^{2m_{12}y}+C_{96}e^{2m_2y}+\\&C_{97}e^{2m_6y}+C_{98}e^{(m_2+m_6)y}+C_{99}e^{2m_2y}+C_{100}e^{2m_2y}+C_{101}e^{(m_2+m_6)y}+C_{102}e^{2m_2y}+\\&C_{103}e^{(m_2+m_6)y}+C_{104}e^{(m_2+m_6)y}+C_{105}e^{(m_6+m_{10})y}+C_{106}e^{m_{12}y}+C_{107}e^{m_{14}y}+C_{108}e^{m_{16}y}+\\&C_{109}e^{2m_2y}+C_{110}e^{(m_2+m_6)y}+C_{111}e^{(m_2+m_6)y}+C_{112}e^{2m_4y}+C_{113}e^{(m_2+m_6)y}+C_{114}e^{2m_2y}+\\&C_{115}e^{(m_2+m_{10})y}+C_{116}e^{(m_2+m_6)y}+C_{117}e^{(m_6+m_{10})y}+C_{118}e^{m_{12}y}+C_{119}e^{m_{20}y}+C_{120}e^{2m_2y}+\end{aligned}$$

$$
\begin{aligned}
&C_{121}e^{(m_2+m_6)y}+C_{122}e^{2m_2y}+C_{123}e^{(m_2+m_{10})y}+C_{124}e^{2m_2y}+C_{125}e^{(m_2+m_4)y}+C_{126}e^{(m_2+m_{18})y}+\\
&\quad C_{127}e^{2m_2y}+C_{128}e^{2m_2y}+C_{129}e^{(m_2+m_6)y}+C_{130}e^{(m_2+m_8)y}+C_{131}e^{(m_2+m_6)y}+\\
&C_{132}e^{(m_2+m_6)y}+C_{133}e^{(m_2+m_6)y}+C_{134}e^{(m_6+m_{10})y}+C_{135}e^{(m_2+m_6)y}+C_{136}e^{(m_4+m_6)y}+\\
&C_{137}e^{(m_6+m_8)y}+C_{138}e^{(m_2+m_6)y}+C_{139}e^{(m_2+m_6)y}+C_{140}e^{2m_6y}+C_{141}e^{(m_2+m_8)y}+C_{142}e^{2m_2y}+\\
&C_{143}e^{(m_2+m_8)y}+C_{144}e^{2m_2y}+C_{145}e^{(m_2+m_{10})y}+C_{146}e^{2m_2y}+C_{147}e^{(m_2+m_4)y}+C_{148}e^{(m_2+m_8)y}+\\
&C_{149}e^{2m_2y}+C_{150}e^{2m_2y}+C_{151}e^{(m_2+m_6)y}+C_{152}e^{(m_2+m_{18})y}+C_{153}e^{(m_2+m_{10})y}+C_{154}e^{(m_8+m_{10})y}+\\
&C_{155}e^{(m_2+m_{10})y}+C_{156}e^{2m_{10}y}+C_{157}e^{(m_2+m_{10})y}+C_{158}e^{(m_4+m_{10})y}+C_{159}e^{(m_8+m_{10})y}+\\
&C_{160}e^{(m_2+m_{10})y}+C_{161}e^{(m_2+m_{10})y}+C_{162}e^{(m_6+m_{10})y}+C_{163}e^{(m_{10}+m_{18})y}+C_{164}e^{2m_2y}+C_{165}e^{2m_6y}\\
&+C_{166}e^{(m_2+m_6)y}+C_{167}e^{2m_4y}+C_{168}e^{(m_2+m_6)y}+C_{169}e^{2m_2y}+C_{170}e^{(m_2+m_{10})y}+C_{171}e^{(m_2+m_6)y}\\
&+C_{172}e^{(m_6+m_{10})y}+C_{173}e^{m_{12}y}+C_{174}e^{m_{20}y}+C_{175}e^{2m_2y}+C_{176}e^{(m_2+m_6)y}+C_{177}e^{2m_2y}+\\
&C_{178}e^{(m_2+m_{10})y}+C_{179}e^{2m_2y}+C_{180}e^{(m_2+m_4)y}+C_{181}e^{(m_2+m_8)y}+C_{182}e^{2m_2y}+C_{183}e^{2m_2y}+\\
&C_{184}e^{(m_2+m_6)y}+C_{185}e^{(m_2+m_8)y}+C_{186}e^{(m_2+m_6)y}+C_{187}e^{(m_2+m_6)y}+C_{188}e^{(m_2+m_6)y}+\\
&C_{189}e^{(m_6+m_{10})y}+C_{190}e^{(m_2+m_6)y}+C_{191}e^{(m_4+m_6)y}+C_{192}e^{(m_8+m_6)y}+C_{193}e^{(m_2+m_6)y}\\
&+C_{194}e^{(m_2+m_6)y}+C_{195}e^{2m_6y}+C_{196}e^{(m_2+m_8)y}+C_{197}e^{2m_2y}+C_{198}e^{(m_2+m_8)y}+C_{199}e^{2m_2y}+\\
&C_{200}e^{(m_2+m_{10})y}+C_{201}e^{2m_2y}+C_{202}e^{(m_2+m_4)y}+C_{203}e^{(m_2+m_8)y}+C_{204}e^{2m_2y}+C_{205}e^{2m_2y}+\\
&C_{206}e^{(m_2+m_6)y}+C_{207}e^{(m_2+m_8)y}+C_{208}e^{(m_2+m_{10})y}+C_{209}e^{(m_8+m_{10})y}+C_{210}e^{(m_2+m_{10})y}+\\
&C_{211}e^{2m_{10}y}+C_{212}e^{(m_2+m_{10})y}+C_{213}e^{(m_4+m_{10})y}+C_{214}e^{(m_8+m_{10})y}+C_{215}e^{(m_2+m_{10})y}+\\
&C_{216}e^{(m_2+m_{10})y}+C_{217}e^{(m_6+m_{10})y}+C_{218}e^{(m_{10}+m_{18})y}+C_{219}e^{2m_2y}+C_{220}e^{2m_6y}+C_{221}e^{(m_2+m_6)y}\\
&+C_{222}e^{2m_2y}+C_{223}e^{2m_{10}y}+C_{224}e^{(m_2+m_6)y}+C_{225}e^{2m_2y}+C_{226}e^{(m_2+m_{10})y}+C_{227}e^{(m_2+m_6)y}+\\
&C_{228}e^{(m_6+m_{10})y}+C_{229}e^{m_{12}y}+C_{230}e^{m_{14}y}+C_{231}e^{m_{22}y}+C_{232}e^{2m_2y}+C_{233}e^{m_{24}y}
\end{aligned}
\tag{2.50}
$$

Now,

$$u_0(y,t)=u_{00}+Ecu_{01} \tag{2.51}$$

in which $u_{00}$ can be had from (2.43) and $u_{01}$ is given in (2.46). Again, using the value of $u_{10}$ from (2.47) and $u_{11}$ from (2.50) we obtain

$$u_1(y,t)=u_{10}+Ecu_{11} \tag{2.52}$$

Similarly, with the help of the value of $\theta_{00}$ from (2.39) and the value of $\theta_{01}$ from (2.44) we can have

$$\theta_0(y,t)=\theta_{00}+Ec\theta_{01} \tag{2.53}$$

Recalling the value of $\theta_{10}$ from (2.40) and that of $\theta_{11}$ from (2.48) we obtain

$$\theta_1(y,t)=\theta_{10}+Ec\theta_{11} \tag{2.54}$$

Also, utilizing the value of $\varphi_{00}$ from (2.41) and that of $\varphi_{01}$ from (2.45) we have

$$\varphi_0(y,t)=\varphi_{00}+Ec\varphi_{01} \tag{2.55}$$

Finally, on using the value of $\varphi_{10}$ from (2.42) and that of $\varphi_{11}$ from (2.49) we have

$$\varphi_1(y,t)=\varphi_{10}+Ec\varphi_{11} \tag{2.56}$$

## 3. RESULTS AND DISCUSSION

To discuss the physical significance of various parameters involved in the results, the numerical calculations are carried out. The effects of the various parameters entering in the governing equations on the velocity, temperature and concentration are shown through graphs as detailed below:

Fig.1: In this figure, velocity profiles are depicted for various values of solutal Grashoff number $Gc$ which is defined by the ratio of the species buoyancy force to the viscous hydrodynamic force. It is noticed that when buoyancy force dominates the viscous hydrodynamic force, the velocity increases. Velocity profiles increases to a peak value near the plate then it decreases. It can be said that velocity profiles converge pointwise.

Fig.2: It shows the influence of chemical reaction parameter on the velocity field. Increase in $Rc$ results in an increment in velocity.

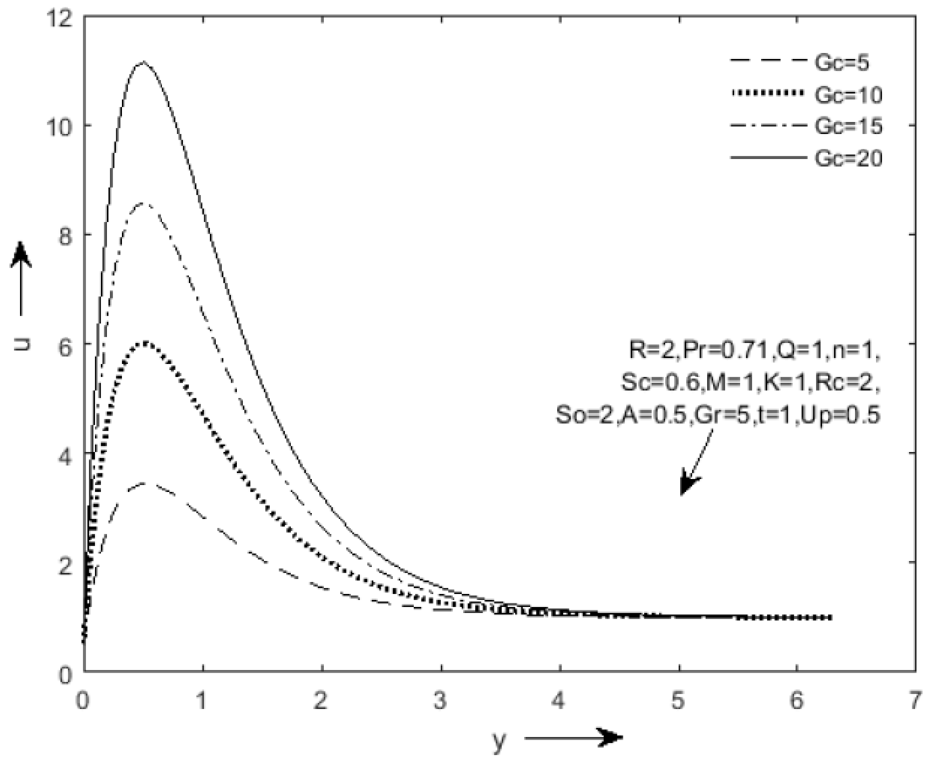


**Figure 1: Velocity profiles for different values of $Gc$**

Fig. 3: Here velocity profiles are depicted against $y$ for various values of the Prandtl number $\Pr$. It shows an oscillatory effect. At the vicinity of the plate the velocity increases with increasing values of $\Pr$. But at certain values of $y$ the profiles intersect and then lead to the opposite effect.

In Fig. 4, It is noticed that increasing values of $Gr$ enhance the velocity.

Fig.5: It shows the oscillatory values of velocity. When $Q$ increases the velocity increases at first and then decreases.

Fig.6: Here concentration profiles are depicted for different values of $Rc$. It can be easily seen that the concentration decreases with increase in $Rc$.

In Fig.7 and Fig.8 concentration is influenced equally by $Sc$ and $So$ respectively. The concentration increases with increasing values of these parameters near the plate. After certain value of $y$, the concentration decreases when these parameters increase.

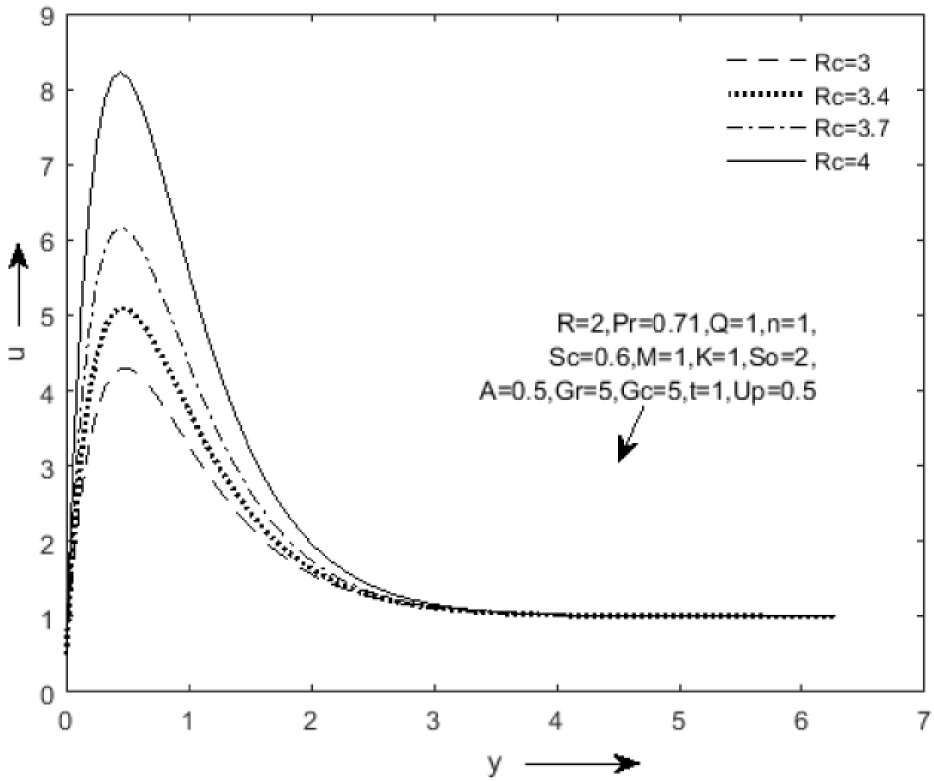


**Figure 2: Velocity Profiles for different values of** $Rc$

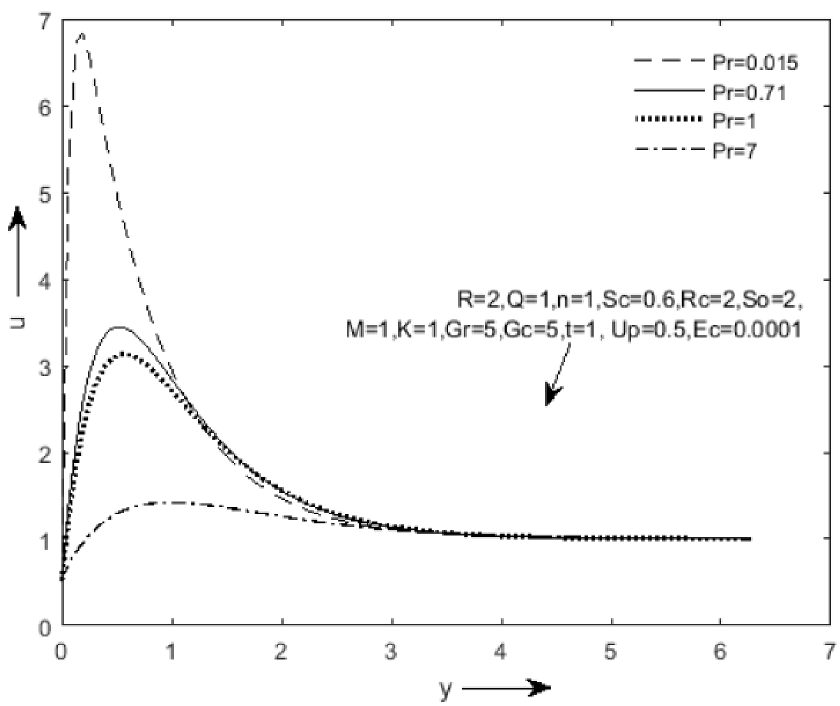


**Figure 3: Velocity Profiles for different values of** $\Pr$

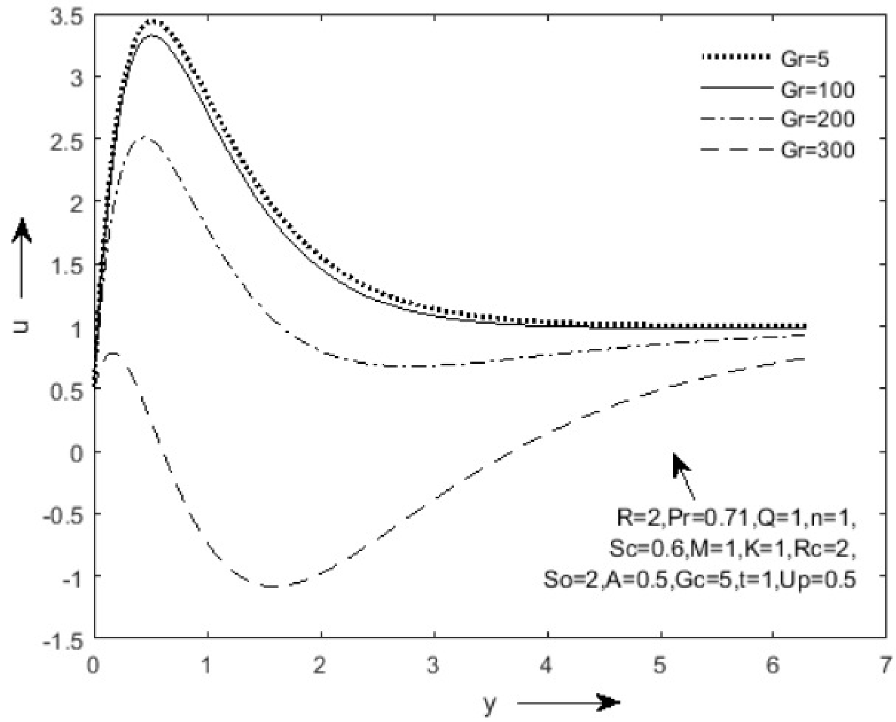


**Figure 4: Velocity Profiles for different values of** $Gr$

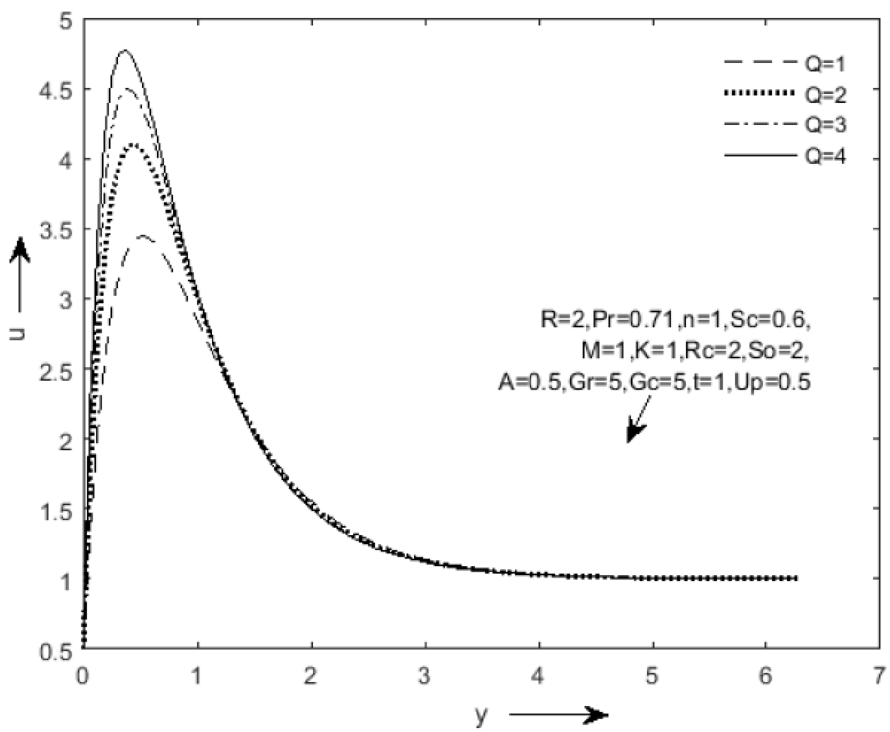


**Figure 5: Velocity Profiles for different values of** $Q$

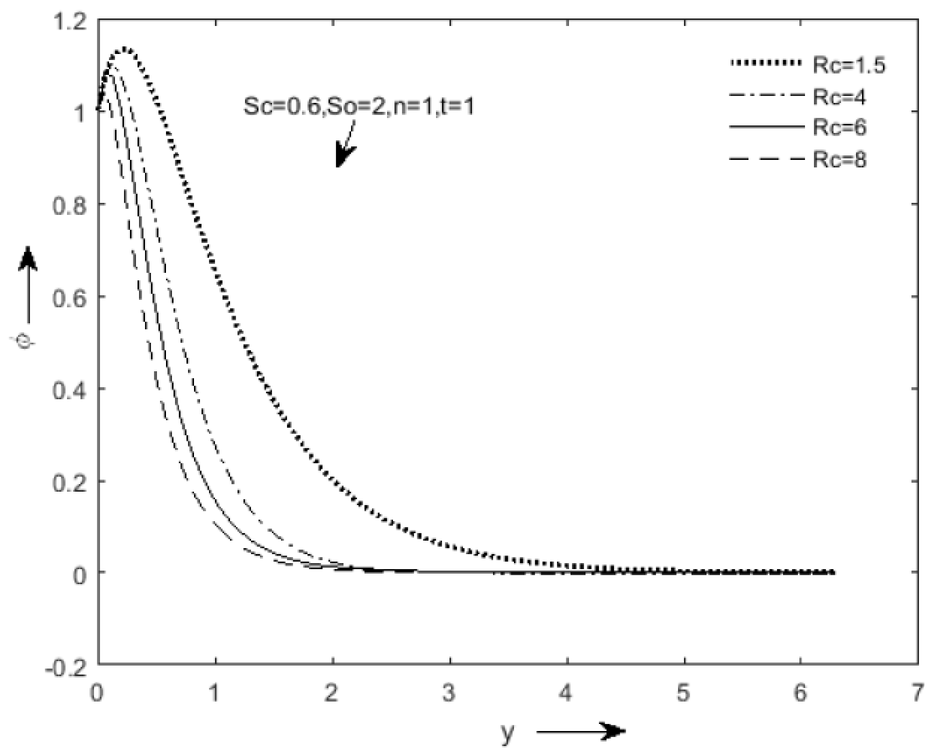


**Figure 6: Concentration Profiles for different values of** $Rc$

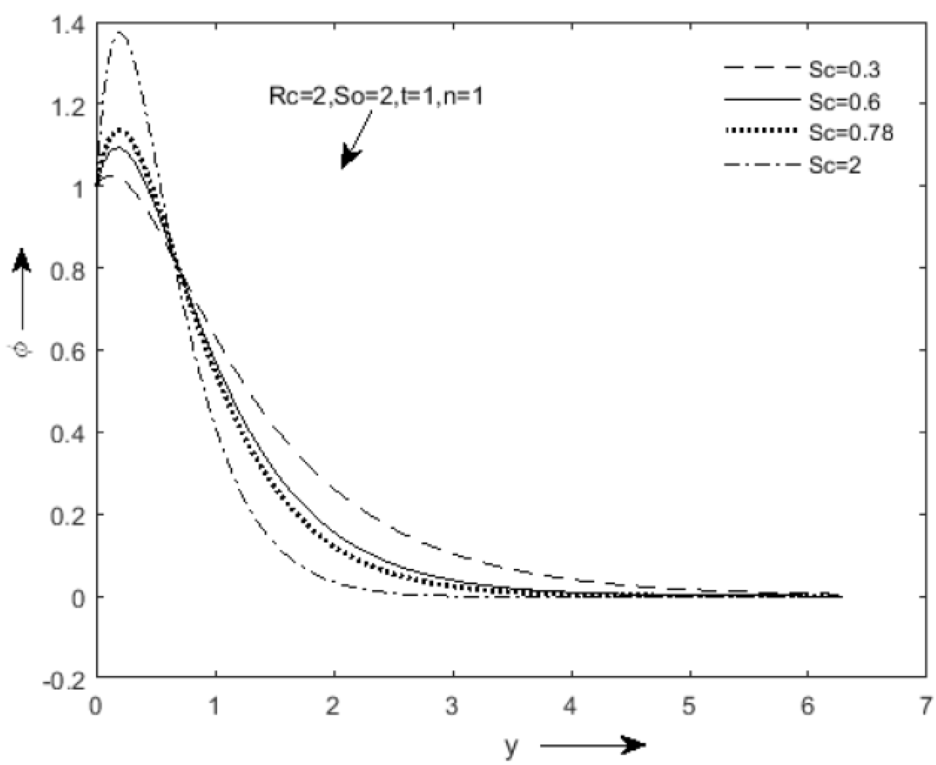


**Figure 7: Concentration Profiles for different values of** $Sc$

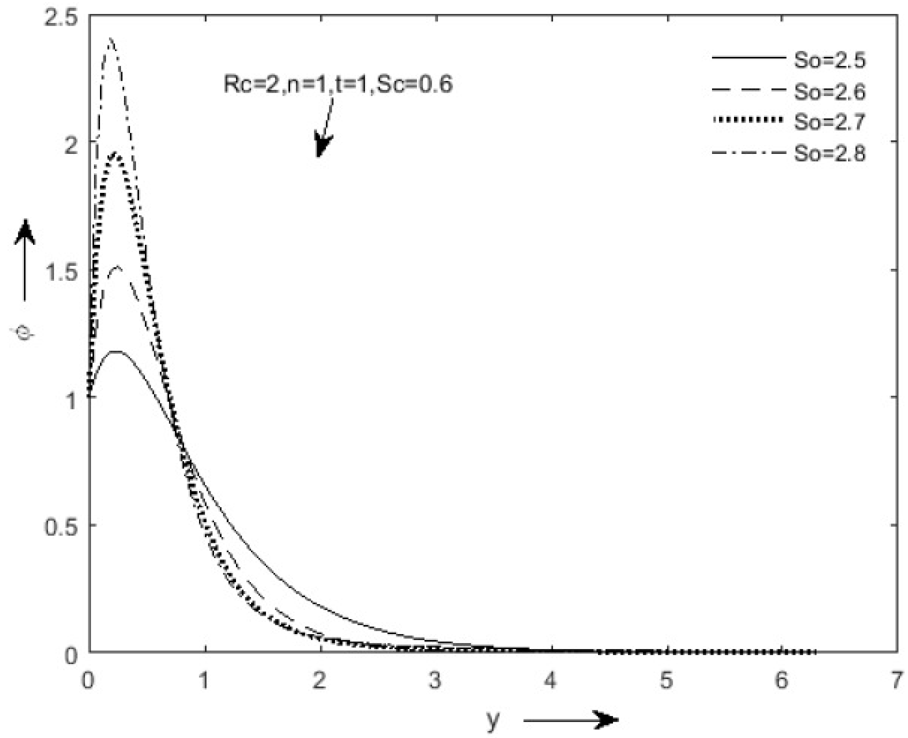


**Figure 8: Concentration Profiles for different values of** $So$

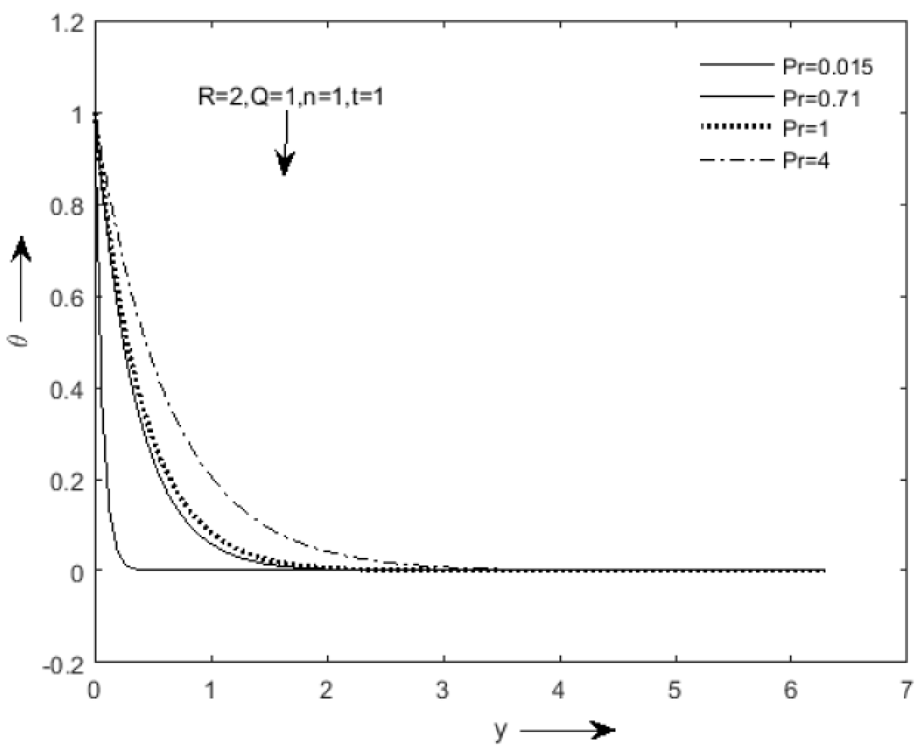


**Figure 9: Temperature Profiles for different values of** $\Pr$

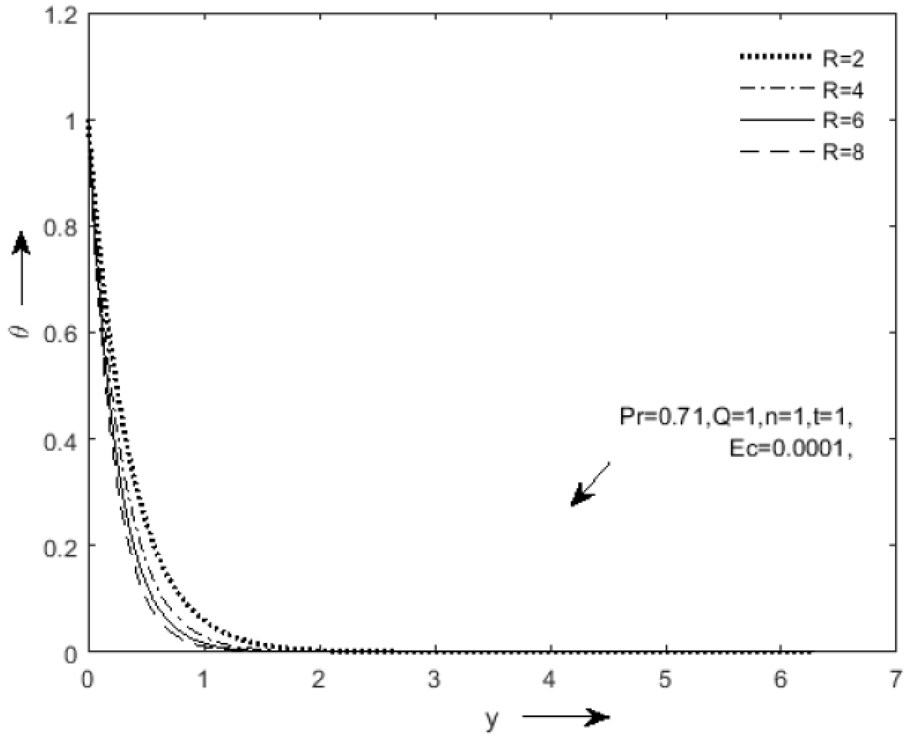


**Figure 10: Temperature Profiles for different values of $R$**

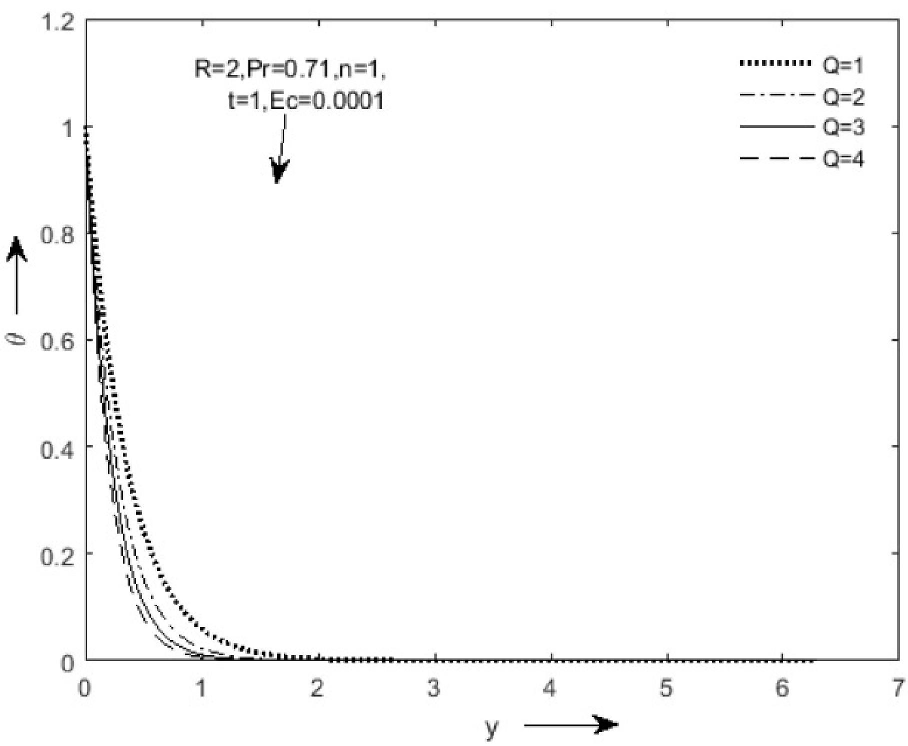


**Figure 11: Temperature Profiles for different values of $Q$**

In Fig.9 the temperature profiles are drawn against $y$ for different values of $\Pr$. It shows that values of $\Pr$ are directly proportional to the temperature. From the Figs. 10 and 11 it is noticed that temperature decreases with increasing values of $R$ and $Q$.

## 4. CONCLUSION

Here unsteady MHD free convection flow with viscous dissipation and chemical reaction is studied. Due to the addition of viscous dissipation, we find a non linearity in our energy equation. So we use the perturbation technique two times to solve the whole system. The different results are discussed through graphs. Finally some conclusions are drawn as follows.

- The parameters $Gc$, $Gr$ and $Rc$ enhance the velocity.
- The concentration decreases by the increment of chemical reaction parameter.
- The enhancing values of the parameters $R$ and $Q$ result in a decrease in the temperature. But $\Pr$ shows a reverse effect on temperature.
- In case of velocity $\Pr$ and $Q$ show the oscillatory effect.
- Similarly $Sc$ and $So$ show oscillatory effects in case of concentration.

## REFERENCES

[1]. Sapieszko, R.S. and Matijevi, E. (1980). Preparation of well-defined colloidal particles by thermal decomposition of metal chelates. I. Iron oxides, *J. Collo. Interf. Sci.*, 74(2), 405-422.

[2]. Taniguchi, S. and Kikuchi, A. (1992). Flow control of liquid iron by magnetic shield in a high frequency induction furnace, *Tetsu-to-Hagane*, 78(5), 753-760.

[3]. Forsberg, C.W. (2003). Hydrogen, nuclear energy and the advanced high-temperature reactor, *Int. J. Hydro. Energy*, 28(10), 1073-1081.

[4]. Yildiz, B. and Kazimi, M.S. (2006). Efficiency of hydrogen production systems using alternative nuclear energy technologies, *Int. J. Hydro. Energ*, 31(1), 77-92.

[5]. Gupta, A.S. (9160). Steady and transient free convection of an electrically conducting fluid from a vertical plate in the presence of a magnetic field, *Applied Scientific Research*, Vol. 9, No. 1, 319–333.

[6]. Singh, A.K. and Kumar, N. (1984). Free-convection flow past an exponentially accelerated vertical plate, *Astrophysics and Space Science*, Vol. 98, No. 2, 245–248.

[7]. Jha, B.K., Prasad, R. and Rai, S. (1991). Mass transfer effects on the flow past an exponentially accelerated vertical plate with constant heat flux, *Astrophysics and Space Science*, Vol. 181, No. 1, 125–134.

[8]. Ahmed, S.F., Das, M.K. and Ali, L.E. (2015). Analytical study on unsteady MHD free convection and mass transfer flow past a vertical porous plate, *American Journal of Applied Mathematics*, Vol.3, No.2,.64-74.

[9]. Chamkha, A.J. and Aly, A.M. (2010). Heat and mass transfer in stagnation point flow of a polar fluid towards a stretching surface in porous media in the presence of Soret, Dufour and chemical reaction effects, *Chem. Eng. Commun.*, 198 (2), 214–234.

[10]. Aurangzaib, A.R.M., Kasim, N.F. and Shafie, Mohammad S. (2012). Effect of thermal stratification on MHD free convection with heat and mass transfer over an unsteady stretching surface with heat source, Hall current and chemical reaction, *Int. J. Adv. Eng. Sci. Appl. Math.*, 4 (3), 217–225.

[11]. Abd El-Aziz, M. (2014). Effect of time-dependent chemical reaction on stagnation point flow and heat transfer over a stretching sheet in a nanofluid, *Phys. Sci.*, 89, Article ID 085205.

[12]. Pal, D. and Mandal, G. (2014). Influence of thermal radiation on mixed convection heat and mass transfer stagnation-point flow in nanofluids over stretching/shrinking sheet in a porous medium with chemical reaction, *Nuclear Eng. Design*, 273, 644–652.

[13]. Pal, D. and Mandal, H. (2011). MHD non-Darcian mixed convection heat and mass transfer over a non- linear stretching sheet with Soret–Dufour effects and chemical reaction, *Int. Commun. Heat Mass Transf.*, 38, 463–467.

[14]. Mahajan, R.L. and Gebhart, B.B. (1989). Viscous dissipation effects in Buoyancy – Induced flows, *Int. J. Heat Mass Transfer*, 32, 7, 1380 – 1382.

[15]. Israel – Cookey, C., Ogulu, A. and Omubo – Pepple, V.M. (2003). The influence of viscous dissipation and radiation on unsteady MHD free convection flow past an infinite heated vertical plate in a porous medium with time depedent suction, *Int. J. Heat Mass Transfer*, 46, 13, 2305 – 2311.

[16]. Zueco Jordan, J. (2007). Network Simulation Method Applied to Radiation and Dissipation Effects on MHD Unsteady Free Convection over Vertical Porous Plate, *Appl. Math. Modelling*, 31 , 20, 2019 – 2033.

[17]. Suneetha, S., Bhaskar Reddy, N. and Ramachandra Prasad, V. (2008). The thermal radiation effects

on MHD free convection flow past an impulsively started vertical plate with variable surface temperature and concentration, *Journal of Naval Architecture and Marine Engineering*, 2, 57– 70.

[18]. Suneetha, S., Bhaskar Reddy, N. and Ramachandra Prasad, V. (2009). Radiation and mass transfer effects on MHD free convection flow past an impulsively started isothermal vertical plate with dissipation, *Thermal Science*, 13 , 2, 71 – 181.

[19]. Kumar. Hitesh (2009). Radiative Heat Transfer with Hydro magnetic flow and viscous dissipation over a stretching surface in the presence of variable heat flux, *Thermal Science*, 13, 2, 163 – 169.