

DISPERSION OF SOLUTE WITH CHEMICAL REACTION IN BLOOD FLOW

Nirmala P. Ratchagar¹, R.VijayaKumar^{2,*}

Authors Affiliation:

¹Department of Mathematics, Annamalai University, Annamalai Nagar, Tamil Nadu 608002, India.
E-mail: nirmalapasala@yahoo.co.in

²Mathematics Section, Faculty of Engineering and Technology, Annamalai University,
Annamalai Nagar, Chidambaram, Tamil Nadu 608002, India.
Department of Mathematics, Periyar Govt. Arts College, Cuddalore 607001, India

*Corresponding Author:

R. VijayaKumar, Mathematics Section, Faculty of Engineering and Technology, Annamalai
University, Annamalai Nagar, Chidambaram, Tamil Nadu 608002, India.

E-mail: rathirath_viji@yahoo.co.in

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Abstract:

A mathematical model is developed to study the influence of an externally applied magnetic field and chemical reaction on the flow characteristics of blood in the presence of mild stenosis. The equations of momentum are solved under appropriate boundary conditions using Hankel transform. Taylor's dispersion model [17] is used to obtain dispersion of solute in blood flow. The effects of various parameters entering into the problems are discussed numerically and explained graphically.

Keywords: Chemical reaction, magnetic field, Taylor dispersion model.

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1. INTRODUCTION

Many researchers considered blood as a homogeneous medium in order to simplify the analytical treatment of the transport problems. Substrates within the capillary are often transported in two forms (i) those that dissolve in the plasma and (ii) those that bound to some transporting protein. For example, oxygen and carbon dioxide bind to the hemoglobin of red cells, and serum albumin is a carrier for many small molecules. Only the substrates which are dissolved in plasma are available for immediate transfer across the capillary wall and into the surrounding tissue. These are in kinetic equilibrium with binding protein throughout the macrocirculation and also in parts in microcirculation. Thus, loss of substrates due to diffusion in tissue creates a kinetic imbalance and initiate release from binding protein.

The study of blood flow through arteries is of considerable importance in many cardiovascular diseases particularly arteriosclerosis (stenosis) which are responsible for the death of people, which are closely related to the nature of blood movement and the dynamic behavior of blood vessel. Flow and diffusion through capillary tissue exchange system has been identified as one of the thrust areas of research in the last decades. In narrow capillary, at times, the radial transport becomes much larger as compared to the axial transport and it contributes to the development of atherosclerotic deposition. The problem of flow and diffusion becomes much more difficult through a capillary with stenosis at some region. From medical survey it is a well known fact that more than eighty percent of the total deaths of human beings are due to the diseases of blood vessel walls.

Many authors [4,5,6,7] proposed various representative models for blood in narrow capillaries. Tandon et al. [15] have developed a model consisting of the viscous fluid representation which is identically same as the suspending medium of the blood. Other models have also been proposed to discuss the nutrition transport in capillaries, but no work has been undertaken except Tandon et al. [16] in very narrow capillary. Shukla et al. [12, 13] have studied the effect of stenosis on the resistance to blood flow through artery by considering the behavior of blood as a power law fluid a Casson fluid. Srivastava and Saxena [14] presented the effect of stenosis on the blood flow, when blood is a two-layered model satisfying a core region of suspension of all erythrocytes assumed to be a Casson fluid and the peripheral layer of plasma as a Newtonian fluid.

Layek and Mukhopadhyay [10] and Kumar et al. [9] worked on a mathematical model to study the flow through a variable shape stenosed artery under the influence of magnetic field and demonstrated the effect of stenosis shape and magnetic field on the resistance to the flow. Haldar and Ghosh [7], Mekheimer and El Kot[11] mathematically analyzed the steady blood flow through a multi-stenotic artery under the effects of magnetic field and porosity. Eldesoky [8] studied the slip effect of unsteady magnetic field on pulsatile blood flow through porous medium in an artery under the effect of body acceleration. Bali and Awasthi [1] studied the Newtonian blood flow through the tapered arteries. Tripathi[18] investigated a mathematical model for blood flow through an inclined artery under the influence of an inclined magnetic field. Bhatnagar et. al.[2] worked on a numerical analysis for the effect of slip velocity and stenosis shape on Non-Newtonian flow of blood. Bunony et. al.[3] investigate an oscillatory blood flow in an indented artery with a heat source in the presence of magnetic field.

The main objective of this paper is to study the dispersion of solute following the Taylor's dispersion model[17]. In the present work, the effect of an externally applied magnetic field with chemical reaction over the flow characteristics in an inclined circular channel to study the nutritional transport of blood from capillary to tissues in mild stenosed arteries is considered.

2. MATHEMATICAL FORMULATION

The blood flow is modeled to be steady, laminar and the nature of blood is incompressible, homogenous flowing in the z direction through an inclined stenosed circular artery. The physical model of the problem is shown in Figure 1. The blood flowing in the tube is assumed to be a suspension of red blood cells in the plasma.

In deriving the governing equation and the corresponding boundary conditions the following assumptions are made:

- (i) The flow is Newtonian, viscous and unidirectional.
- (ii) Stenosis developed in the artery in an axially symmetric manner and depends upon the axial distance and the height of its growth.
- (iii) The maximum height of the stenosis is much less as compared to the length and unobstructed radius of the artery i.e., stenosis is mild.
- (iv) Radial velocity in the stenotic region is very small in comparison to the axial velocity.
- (v) A uniform magnetic field is applied normal to the direction of the blood flow.
- (vi) Longitudinal diffusion is much less than the transverse diffusion, which implies

$$\frac{\partial^2 C'}{\partial z'^2} \ll \left(\frac{\partial^2 C'}{\partial r'^2} + \frac{1}{r'} \frac{\partial C'}{\partial r'} \right)$$

Using the above assumption the governing equations for incompressible flow of Newtonian fluid in cylindrical coordinates are

Continuity equation

$$\frac{\partial u_z}{\partial z'} = 0$$

Momentum equations are

$$\rho g \sin \alpha = -\frac{\partial p'}{\partial z'} - \frac{1}{r'} \frac{\partial}{\partial r'} (r' \tau_{rz}) - B_0^2 \sigma_0 u - \frac{\mu}{\kappa'} u \quad (1)$$

$$0 = \frac{\partial p'}{\partial r'} \quad (2)$$

The concentration equation for the solute is expressed as follows:

$$\frac{\partial C'}{\partial t} + u \frac{\partial C'}{\partial z'} = D \left(\frac{\partial^2 C'}{\partial r'^2} + \frac{1}{r'} \frac{\partial C'}{\partial r'} \right) - K_1 C' \quad (3)$$

where, ρ is the fluid density, g is the acceleration due to gravity, k' is the permeability of porous medium, u is the axial(z) component of the velocity, p' is the pressure, $B_0 (= \mu' H_0)$ the electro-magnetic induction, μ' the magnetic permeability, H_0 the intensity of magnetic field, σ_0 is the conductivity of fluid, C' represents the concentration of the solute and D the diffusion coefficient for the solute under consideration in the blood.

The constitutive equation of a Newtonian fluid (blood) is given by

$$\tau_{rz} = \mu \frac{\partial u}{\partial r'} \quad (4)$$

where, μ is the coefficient of viscosity of blood.

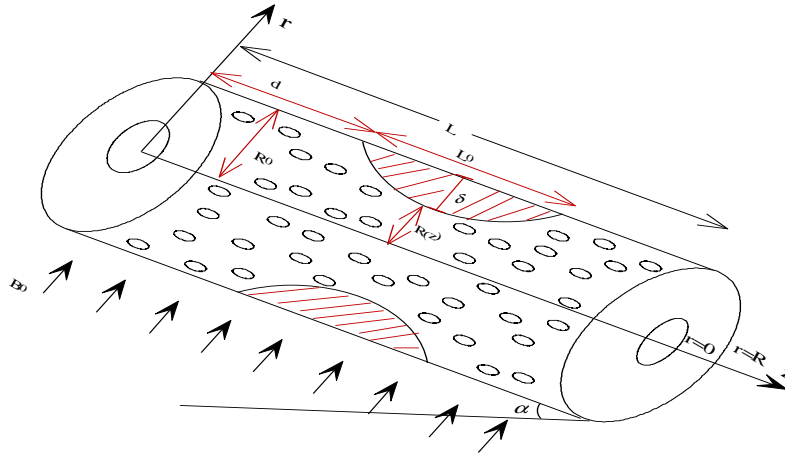


Figure 1: Geometry of an inclined artery with stenosis.

Stenosis geometric is idealized in the typical fashion by a cosine expression as (Tandon et al. [15, 16])

$$R'(z') = \begin{cases} R_0 - \frac{\delta'}{2} \left[1 + \cos \frac{2\pi}{L_0'} \left(z' - d' - \frac{L_0'}{2} \right) \right] & \text{if } d' < z' < d' + L_0' \\ R_0, & \text{otherwise} \end{cases} \quad (5)$$

where, $R'(z')$ is the radius of the constricted region, L_0' is the length of stenosis, d indicates its location and

$$\frac{\delta'}{R_0} \ll 1.$$

To solve the equations (1)-(3), the following boundary conditions are introduced

$$\frac{\partial u}{\partial r'} = 0 \quad \text{at} \quad r' = 0 \quad (6)$$

$$u = 0 \quad \text{at} \quad r' = R'(z') \quad (7)$$

$$\frac{\partial C'}{\partial r'} = 0 \quad \text{at} \quad r' = 0 \quad (8)$$

$$D \frac{\partial C'}{\partial r'} = VNC' \quad \text{at} \quad r' = R'(z') \quad (9)$$

where, N is the retention parameter.

Introducing the nondimensional variables as follows

$$U = \frac{u'}{u_0}, r = \frac{r'}{R_0}, z = \frac{z'}{R_0}, p = \frac{p'}{\rho u_0^2}, \text{Re} = \frac{\rho u_0 R_0}{\mu}, R = \frac{R'}{R_0}, F_r = \frac{u_0^2}{g R_0}, k = \frac{k'}{R_0},$$

$$M^2 = \frac{B_0^2 R^2 \sigma}{\mu}, \beta^2 = \frac{R_0^2 K_1}{D}$$

Using the non-dimensional variables, equation (1) reduces to

$$\frac{\partial^2 U}{\partial r^2} + \frac{1}{r} \frac{\partial U}{\partial r} - \left(M^2 + \frac{1}{k} \right) U = Q \quad (10)$$

where, $Q = \text{Re} \left(\frac{\sin(\alpha)}{F_r} - p \right)$, M^2 is the square of Hartmann number, Fr is the Froude number, Re is the

Reynolds number.

The boundary conditions (6) and (7) in dimensionless form can be written as

$$U = 0 \quad \text{at} \quad r = R(z) \quad (11)$$

$$\frac{\partial U}{\partial r} = 0 \quad \text{at} \quad r = 0 \quad (12)$$

The geometry of the stenosis in dimensionless form is given by

$$R(z) = \begin{cases} 1 - \frac{\delta}{2R_0} \left[1 + \cos \frac{2\pi}{L_0} \left(z - d - \frac{L_0}{2} \right) \right] & \text{if } d < z < d + L_0 \\ 1, & \text{otherwise} \end{cases} \quad (13)$$

To solve equation (3), we use the following non-dimensional variables:

$$r = \frac{r'}{R_0}, C = \frac{C'}{C_0}, \xi = \frac{z - Ut}{L}$$

Hence the equation (3) becomes,

$$\frac{1}{t'} \frac{\partial C}{\partial t} + \frac{\gamma}{L} \frac{\partial C}{\partial z} = \frac{D_1}{R_0^2} \left(\frac{\partial^2 C}{\partial r^2} + \frac{1}{r} \frac{\partial C}{\partial r} \right) - K_1 C \quad (14)$$

If the Taylor's longitudinal conditions are valid in this problem, the partial equilibrium may be assumed at any cross-section of the artery and the variation in C with r is obtained from equation (14) takes the form,

$$\frac{\partial^2 C}{\partial r^2} + \frac{1}{r} \frac{\partial C}{\partial r} = \frac{\gamma R_0^2}{D_1 L} \frac{\partial C}{\partial \xi} - \beta^2 C \quad (15)$$

with the boundary conditions:

$$\frac{\partial C}{\partial r} = 0 \quad \text{at} \quad r = 0 \quad (16)$$

$$D \frac{\partial C}{\partial r} = VNC \quad \text{at} \quad r = R(z) \quad (17)$$

3. METHOD OF SOLUTION

3.1. Velocity Distribution

Solving equation (10) with the boundary conditions (11) and (12), we get

We define Hankel transform as,

$$\begin{aligned}
 U^* &= \int_0^{R(z)} U(r) r J_0(s_i r) dr \\
 &= \frac{QR J_1(s_i R)}{s_i \left(s_i^2 + M^2 + \frac{1}{k} \right)}
 \end{aligned} \quad (18)$$

Taking the inverse Hankel transform, we obtain the velocity distribution,

$$U = \frac{2Q}{R} \sum_{i=1}^{\infty} \frac{J_0(s_i r)}{s_i \left(s_i^2 + M^2 + \frac{1}{k} \right) J_1(s_i R)} \quad (19)$$

The average velocity is given by

$$\bar{U} = \frac{2}{R^2} \int_0^{R(z)} r U dr = \frac{4Q}{R^2} \sum_{i=1}^{\infty} \frac{1}{s_i^2 \left(s_i^2 + M^2 + \frac{1}{k} \right)}$$

Relative velocity γ as

$$\gamma = U - \bar{U} = \frac{2Q}{R^2} \sum_{i=1}^{\infty} \frac{1}{s_i \left(s_i^2 + M^2 + \frac{1}{k} \right)} \left(\frac{J_0(s_i r)}{J_1(s_i R)} - \frac{2}{R s_i} \right) \quad (20)$$

3.2. Concentration Distribution

To solve the equation (15), we use the boundary conditions (16) and (17), hence, we get

$$C = \frac{4QDQ}{R^3} \sum_{i=1}^{\infty} \left[\frac{1}{RVNJ_0(s_i R) - D_1 s_i^2 - D_1 \beta^2} \sum_{i=1}^{\infty} \frac{J_1(s_i R)}{s_i^2 \left(s_i^2 + M^2 + \frac{1}{k} \right)} \left(\frac{R_0^2}{2} - \frac{2}{s_i^2} \right) \right] \frac{J_0(s_i r)}{J_1(s_i R)}$$

$$\text{where, } Q = \frac{R_0^2}{D_1 L} \frac{\partial C}{\partial \xi}.$$

The volumetric rate at which the solute is transported across a section of the artery of unit breadth is

$$M = 2\pi R_0^2 \int_0^{R(z)} C \gamma r dr. \quad (21)$$

Following Taylor [17], we assume that the variations of C with r are small compared with those in the

longitudinal direction and if C_m is the mean concentration over a section, $\frac{\partial C}{\partial \xi}$ is indistinguishable from $\frac{\partial C_m}{\partial \xi}$

so that (21) can be written as

$$M = K_1^f \frac{\partial C_m}{\partial \xi} \quad (22)$$

$$\text{where, } K_1^f = 2\pi R_0^2 \int_0^{R(z)} C_1 \gamma r dr.$$

The fact that no material is lost in the process is expressed by the continuity equation for C_m

Namely,

$$\frac{\partial C}{\partial \xi} = - \frac{\partial C_m}{\partial t} \quad (23)$$

where $\frac{\partial}{\partial t}$ represents differentiation with respect to time at point, and ξ is constant. Using (22), equation (23), becomes

$$\frac{\partial C_m}{\partial t} = D^* \frac{\partial^2 C_m}{\partial \xi^2} \quad (24)$$

which is the equation governing the longitudinal dispersion. Equation (24) implies that C_m is dispersed relative to a plane which moves with average velocity exactly as though, it has been diffused by a process which obeys the same law as the molecular diffusion but with a relative diffusion coefficient.

4. RESULTS AND DISCUSSION

The results of the analysis for different values of the Hartmann number (M), stenosis height (δ) and Reynolds number (Re) for velocity, concentration and dispersion coefficient are obtained analytically and the numerical values have been computed in the figures 2 to 11 using MATHEMATICA 8.0. The analysis for normal and diseased inclined artery with chemical reaction associated with stenosis due to the local deposition of lipids which help in identification, diagnosis and treatment of many cardiovascular disorders.

Figures 2 to 5 shows that the variation of the velocity with axial distance r for different values of Hartmann number (M), stenosis height (δ), Reynolds number (Re) and permeability respectively. Figure 2 depicts that, when the Hartmann number increases for the fixed value of $\delta = 0.5$ and $Re=0.2$, the blood flow decreases and the profiles get flatter and approach those of the plug flow. From the Figures 3, 4 and 5, we observe that the velocity profile is parabolic and the velocity of the blood flow increases with increase in stenosis height, Reynolds number and permeability of porous medium. Velocity profiles are similar in stenotic and non-stenotic regions, but the value is greater at the stenotic region than compared with the non-stenotic region. Figures 6 and 7 depict the concentration with r for different values of Hartmann number (M) and chemical reaction rate coefficient (β). Clearly, the concentration in the capillary region decreases with increase in the Hartmann number (M) and chemical reaction rate coefficient (β).

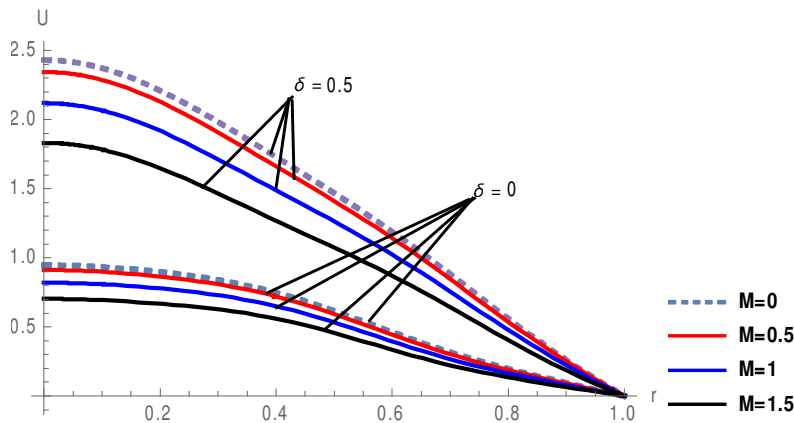


Figure 2: Effects of Hartmann number M on velocity profile.

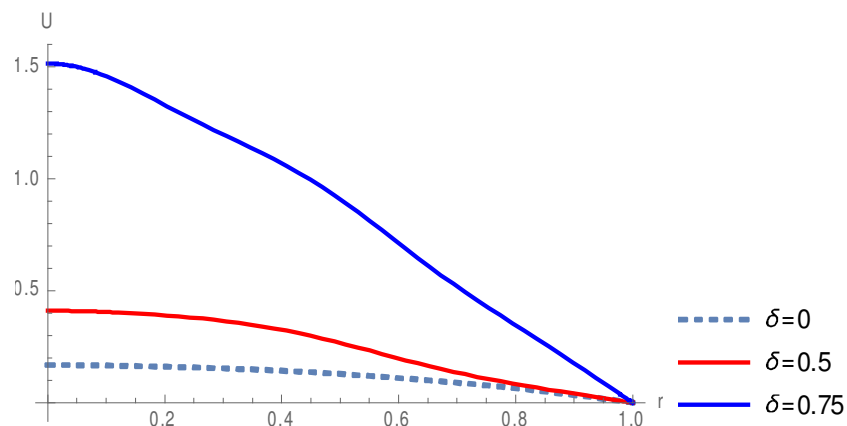


Figure 3: Variation of velocity (U) with radial axis (r) for different values of height stenosis δ .

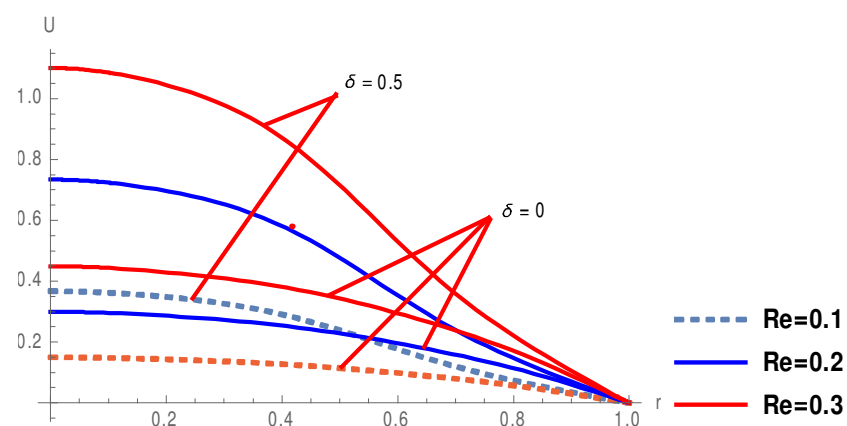


Figure 4: Variation of axial velocity (U) with radial axis (r) and height of stenosis (δ) for different values of Reynolds number (Re).

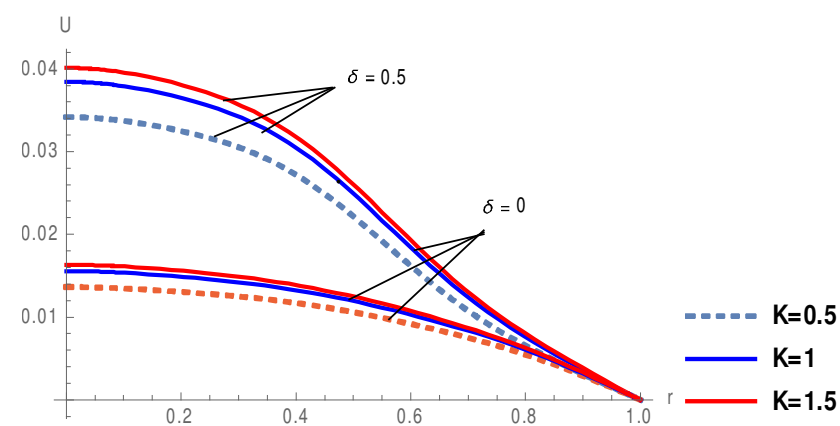


Figure 5: Variation of axial velocity (U) with r and height of stenosis (δ) for different values of permeability (K).

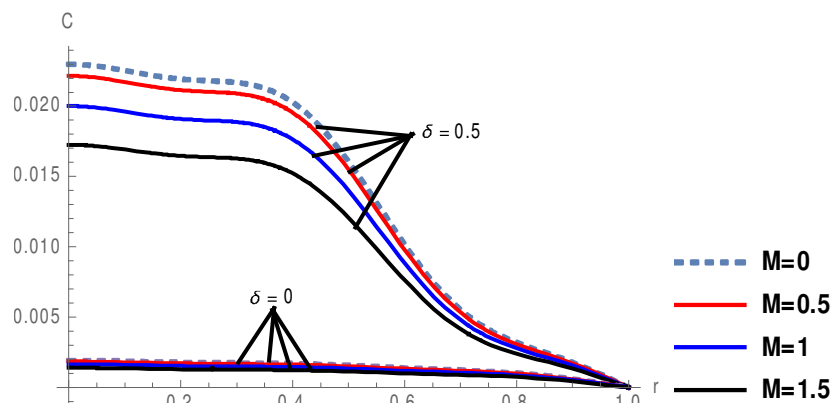


Figure 6: Concentration distribution versus r for different values of M .

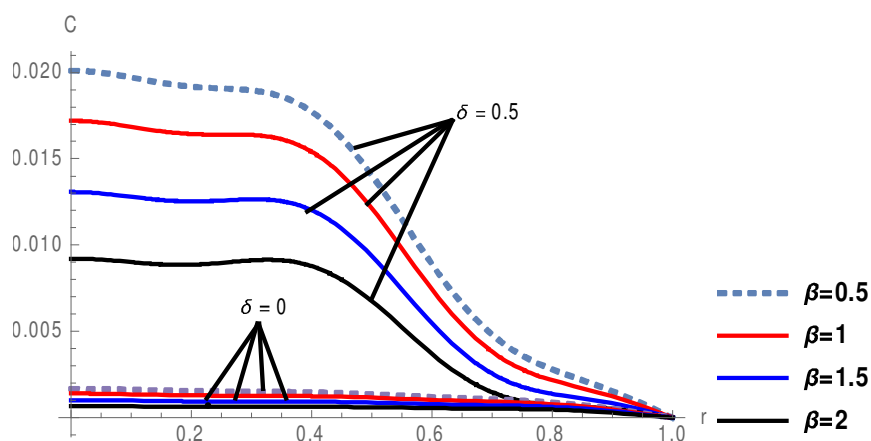


Figure 7: Concentration distribution versus r for different values of β .

The diffusion of large and small molecular weight nutrients with the capillary for different values of Re and δ is also studied. The large molecular weight nutrients face more resistance within the capillary region to diffuse into the tissue and therefore the cells of the deeper regions are deprived from getting sufficient nutrition. Figures 8 and 9 are the plots for dispersion coefficient D^* against Re and M for different values of Hartmann number (M) and chemical reaction rate coefficient (β) in the stenotic and non-stenotic regions. It is clear that D^* decreases with increase in the Hartmann number (M) and chemical reaction rate coefficient (β). Figures 10 and 11 represent the D^* in capillary region with M for different values of δ and Re . It is observed that D^* increases with increase in δ and Re . It is similar in stenotic and non-stenotic region. The above results are very useful for analysis so that the regions of the artery which are affected with stenosis get the proper and sufficient nutrients in the stenotic region.

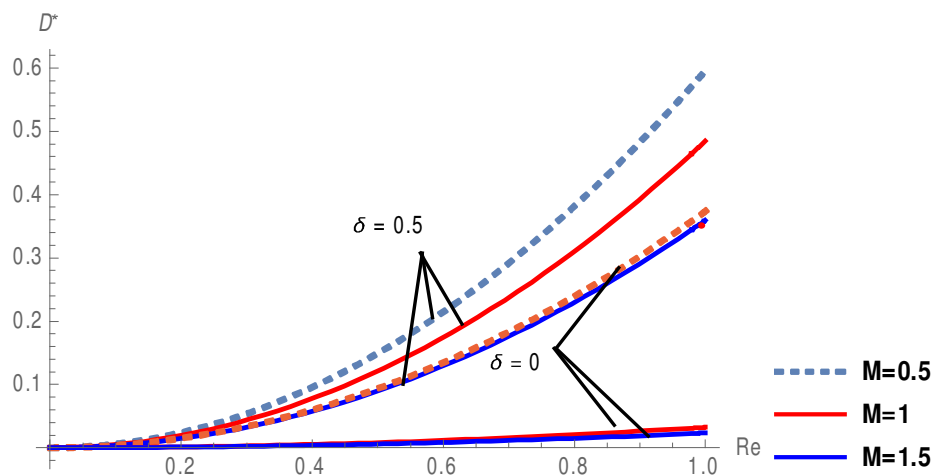


Figure 8: Dispersion coefficient D^* versus Reynold's number for different values Hartmann number M .

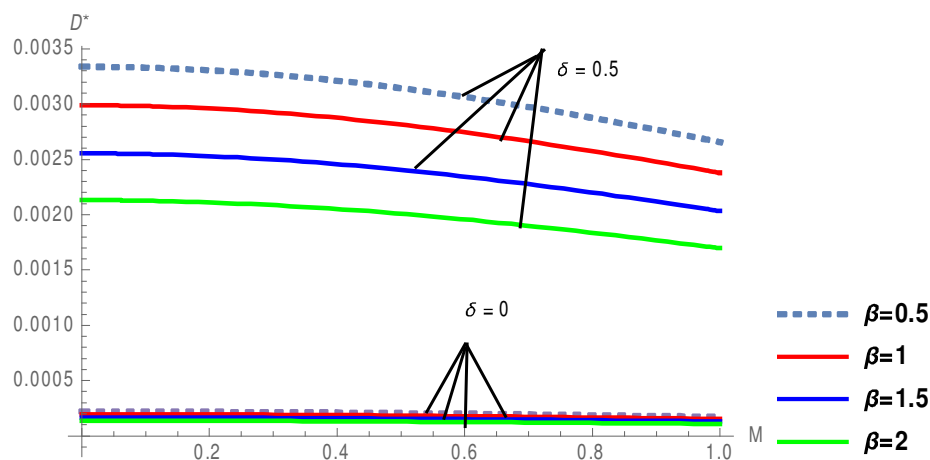


Figure 9: Dispersion coefficient D^* versus Hartmann number M for different values reaction rate.

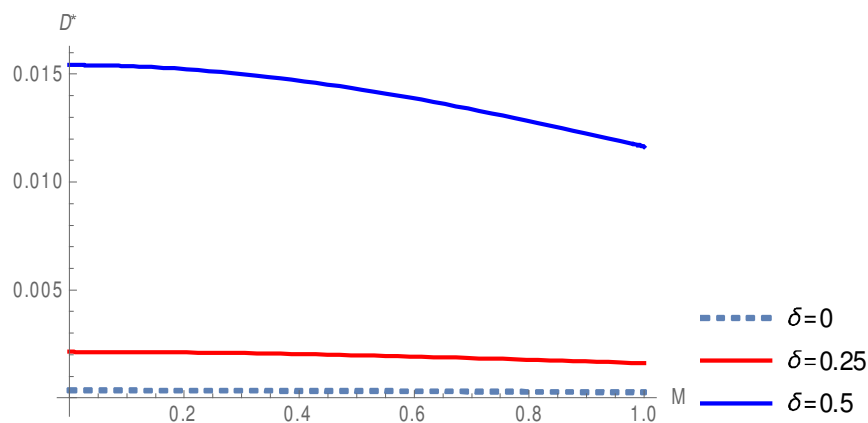


Figure 10: Dispersion coefficient D^* versus Hartmann number M for different values of height of stenosis.

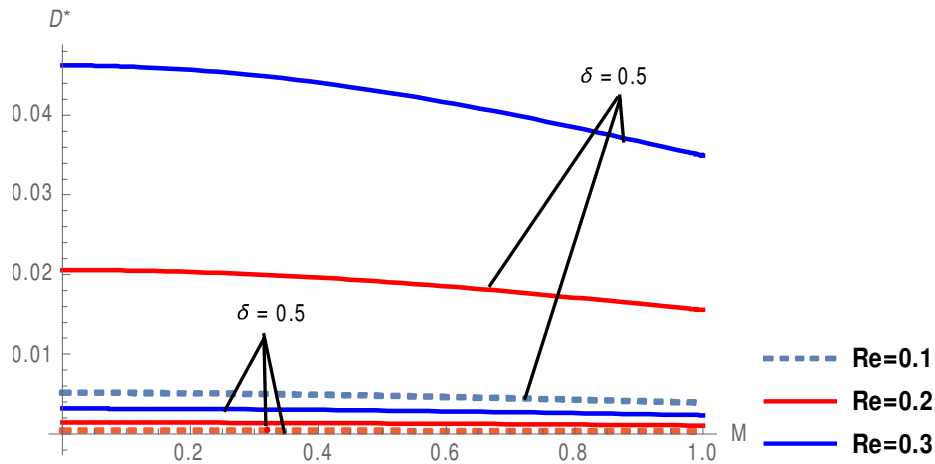


Figure 11: Dispersion coefficient D^* versus Hartmann number M for different values of Reynolds number and for fixed value of reaction rate $\beta = 0$.

5. CONCLUSION

In this paper, we have studied the dispersion of solute in capillary in normal and stenotic state depending on the variation of parameter with chemical reaction. The concentration of the blood flow decreases with increase of chemical reaction rate for both the stenosed artery and the non-stenosed artery. The present study provides valuable information for medical practitioners who seek to understand the flow of blood under stenosis conditions as well as for treating hypertension patients using magnetic therapy. Further, an important conclusion arrived at in this study is that those cells which are situated at the deeper regions of the stenotic affected area of the tissue are deprived of the nutrients, therefore, by improving the nutrient supply of these cells the progression of the disease can be controlled thereby the sufferings of the patient can be alleviated.

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