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A SIMPLISTIC METHOD TO WORK OUT THE EOQ/EPQ WITH SHORTAGES BY APPLYING ALGEBRAIC METHOD AND ARITHMETIC GEOMETRIC MEAN INEQUALITY IN FUZZY ATMOSPHERE

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Abstract:
In this research, we discuss a new method to appraise the EOQ/EPQ with shortages by manipulating algebraic method and Arithmetic Geometric Mean (AGM) inequality in fuzzy surroundings. This article proposes to find both the optimal lot size and optimal shortage level without any derivatives. This method is very simple to analogize with other procedure to derive EOQ/EPQ. Here the demand rate and shortage cost are taken as trapezoidal fuzzy number. The effectiveness of the proposed method is illustrated by means of numerical examples.

Keywords: Algebraic method, Inventory, Economic Order Quantity (EOQ), Economic Production Quantity (EPQ), Arithmetic Geometric Mean (AGM) inequality, Cost comparison optimization, Shortage level, Trapezoidal fuzzy number.

2010 Mathematics Subject Classification - 03E72, 90C05, 90C33, 90B05

1. INTRODUCTION

In the recent times Minner [8], Wee et al. [13] and Teng [12] proposed methods without using derivatives to find EOQ and EPQ model. Prior to this Grubbstrom and Erdem [7] also gave some results to evaluate EOQ and EPQ model without using derivatives followed by Ronald et al. [11] who introduced a simple method to find the shortages level for EOQ and EPQ model. Chang et al. [4] also introduced a simple method to implement the Arithmetic Geometric Mean (AGM) inequality to find the same concept in EOQ and EPQ models with and without shortages. Cardenas– Barron [2] tried to improve the algebraic method to solve the EOQ and EPQ models with backorders and improved the method by replacing their sophisticated algebraic skill. Simultaneously he [3] also proposed an optimization approach: the arithmetic-geometric mean (AGM) inequality and the Cauchy-Bunyakovsky-Schwarz (CBS) inequality. He used the AGM and CBS inequalities to derive the EOQ and EPQ models with backorders. Apart from these, the authors [9,10] also introduced the fuzzy concepts to those methods to simplify the real life problems. In this paper, we apply AGM inequality and Algebraic method to simplify the EOQ and EPQ models with shortages in fuzzy background. The effectiveness of the proposed method is illuminated by means of certain numerical examples.

2. PRELIMINARIES

2.1 Definition

A fuzzy set $\tilde{A}$ is defined by $\tilde{A}= \{(x, \mu_A(x)): x\in A, \mu_A(x)\in [0,1] \}$. In the pair $(x, \mu_A(x))$, the first element x belongs to the classical set A, the second element $\mu_A(x)$ belongs to the closed interval [0, 1] called the Membership function.

2.2 Trapezoidal Fuzzy Number

A Trapezoidal fuzzy number $\tilde{A}$ (shown graphically in the Fig. 1) is denoted as, $\tilde{A} = (a_1, a_2, a_3, a_4)$ and is defined by the membership function as,

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{(x-a_1)}{(a_2-a_1)} & \text{for } a_1 \le x \le a_2, \\ 1 & \text{for } a_2 \le x \le a_3, \\ \dfrac{(a_4-x)}{(a_4-a_3)} & \text{for } a_3 \le x \le a_4, \\ 0, & \text{otherwise.} \end{cases}$$

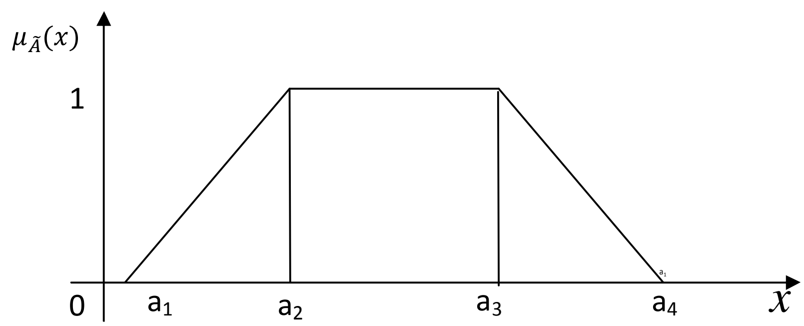

Figure 1: Graphical Representation of the trapezoidal number.

2.3 The Fuzzy Arithmetic Operations on Trapezoidal Fuzzy Number under Function Principle

Let $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ be two trapezoidal fuzzy numbers with the condition that $a_1 \le a_2 \le a_3 \le a_4$ and $b_1 \le b_2 \le b_3 \le b_4$. If $a_1 = a_2 = a_3 = a_4$ and $b_1 = b_2 = b_3 = b_4$ then $\tilde{A}$ and $\tilde{B}$ are crisp numbers. Then the fuzzy arithmetic operations under the function principle are given by,

Addition:

$\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$, where $a_1, a_2, a_3, a_4, b_1, b_2, b_3$ and b_4 are any real numbers.

Subtraction:

$\tilde{A} - \tilde{B} = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1)$, where $a_1, a_2, a_3, a_4, b_1, b_2, b_3$ and b_4 are any real numbers.

Multiplication:

$$\tilde{A} \cdot \tilde{B} = \begin{pmatrix} \min(a_1b_1, a_1b_4, a_4b_1, a_4b_4), \min(a_2b_2, a_2b_3, a_3b_2, a_3b_3), \\ \max(a_2b_2, a_2b_3, a_3b_2, a_3b_3), \max(a_1b_1, a_1b_4, a_4b_1, a_4b_4) \end{pmatrix}$$

If $a_1, a_2, a_3, a_4, b_1, b_2, b_3$ and b_4 are all nonzero positive real numbers, then

$$\tilde{A} \cdot \tilde{B} = (a_1 \cdot b_1, a_2 \cdot b_2, a_3 \cdot b_3, a_4 \cdot b_4)$$

Scalar Multiplication:

Let $\lambda \in R$, then $\lambda\tilde{A} = (\lambda a_1, \lambda a_2, \lambda a_3, \lambda a_4);\ \lambda \ge 0$

$$\lambda\tilde{A} = (\lambda a_4, \lambda a_3, \lambda a_2, \lambda a_1);\ \lambda < 0$$

Division:

$$\frac{\tilde{A}}{\tilde{B}} = \begin{pmatrix} \min\left(\frac{a_1}{b_1}, \frac{a_1}{b_4}, \frac{a_4}{b_1}, \frac{a_4}{b_4}\right), \min\left(\frac{a_2}{b_2}, \frac{a_2}{b_3}, \frac{a_3}{b_2}, \frac{a_3}{b_3}\right), \\ \max\left(\frac{a_2}{b_2}, \frac{a_2}{b_3}, \frac{a_3}{b_2}, \frac{a_3}{b_3}\right), \max\left(\frac{a_1}{b_1}, \frac{a_1}{b_4}, \frac{a_4}{b_1}, \frac{a_4}{b_4}\right) \end{pmatrix}$$

If $a_1, a_2, a_3, a_4, b_1, b_2, b_3$ and b_4 are all nonzero positive real numbers, then

$$\frac{\tilde{A}}{\tilde{B}} = \left(\frac{a_1}{b_4}, \frac{a_2}{b_3}, \frac{a_3}{b_2}, \frac{a_4}{b_1}\right)$$

2.4 Graded Mean Integration Representation Method

Defuzzification is a process of transforming fuzzy values to crisp values. Defuzzification methods have been widely studied for some years and were applied to fuzzy systems. The major idea behind these methods was to obtain a typical value from a given set according to some specified characters. Defuzzification method provides a correspondence from the set of all fuzzy sets into the set of all real numbers.

Let $\tilde{A} = (a_1, a_2, a_3, a_4)$ be a trapezoidal fuzzy number, then the defuzzified value $\tilde{A}$ using the graded mean integration representation method [4] is given by

$$P(\tilde{A}) = \frac{a_1 + 2a_2 + 2a_3 + a_4}{6}$$

This method is also used for ranking the triangular fuzzy numbers to choose which is minimum and maximum.

3. Derivation of the optimal lot size and the Shortage level for EOQ/EPQ models with Shortages:

This section gives the details of the Algebraic method and the AGM method used to evaluate the optimal solutions of EOQ and EPQ models with shortages. The method is created by combining the Algebraic method and the Arithmetic Geometric Mean (AGM) inequality.

The Algebraic Method: Let a_1 and a_2 be two positive real numbers and x be the decision variable, thus

$$a_1 x^2 - a_2 x = a_1\left(x - \frac{a_2}{2a_1}\right)^2 - \frac{a_2^{\,2}}{4a_1}$$

The AGM inequality: Let the positive real numbers be $a_1, a_2, a_3, \ldots a_n$ such that,

$\frac{a_1 + a_2 + a_3 + \ldots + a_n}{n} \geq (a_1.a_2.a_3\ldots a_n)^{1/n}$ the equation holds only if $a_1 = a_2 = a_3 = \ldots = a_n$.

Notations: The following notations are used throughout the paper.

d The demand rate per unit time
q Lot size per order (Production rate)
C_1 Inventory carrying cost (holding cost)
C_2 The shortage cost per item1
C_3 Setup cost per order (Ordering cost)
k The production rate per unit time12
q_1 The shortage level

3.1 The EOQ model with shortages

The total inventory cost for the EOQ with shortage is of the form

$$TC(q, q_1) = \frac{C_3 \tilde{d}}{q} + \frac{1}{2} C_1\left(\frac{q_1^{\,2}}{q}\right) + \frac{1}{2}\tilde{C}_2\left[\frac{(q - q_1)^2}{q}\right] \tag{1}$$

where $\tilde{d} = (d_1, d_2, d_3, d_4)$, $\tilde{C}_2 = (C_{21}, C_{22}, C_{23}, C_{24})$.

(1) can be written as

$$TC(q, q_1) = \frac{C_3 \tilde{d}}{q} + \frac{\tilde{C}_2(q^2 - 2qq_1 + q_1^{\,2})}{2q} + \frac{C_1 q_1^{\,2}}{2q}$$

$$= \frac{C_3 \tilde{d}}{q} + \frac{\tilde{C}_2 q^2}{2q} - \frac{\tilde{C}_2 2qq_1}{2q} + \frac{\tilde{C}_2 q_1^{\,2}}{2q} + \frac{C_1 q_1^{\,2}}{2q}$$

$$=\frac{C_3\tilde{d}}{q}+\frac{\tilde{C}_2 q}{2}+\frac{(C_1+\tilde{C}_2)q_1{}^2}{2q}-\tilde{C}_2 q_1 \tag{2}$$

Applying the Algebraic method in (2), the total inventory cost can be written as

$$TC(q,q_1)=\frac{C_3\tilde{d}}{q}+\frac{\tilde{C}_2 q}{2}+\frac{C_1+\tilde{C}_2}{2q}\left(q_1-\frac{\tilde{C}_2 q}{C_1+\tilde{C}_2}\right)^2-\frac{\tilde{C}_2{}^2 q}{2(C_1+\tilde{C}_2)}$$

$$=\frac{C_3\tilde{d}}{q}+\frac{C_1\tilde{C}_2 q}{2(C_1+\tilde{C}_2)}+\frac{(C_1+\tilde{C}_2)}{2q}\left(q_1-\frac{\tilde{C}_2 q}{C_1+\tilde{C}_2}\right)^2 \tag{3}$$

(3) has the minimum value when $q_1=\dfrac{\tilde{C}_2 q}{C_1+\tilde{C}_2}$. (4)

Thus from (3) we get

$$TC(q)=\frac{C_3\tilde{d}}{q}+\frac{C_1\tilde{C}_2 q}{2(C_1+\tilde{C}_2)} \tag{5}$$

On applying the AGM method in (5) we get,

$$TC(q)=\frac{\dfrac{2C_3\tilde{d}}{q}+\dfrac{C_1\tilde{C}_2 q}{(C_1+\tilde{C}_2)}}{2}\geq\sqrt{\frac{2C_3\tilde{d}}{q}\frac{C_1\tilde{C}_2 q}{(C_1+\tilde{C}_2)}}\geq\sqrt{\frac{2C_1\tilde{C}_2 C_3\tilde{d}}{C_1+\tilde{C}_2}} \tag{6}$$

$TC(q)$ has the minimum value when

$$TC(q)=\sqrt{2C_1C_3\tilde{d}\left(\frac{\tilde{C}_2}{C_1+\tilde{C}_2}\right)}$$

with the equality holding iff

$$\frac{C_3\tilde{d}}{q}=\frac{C_1\tilde{C}_2 q}{2(C_1+\tilde{C}_2)} \tag{7}$$

Or, iff the product of two constant functions is constant. On solving (7), we get the optimal lot size

$$q^*=\sqrt{\frac{2C_3\tilde{d}}{C_1}\left(\frac{C_1+\tilde{C}_2}{\tilde{C}_2}\right)}$$

From (4) we get the optimal shortage level

$$q_1{}^*=\frac{\tilde{C}_2 q^*}{C_1+\tilde{C}_2}=\sqrt{\frac{2C_3\tilde{d}}{C_1}\left(\frac{\tilde{C}_2}{C_1+\tilde{C}_2}\right)}$$

From (6) we get the optimal total cost

$$TC^*(q,q_1)=\sqrt{2C_1C_3\tilde{d}\left(\frac{\tilde{C}_2}{C_1+\tilde{C}_2}\right)}$$

where $\tilde{d}=(d_1,d_2,d_3,d_4)$, $\tilde{C}_2=(C_{21},C_{22},C_{23},C_{24})$.

3.2. The EPQ model with Shortages

Here we take $\rho=\dfrac{k-\tilde{d}}{k}$, the total inventory cost for the EPQ model with shortage is of the form

$$TC(q,q_1)=\frac{C_3\tilde{d}}{q}+\frac{1}{2}C_1\left(\frac{q_1{}^2}{q\rho}\right)+\frac{1}{2}\tilde{C}_2\left[\frac{(q\rho-q_1)^2}{q\rho}\right] \tag{8}$$

where $\tilde{d}=(d_1,d_2,d_3,d_4)$, $\tilde{C}_2=(C_{21},C_{22},C_{23},C_{24})$.

Now (8) can be written as

$$TC(q,q_1)=\frac{C_3\tilde{d}}{q}+\frac{\tilde{C}_2(q^2\rho^2-2\rho q q_1+q_1{}^2)}{2q\rho}+\frac{C_1 q_1{}^2}{2q\rho}$$

$$=\frac{C_3\tilde{d}}{q}+\frac{\tilde{C}_2 q^2\rho^2}{2q\rho}-\frac{\tilde{C}_2 2\rho q q_1}{2q\rho}+\frac{\tilde{C}_2 q_1{}^2}{2q\rho}+\frac{C_1 q_1{}^2}{2q\rho}$$

$$= \frac{C_3\tilde{d}}{q} + \frac{\tilde{C}_2 q\rho}{2} + \frac{(C_1+\tilde{C}_2)q_1^{\,2}}{2q\rho} - \tilde{C}_2 q_1 \tag{9}$$

Applying the Algebraic method in (9), the total inventory cost can be written as

$$TC(q,q_1) = \frac{C_3\tilde{d}}{q} + \frac{\tilde{C}_2 q\rho}{2} + \frac{C_1+\tilde{C}_2}{2q\rho}\left(q_1 - \frac{\tilde{C}_2 q\rho}{C_1+\tilde{C}_2}\right)^2 - \frac{\tilde{C}_2^{\,2} q\rho}{2(C_1+\tilde{C}_2)}$$

$$= \frac{C_3\tilde{d}}{q} + \frac{C_1\tilde{C}_2 q\rho}{2(C_1+\tilde{C}_2)} + \frac{(C_1+\tilde{C}_2)}{2q\rho}\left(q_1 - \frac{\tilde{C}_2 q\rho}{C_1+\tilde{C}_2}\right)^2 \tag{10}$$

(10) has the minimum value when $q_1 = \dfrac{\tilde{C}_2 q\rho}{C_1+\tilde{C}_2}$ (11)

Thus from (10), we get

$$TC(q) = \frac{C_3\tilde{d}}{q} + \frac{C_1\tilde{C}_2 q\rho}{2(C_1+\tilde{C}_2)} \tag{12}$$

Applying AGM method in (12) we get,

$$TC(q) = \frac{\dfrac{2C_3\tilde{d}}{q} + \dfrac{C_1\tilde{C}_2 q\rho}{(C_1+\tilde{C}_2)}}{2} \geq \sqrt{\frac{2C_3\tilde{d}}{q}\,\frac{C_1\tilde{C}_2 q\rho}{(C_1+\tilde{C}_2)}} \geq \sqrt{\frac{2C_1\tilde{C}_2 C_3\tilde{d}\rho}{C_1+\tilde{C}_2}} \tag{13}$$

$TC(q)$ has the minimum value when

$$TC(q) = \sqrt{2C_1C_3\tilde{d}\rho\left(\frac{\tilde{C}_2}{C_1+\tilde{C}_2}\right)}$$

with the equality holding iff

$$\frac{C_3\tilde{d}}{q} = \frac{C_1\tilde{C}_2 q\rho}{2(C_1+\tilde{C}_2)} \tag{14}$$

iff the product of two constant functions is constant. On solving eqn. (14), we get the optimal lot size

$$q^* = \sqrt{\frac{2C_3\tilde{d}}{C_1\rho}\left(\frac{C_1+\tilde{C}_2}{\tilde{C}_2}\right)} = \sqrt{\frac{2C_3\tilde{d}}{C_1}\left(\frac{k}{k-\tilde{d}}\right)\left(\frac{C_1+\tilde{C}_2}{\tilde{C}_2}\right)}$$

From eqn. (11), we get the optimal shortage level

$$q_1^* = \frac{\tilde{C}_2\rho q^*}{C_1+\tilde{C}_2} = \sqrt{\frac{2C_3\tilde{d}\rho}{C_1}\left(\frac{\tilde{C}_2}{C_1+\tilde{C}_2}\right)} = \sqrt{\frac{2C_3\tilde{d}}{C_1}\left(\frac{k-\tilde{d}}{k}\right)\left(\frac{\tilde{C}_2}{C_1+\tilde{C}_2}\right)}$$

From (13), we get the optimal total cost

$$TC^*(q,q_1) = \sqrt{2C_1C_3\tilde{d}\rho\left(\frac{\tilde{C}_2}{C_1+\tilde{C}_2}\right)} = \sqrt{2C_1C_3\tilde{d}\left(\frac{k-\tilde{d}}{k}\right)\left(\frac{\tilde{C}_2}{C_1+\tilde{C}_2}\right)}$$

where $\tilde{d} = (d_1,d_2,d_3,d_4)$, $\tilde{C}_2 = (C_{21},C_{22},C_{23},C_{24})$.

4. NUMERICAL EXAMPLE

4.1 Problem on EOQ with Shortages:

A furniture dealer sells special typist chairs. Each purchase order costs Rs. 50 to the dealer and the holding cost amounts to Rs. 80 per chair per year. The dealer sells around 90 chairs per month. He has estimated a shortage cost of Rs. 20 per chair per year approximately.

i. What is the EOQ for the chair?

ii. When the supply of the chairs comes, how many chairs on an average are expected to be delivered to the customer immediately?

Solution:

Here $d = 90$ Chairs per month.

$\tilde{d} = (d_1,d_2,d_3,d_4) = (84,88,92,96)$ Chairs per month,

$= (1008,1056,1104,1152)$ Chairs per year.

$C_1 = Rs.80$ per chair per year.

$C_2 = Rs.20$ per chair per year approximately.

$= (C_{21}, C_{22}, C_{23}, C_{24}) = (14,18,22,26)$

$C_3 = Rs.50$

i) The optimal ordering quantity is

$$q^* = \sqrt{\frac{2C_3\tilde{d}}{C_1}\left(\frac{C_1+\tilde{C}_2}{\tilde{C}_2}\right)} = \sqrt{\frac{2C_3(d_1,d_2,d_3,d_4)}{C_1}\left(\frac{C_1+(C_{21},C_{22},C_{23},C_{24})}{(C_{21},C_{22},C_{23},C_{24})}\right)}$$

$$= \sqrt{\frac{2(50)(1008,1056,1104,1152)}{80}\left(\frac{80+(14,18,22,26)}{(14,18,22,26)}\right)}$$

$= (67.4937, 76.6808, 88.4310, 97.6729)$

The defuzzified value is

$$= \frac{67.4937 + 2(76.6808) + 2(88.4310) + 97.6729}{6} = \frac{495.3902}{6} = 82.5650$$

That is, the dealer needs to purchase around 82 chairs.

ii) The optimal time interval is

$$t = \frac{q^*}{\tilde{d}} = \sqrt{\frac{2C_3}{\tilde{d}C_1}\left(\frac{C_1+\tilde{C}_2}{\tilde{C}_2}\right)} = \sqrt{\frac{2C_3}{(d_1,d_2,d_3,d_4)C_1}\left(\frac{C_1+(C_{21},C_{22},C_{23},C_{24})}{(C_{21},C_{22},C_{23},C_{24})}\right)}$$

$$= \sqrt{\frac{2\times 50}{(1008,1056,1104,1152)80}\left(\frac{80+(14,18,22,26)}{(14,18,22,26)}\right)}$$

$= (0.062, 0.073, 0.080, 0.092)$ Years

The defuzzified value is

$$= \frac{0.062 + 2(0.073) + 2(0.080) + 0.092}{6} = 0.077 \text{ years}$$

That is, $0.077 \times 365 = 28.106$ days

iii) The maximum inventory level is

$$q_2^* = q^*\left(1 - \frac{C_1}{C_1+\tilde{C}_2}\right) = \sqrt{\frac{2C_3\tilde{d}}{C_1}\left(\frac{\tilde{C}_2}{C_1+\tilde{C}_2}\right)}$$

$$= \sqrt{\frac{2(50)(1008,1056,1104,1152)}{80}\left(\frac{(14,18,22,26)}{80+(14,18,22,26)}\right)}$$

$= (12.90, 15.24, 17.58, 19.97)$

The defuzzified value is

$$= \frac{12.90 + 2(15.24) + 2(17.58) + 19.97}{6} = \frac{98.51}{6} = 16.41$$

That is, 16 chairs on an average are expected to be delivered to the customer immediately.

4.2 Problem on EPQ with Shortage

The stockist has to supply around 400 units of a product every Monday to his customers and it can produce 80 such items per day. He produces at the rate of Rs.50 per unit. The cost of ordering and transportation is Rs.75 per order. The cost of carrying inventory is estimated at Rs.5 per unit per week approximately. Find

i. The economic lot size,

ii. The optimal total cost and

iii. The shortage level.

Solution:

Here $d = 400$ units per week,

$\tilde{d} = (370, 390, 410, 430)$ per week.

Holding cost $C_1 = 8\%$ per year of the cost of the product

$= \left(\frac{8}{100} \times \frac{50}{52}\right)$ per week

$$= \text{Rs.}\left(\frac{1}{13}\right) \text{per week.}$$

Shortage Cost $C_2 = Rs.5$ per unit per week

$$\tilde{C}_2 = (C_{21}, C_{22}, C_{23}, C_{24}) = (3.5, 4.5, 5.5, 6.5).$$

Setup Cost $C_3 = Rs.75$ per order

Production Rate $k = Rs.80$ per day

$$= 80 \times 7 = Rs.560 \text{ per week.}$$

i) The economic lot size is given by

$$q^* = \sqrt{\frac{2C_3\tilde{d}}{C_1}\left(\frac{k}{k-\tilde{d}}\right)\left(\frac{C_1+\tilde{C}_2}{\tilde{C}_2}\right)}$$

$$= \sqrt{\frac{2(75)(370,390,410,430)}{(1/13)}\left(\frac{560}{560-(370,390,410,430)}\right)\left(\frac{(1/13)+(3.5,4.5,5.5,6.5)}{(3.5,4.5,5.5,6.5)}\right)}$$

$= (1081.76, 1443.88, 1923.29, 2605.07)$

The defuzzified value is

$$= \frac{1081.76 + 2(1443.88) + 2(1923.29) + 2605.07}{6} = \frac{10421.17}{6} = 1736.86$$

ii) The optimum total cost is given by

$$TC^*(q, q_1) = \sqrt{2C_1C_3\tilde{d}\left(\frac{k-\tilde{d}}{k}\right)\left(\frac{\tilde{C}_2}{C_1+\tilde{C}_2}\right)}$$

$$= \sqrt{2(1/13)(75)(370,390,410,430)\left(\frac{560-(370,390,410,430)}{560}\right)\left(\frac{(3.5,4.5,5.5,6.5)}{(1/13)+(3.5,4.5,5.5,6.5)}\right)}$$

$= (27.9785, 33.5077, 39.0257, 45.7454)$

The defuzzified value is

$$= \frac{27.9785 + 2(33.5077) + 2(39.0257) + 45.7454}{6} = \frac{218.7907}{6} = 36.4651$$

iii) The shortage level is given by

$$q_1^* = \sqrt{\frac{2C_3\tilde{d}}{C_1}\left(\frac{k-\tilde{d}}{k}\right)\left(\frac{\tilde{C}_2}{C_1+\tilde{C}_2}\right)}$$

$$= \sqrt{\frac{2(75)(370,390,410,430)}{(1/13)}\left(\frac{560-(370,390,410,430)}{560}\right)\left(\frac{(3.5,4.5,5.5,6.5)}{(1/13)+(3.5,4.5,5.5,6.5)}\right)}$$

$= (363.7205, 480.5079, 507.3336, 594.6898)$

The defuzzified value is

$$= \frac{363.7205 + 2(480.5079) + 2(507.3336) + 594.6898}{6} = \frac{2934.0933}{6} = 489.0156$$

5. CONCLUSION

We have two optimization methods to compute the EOQ/EPQ models. These are cost comparisons and AGM inequality. However, neither optimization method attempted to derive the optimal shortage level. Main work of this paper is to introduce an alternative method for deriving EOQ/EPQ models when shortages are allowed in fuzzy environment. Compared to the other methods our proposed method is simple and derives both the optimal lot size and the shortage level. This method is also simpler than the algebraic method. Finally this method is very simple and accessible to the learning of inventory models in fuzzy environment for the readers who lack the knowledge of calculus.

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