Original Article

Available online at
www.bpasjournals.com

Bulletin of Pure and Applied Sciences.
Vol. 38E (Math & Stat.), No.1, 2019. P.405-412
Print version ISSN 0970 6577
Online version ISSN 2320 3226
DOI: 10.5958/2320-3226.2019.00044.4

DYNAMICS OF DUST PARTICLES IN THE AMOSPHERE USING MATHEMATICAL MODELLING

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Received on 25.01.2019 Revised on 28.05.2019 Accepted on 06.06.2019

Abstract:
This paper aims at presenting exact solutions for velocity profile and skin friction coefficient for a fluid in the presence and absence of couple stress. Analytical method is carried out herewith an applied electric field. The results are presented through graphs in each case. Graphs are plotted by giving numerical values to the parameters to depict the effects of velocity and skin friction. It is shown that the velocity increases with the increase in electric number for both with and without couple stress and the skin friction coefficient decreases with increase in electric number in the absence of couple stress parameter.

Keywords: couple stress fluid, velocity profiles, atmospheric fluid, skin friction coefficient.

2010 Mathematics Subject Classification: 76W05, 76W99.

1. INTRODUCTION

Dust particles are materials in the air that can have adverse effects on human beings and eco- system. Dust in the air can aggravate air passages and lead to climate change, global warming, harmful effects on human health and environment. The behavior of air pollutants in the atmosphere is governed by the process of molecular diffusion and convection. It is mainly affected by various meteorological conditions such as wind, temperature inversion, foggy atmosphere, etc. This process is also affected by several removal mechanisms such as chemical transformation, dry deposition, greenbelt plantation, etc.[1]. Recently, theoretical analysis has been initiated to investigate the dynamic behavior of secondary aerosols by chemical reactions in atmosphere. Reactions of gases on aerosol surfaces play an important role in the chemistry of the troposphere.

Electric fields are used extensively in different industrial problems, particularly in those concerned with chemical, electrical, electronic and drug industries for various separation processes. Schmidt [30] performed experiments with different aerosols and observed that the application of an electric field had the effect of reducing sedimentation time. Liu and Lin [20], used a trajectory analysis for estimating the aggregation rates and found that the electric fields can enhance the gravitational settling of charged particles.

Atmospheric electricity abounds in the environment; some traces of it are found less than four feet from the surface of the earth, but on attaining greater height it becomes more apparent. Benjamin Franklin [3] was the first to design an experiment to prove the electrical nature of lightening. Lemonnier [19] discovered that even when there are no clouds, in the so-called fair weather condition, a weak electrification exists in the atmosphere. He also found some evidence that the electrification varied from the night to the day. The greater the altitude the more atmospheric electricity abounds. The presence of the earth's surface influences the concentration of ions, aerosols and radioactive particles, through its control over the wind, temperature and water vapor distributions. Several investigations are carried out on the problem of hydrodynamic flow of a viscous incompressible fluid considering various variations in the problem. Mention may be made of the studies of Greenspan and Howard [10,11], Holton [16], Hayat and Hutter [14]), and Guria et al., [12,13]. The problem of electrohydrodynamic flow of fluids is studied by many researchers viz. Ng et al.,[27,28,29], Haque and Arajs [15] .These studies have motivated to investigate the velocity profiles and skin friction of aerosols in a poorly conducting atmospheric fluid with and without couple stress through a horizontal channel under the effect of electric field. Malashetty et al., [21,22] have discussed the effect of rotation on the set of double diffusive convection in a Darcy porous medium saturated with a couple stress fluid. Das [5] and Das et al. [6] considered the effect of chemical reaction and thermal radiation on heat and mass transfer flow of MHD micropolar fluid. Bakr [2] studied the effects of chemical reaction on MHD free convection and mass transfer flow of a micropolar fluid. Cai-Wan Chang-Jian and Chao-Kuang Chen [4] investigated a dynamic analysis of the rub-impact rotor supported by two couple stress fluid film journal bearings. Gaikwad et al. [8] investigated an analytical study of linear and non-linear double diffusive convection with Soret and Dofour effects in couple stress fluid. The effect of surface roughness on the hydrodynamic lubrication of porous step slider bearings with couple stress fluids, has been studied by Naduvinamani and Siddangouda [24] and Naduvinamani et al. [25,26]. The objective of this paper is to present the mathematical model on dynamic behavior of atmospheric aerosols with and without couple stress under the influence of electric field. The rest of the paper is organized as follows: the second section deals with the mathematical formulation of the problem, the third section contains results and discussions, while the conclusions are given in the fifth section.

2. MATHEMATICAL FORMULATION

Consider a two dimensional geometry as shown in Figure 1. It consists of flow through a horizontal channel extended to infinity on both directions. The *x*-axis is taken along the walls and the *y*-axis perpendicular to it. The couple stress fluid is filled in the channel with embedded electrodes of different potentials at $y = 0$ and $y = h$.

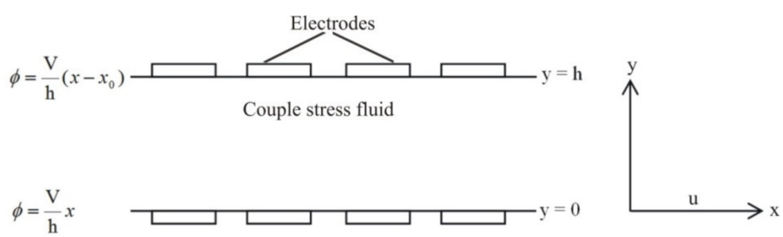

Figure 1: Physical Configuration

In this paper, we make the assumption that the electrical conductivity (σ) is negligibly small and hence the magnetic field is negligible. This assumption makes the electric field $\vec{E}$, to be conservative.

i.e.
$$\vec{E} = -\nabla \phi \tag{1}$$

The basic equations are as below:
The Conversation of mass for an incompressible flow

$$\nabla \cdot \vec{q} = 0 \tag{2}$$

The Conservation of momentum

$$\rho\left(\frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \nabla)\vec{q}\right) = -\nabla p + \mu \nabla^2 \vec{q} - \lambda \nabla^4 \vec{q} + \rho_e \vec{E} \tag{3}$$

where, λ is a couple stress parameter.

The Conservation of charges

$$\frac{\partial \rho_e}{\partial t} + (\vec{q} \cdot \nabla)\rho_e + \nabla \cdot \vec{J} = 0 \tag{4}$$

Maxwell's equation

$$\nabla \cdot \vec{E} = \frac{\rho e}{\in_0} \quad \text{(Gauss's law)} \tag{5}$$

$$\nabla \times \vec{E} = 0 \quad \text{(Faraday's law)} \tag{6}$$

$$\vec{J} = \sigma \vec{E} \quad \text{(Ohm's law)} \tag{7}$$

where $\vec{q}$ is the velocity, p the pressure, ρ the density, $\vec{E}$ the electric field, ρ_e the electric charge density, and $\vec{J}$ represents the current density.

The above equations are solved by using the following boundary conditions on velocity and potential ,

$$\left.\begin{array}{l} u = 0 \text{ and } T = T_1 \text{ at } y = b \\ u = 0 \text{ and } T = T_0 \text{ at } y = 0 \end{array}\right\} \tag{8}$$

$$\text{The couple stress condition, } \frac{d^2u}{dy^2} = 0 \text{ at } y = 0 \text{ and } y = b \tag{9}$$

$$\left.\begin{array}{ll} \varphi = \frac{V}{h} x \text{ at} & y = 0 \\ \varphi = \frac{V}{h}(x - x_0) \text{ at} & y = b \end{array}\right\} \tag{10}$$

The boundary condition on velocity represents the no- slip conditions of the couple stresses at the solid boundaries . In cartesian form, using the above approximation equation (3) becomes

$$0 = -\frac{\partial p}{\partial x} + \mu \nabla^2 u - \lambda \nabla^4 u + \rho_e E_x \tag{11}$$

In a poorly conducting fluid, the electrical conductivity is assumed to vary linearly with temperature and increases with temperature in the form

$$\sigma = \sigma_0 \left[1 + \alpha_h (T_b - T_0)\right] \tag{12}$$

where α_h is the volumetric coefficient of expansion and σ is the electrical conductivity.

We assume the flow is fully developed and unidirectional in the x – direction. This means the velocity is independent of time and all physical quantities except the pressure and concentration are independent of *x*, so that the velocity and temperature will be functions of *y* only. Using the following dimensionless quantities,

$$y* = \frac{y}{h}; \quad u* = \frac{u}{\frac{\nu}{h}} \; E_x^* = \frac{E_x}{\frac{V}{h}}; \; \rho_e^* = \frac{\rho_e}{\frac{\varepsilon_0 V}{h^2}}; \; x* = \frac{x}{h}; \tag{13}$$

where V is electric potential, we get electric potential through the electrodes.

We assume that the fluid with pollutants is isotropic and homogenous so that molecular diffusivity D, viscosity μ are all constants.

$$\frac{d^4u}{dy^4} - a^2 \frac{d^2u}{dy^2} - a^2 W_e P_e \vec{E}_x = a^2 P \tag{14}$$

where $W_e = \frac{\varepsilon_0 V^2}{\mu}, l = \sqrt{\frac{\lambda}{\mu}}, a = \frac{h}{l}$ is the couple stress parameter.

Equation (7) becomes,

$$\nabla \cdot \vec{J} = 0 \tag{15}$$

Using equation (1) we get,

$$\sigma(\nabla^2\phi) + \nabla\phi \cdot \nabla\sigma = 0 \tag{16}$$

The boundary conditions on velocity, couple stress, temperature and electric potential after dimensionless are

$$u = 0 \text{ at } y = 0,1 \tag{17}$$

$$\frac{d^2u}{dy^2} = 0 \text{ at } y = 0,1 \tag{18}$$

$$\left.\begin{array}{l}\theta = 0 \text{ at } y = 0\\ \theta = 0 \text{ at } y = 1\end{array}\right\} \tag{19}$$

$$\left.\begin{array}{ll}\varphi = x \text{ at} & y = 0\\ \varphi = x - x_0 \text{ at} & y = 1\end{array}\right\} \tag{20}$$

In a poorly conducting fluid, σ <<1 and hence any perturbation on it is negligible thus it depends on the conduction temperature T_b namely,

$$\frac{d^2T_b}{dy^2} = 0 \tag{21}$$

with the boundary conditions

$$\left.\begin{array}{l}T_b = T_0 \text{ at } y = 0\\ T_b = T_1 \text{ at } y = b\end{array}\right\} \tag{22}$$

is $\quad T_b - T_0 = \Delta Ty \quad$ (23)

Therefore equation (16) becomes $\sigma_0[1+\alpha_h \Delta Ty] = \sigma_0(1+\alpha y) = \sigma_0 e^{\alpha y}$

$$\sigma \approx e^{\alpha y} \tag{24}$$

where $\alpha = \alpha_h \Delta T$.

Then from (16), on using (24) in it, we get

$$\frac{d^2\phi}{dy^2} + \alpha\frac{d\phi}{dy} = 0 \tag{25}$$

Its solution satisfying the boundary condition (20) is

$$\phi = x - \frac{x_0}{1-e^{-\alpha}}\left[1 - e^{-\alpha y}\right] \tag{26}$$

Using the dimensionless quantities and equation (25), equations (5), (6) and (7) reduce to

$$\rho_e = \nabla.\vec{E} = -\nabla^2\phi = -\frac{x_0\alpha^2 e^{-\alpha y}}{1-e^{-\alpha}}\ ;\ E_x = -1$$

Therefore,

$$\rho_e E_x = \frac{x_0\alpha^2 e^{-\alpha y}}{1-e^{-\alpha}} \tag{27}$$

We consider two cases. First case in the presence of couple stress and the second without couple stress.

Case 1: $a \neq 0$(with couple stress)

The solution of equation (14) satisfying the conditions (17) and (18) is

$$u = k_1 k_2 y - k_3 k_2 y + k_4\left(y - y^2\right) - k_1 + k_1 e^{ay} + k_3 - k_3 e^{ay} - k_5 + k_5 e^{ay} - k_6 2\sinh ay \tag{28}$$

where,

$$k_1 = \left(\frac{a_0}{\alpha^2(\alpha^2 - a^2)}\right); k_2 = \left(1 - e^{-\alpha}\right); k_3 = \left(\frac{a_0}{a^2(\alpha^2 - a^2)}\right); k_4 = \frac{P}{2};$$

$$k_5 = \frac{P}{a^2}; \quad k_6 = \frac{1}{2(\sinh a)}\left(\frac{a_0(e^{-\alpha} - e^{a})}{a^2(\alpha^2 - a^2)} + \frac{a^2 P(e^{a} - 1)}{a^4}\right);$$

Case 2: a= 0 (without couple stress)

To find the velocity from equation (14) satisfying the conditions (17) and (18) is

$$u = \frac{a_0}{\alpha^2} e^{-\alpha} y + \frac{P}{2} y^2 + \frac{a_0}{\alpha^2} y - \frac{P}{2} y - \frac{a_0}{\alpha^2} e^{-\alpha} y - \frac{a_0}{\alpha^2} \tag{29}$$

where $a_0 = \dfrac{w_e x_0 \alpha^2}{e^{-\alpha} - 1}$.

The analytical results for both with and without couple stress are depicted graphically through figures.

To find the skin friction:

In many practical applications it is advantageous to know the skin friction at the boundaries. This can be determined once we know the velocity. The skin friction τ at the walls is defined as

$$\tau = \mu\left(\frac{du}{dy}\right)_{y=0 \text{ and } h}$$

making this dimensionless using the scale for u used earlier we get,

$$\tau = \left(\frac{du}{dy}\right)_{y=0 \text{ and } 1}$$

where $\dfrac{du}{dy}$ can be obtained using (28) and (29).

The analytical results for both cases (with and without couple stress) are represented through figures.

3. RESULTS AND DISCUSSIONS

A mathematical modelling of velocity profiles and the skin friction of atmospheric aerosols with and without couple stress under the influence of electric field is discussed. The graphical illustrations are as follows.

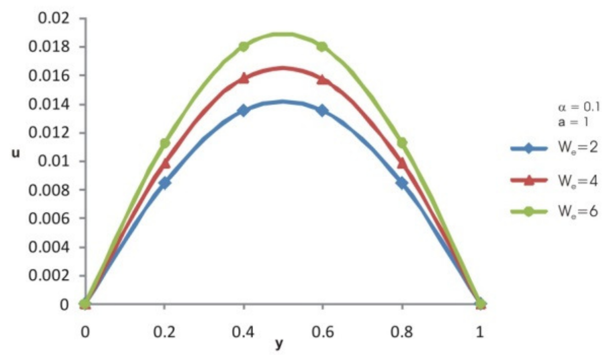

Figure 2: Velocity profiles *u* with *y* for different values of electric values of the electric number in the presence of couple stress.

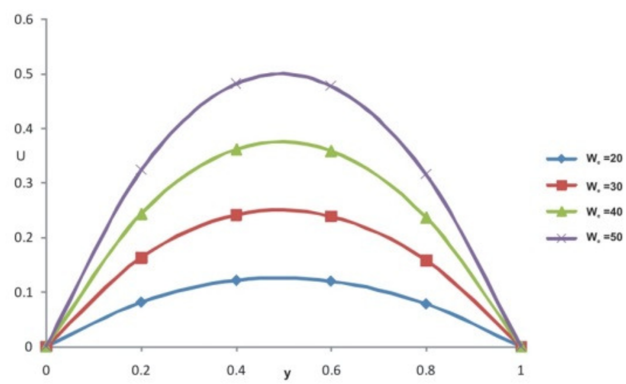

Figure 3: Velocity profiles *u* with *y* for different values of electric number in the absence of couple stress.

Figures 2 and 3 above represent the effects of electric number on the velocity profiles of aerosols in the presence and absence of couple stress respectively. It is observed that the velocity increases with the increase in electric number for both with and without couple stress.

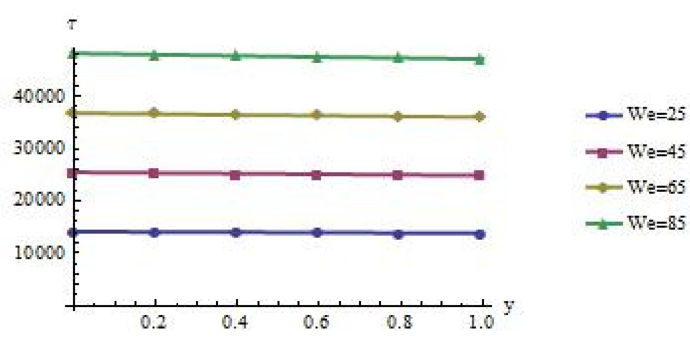

Figure 4: Skin friction coefficient τ with *y* for different values of electric number in the presence of couple stress

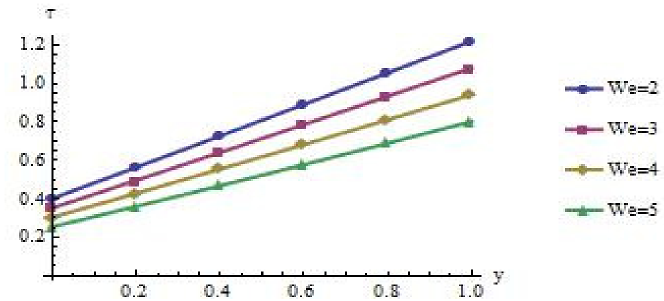

Figure 5: Skin friction coefficient τ with *y* for different values of electric number in the absence of couple stress.

The coefficient of skin friction on the lower and upper surface for different values of electric number are illustrated through the Figures 4and 5. Figure 4 depicts the effects of couple stress on skin friction coefficient with *y*. It is seen that the skin friction coefficient increases with increase in couple stress parameter. From the Figure 5 it is found that the skin friction coefficient decreases with increase in electric number in the absence of couple stress parameter.

5. CONCLUSION

An attempt to obtain the exact solutions of velocity and skin friction coefficient of the fluid in the presence and absence of couple stress fluid is made. The methodology in the present work lends and easy procedure to obtain the results in the fluid flow for couple stress. Some good results are obtained and it is observed that the velocity and skin friction coefficient increases under the influence of electric field in the presence of couple stress but in the absence of couple stress, the skin friction coefficient decreases. The above mathematical model is the representative of effects of electric field and couple stress on the dynamic behavior of aerosols in the atmosphere.

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