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SOME RESULTS ON BIPOLAR FUZZY SELF CENTERED GRAPHS

S. Yahya Mohamed[1,*], N. Subashini[2]

Authors Affiliation:
[1]Assistant Professor, Department of Mathematics, Government Arts College, Tiruchirappalli, Tamil Nadu 620022, India.
E-mail: yahyamd@yahoo.com
[2]Assistant Professor, Department of Mathematics, Saranathan College of Engineering, Tiruchirappalli, Tamil Nadu 620012.
E-mail: yazhinisuba@gmail.com

***Corresponding Author:**
S. Yahya Mohamed, Assistant Professor, Department of Mathematics, Government Arts College, Tiruchirappalli, Tamil Nadu 620022, India.
E-mail: yahyamd@yahoo.com

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Abstract:
The distance and related concepts like eccentricity, radius, diameter, center, periphery, etc. are already defined and used in many applications of graph theory. In this paper, we define a new distance called bipolar fuzzy distance in bipolar fuzzy graphs. Using this distance, we define the concepts of bipolar fuzzy eccentricity, bipolar fuzzy center, etc. and establish the relation between bipolar fuzzy radius and bipolar fuzzy diameter. Also the concept of self centered graphs is generalized to bipolar fuzzy self centered graphs and a necessary condition for a bipolar fuzzy graph to be a bipolar fuzzy self centered graph is obtained. The max-max and min-min composition of the bipolar fuzzy distance matrix is introduced, and we also present an easy check to see whether a given bipolar fuzzy graph is bipolar fuzzy self centered or not.

Keywords: Bipolar fuzzy distance, Bipolar fuzzy eccentricity, Bipolar fuzzy center, Bipolar fuzzy radius, Bipolar fuzzy diameter and Bipolar fuzzy self center.

2010 Mathematics Subject Classification: 05C72.

1. INTRODUCTION

In 1965, L.A. Zadeh [16] initiated the concept of bipolar fuzzy sets as a generalization of fuzzy sets. Bipolar fuzzy sets are an extension of fuzzy sets whose membership degree range is [-1,1]. In a bipolar fuzzy set, the membership degree 0 of an element means that the element is irrelevant to the corresponding property, the membership degree (0,1] of an element indicates that the element somewhat satisfies the property, and the membership degree [-1,0) of an element indicates that the element somewhat satisfies the implicit counter-property. Although bipolar fuzzy sets and intuitionistic fuzzy sets look similar to each other, they are essentially different sets. In many domains, it is important to be able to deal with bipolar fuzzy information. It is noted that positive information represents what is granted to be possible, while negative information represents what is considered to be impossible. This domain has recently motivated new research in several direction. Akram [1] introduced the concept of bipolar fuzzy graphs and defined different operations on it. Bhutani and Rosenfeld introduced the concept of m-strong fuzzy graphs in [5,17] and studied some of their properties. By a bipolar fuzzy graph [6], we mean a pair $G = (A,B)$ where $A = (\mu_A^P, \mu_A^N)$ is a bipolar fuzzy set in V and $B = (\mu_B^P, \mu_B^N)$ is a bipolar fuzzy relation on E such that $\mu_B^P(x,y) \leq \min(\mu_A^P(x), \mu_A^P(y))$ and $\mu_B^N(x,y) \geq \max(\mu_A^N(x), \mu_A^N(y))$ for all $(x,y) \in E$. We call A the bipolar fuzzy vertex set of V, B the bipolar fuzzy edge set of E respectively. Note that B is symmetric bipolar fuzzy relation on A. We use the notation xy for an element of E. Bipolar fuzzy graphs are precise models of all kinds of networks. Sunitha, M.S. and Vijayakumar, A. [9,10] introduced the distance and related concepts like eccentricity, radius, diameter, center, periphery, etc. are already defined and used in many applications of graph theory. Bhattacharya, P. and Suraweera. F [2,3] defined a new distance called bipolar fuzzy distance in bipolar fuzzy graphs. Using this distance, the concepts of bipolar fuzzy eccentricity, bipolar fuzzy center etc. are defined and the relation between bipolar fuzzy radius and bipolar fuzzy diameter is established. Mathew, S. and Sunitha [7] introduced the concept of self centered graphs is generalized to bipolar fuzzy self centered graphs and a necessary condition for a bipolar fuzzy graph to be a bipolar fuzzy self centered graph is obtained. The max-max and min-min composition of the bipolar fuzzy distance matrix is introduced, and presents an easy check to see whether a given bipolar fuzzy graph is bipolar fuzzy self centered or not.

2. PRELIMINARIES

In this section, some basic definitions and preliminary ideas are given which are useful for proving the theorems.

Definition 2.1: A path P of length n is a sequence of distinct nodes $u_0, u_1, u_2, \ldots, u_n$ such that $\mu(u_{i-1}, u_i) > 0, i = 1,2,3, \ldots. n$ and the degree of membership of a weakest edge is defined as its strength. The strength of connectedness between two nodes x and y is defined as the maximum of the strengths of all paths between x and y and is denoted by $CONN_G(x,y)$. An x-y path P is called the strongest x-y path if its strength equals $CONN_G(x,y)$.

Definition 2.2: By a bipolar fuzzy graph, we mean a pair $G : (A, B)$ where $A = (\mu_A^P, \mu_A^N)$ is a bipolar fuzzy set in V and $B = (\mu_B^P, \mu_B^N)$ is a bipolar fuzzy relation on E such that $\mu_B^P(x,y) \leq \min(\mu_A^P(x), \mu_A^P(y))$ and $\mu_B^N(x,y) \geq \max(\mu_A^N(x), \mu_A^N(y))$ for all $(x,y) \in E$. We call A the bipolar fuzzy vertex set of V, B the bipolar fuzzy edge set of E respectively. Note that B is symmetric bipolar fuzzy relation on A. We use the notation xy for an element of E. Thus, $G = (A, B)$ is a bipolar fuzzy graph of $G^* = (V, E)$ if $\mu_B^P(xy) \leq \min(\mu_A^P(x), \mu_A^P(y))$ and $\mu_B^N(xy) \geq \max(\mu_A^N(x), \mu_A^N(y))$ for all $xy \in E$.

Definition 2.3: Let $G : (A,B)$ be a bipolar fuzzy graph. Then the complement of G is denoted as $\bar{G}: (\bar{A}, \bar{B})$ where $\bar{A} = A$ and $\mu_B^P(xy) = \min(\mu_A^P(x), \mu_A^P(y)) - {\mu_B}^P(xy)$ and $\mu_B^N(xy) = \max(\mu_A^N(x), \mu_A^N(y)) - {\mu_B}^N(xy)$ for all $xy \in E$.

3. BIPOLAR FUZZY SELF CENTERED GRAPHS

Definition 3.1: Let $G:(A, B)$ be a bipolar fuzzy graph on $G^*: (V, E)$ then the bipolar fuzzy distance between two nodes $(\mu_A^P(u), \mu_A^N(u))$ and $(\mu_A^P(v), \mu_A^N(v))$ in G is defined and denoted by $d_{bf}^P(u,v) = \frac{min[L(P)*S(P)]}{P}$ and $d_{bf}^N(u,v) = max[L(P)*S(P)]/P$ is a $u - v$ path, $L(P)$ is the length and $S(P)$ is the strength of P and * represents the ordinary product.

Example 3.1: Define $G = (A,B)$ by $({\mu_A}^P(a), {\mu_A}^N(a)) = (0.5, -0.9)$, $({\mu_A}^P(b), {\mu_A}^N(b)) = (0.6, -0.8)$, $({\mu_A}^P(c), {\mu_A}^N(c)) = (0.5, -0.6)$, $({\mu_A}^P(d), {\mu_A}^N(d)) = (0.4, -0.3)$ and $({\mu_B}^P(ab), {\mu_B}^N(ab)) = (0.3, -0.2)$, $({\mu_B}^P(bc), {\mu_B}^N(bc)) = (0.5, -0.4)$, $({\mu_B}^P(cd), {\mu_B}^N(cd)) = (0.2, -0.1)$, $({\mu_B}^P(ad), {\mu_B}^N(ad)) = (0.3, -0.2)$, $({\mu_B}^P(bd), {\mu_B}^N(bd)) = (0.4, -0.3)$.

$(d_{bf}^P(a,b), d_{bf}^N(a,b)) = (0.3, -0.2)$, $(d_{bf}^P(a,c), d_{bf}^N(a,c)) = (0.4, -0.2)$, $(d_{bf}^P(a,d), d_{bf}^N(a,d)) = (0.3, -0.2)$, $(d_{bf}^P(b,c), d_{bf}^N(b,c)) = (0.4, -0.2)$, $(d_{bf}^P(b,d), d_{bf}^N(b,d)) = (0.4, -0.2)$ and $(d_{bf}^P(c,d), d_{bf}^N(c,d)) = (0.2, -0.1)$.

Definition 3.2: Let $G:(A, B)$ be a bipolar fuzzy graph on $G^*:(V,E)$ then the biolar fuzzy eccentricity of a node $(\mu_A^P(u), \mu_A^N(u)) \in V(G)$ is defined and denoted by $e_{bf}^P(u) = \max_{v\in V(G)} d_{bf}^P(u,v)$ and $e_{bf}^N(u) = \min_{v\in V(G)} d_{bf}^N(u,v)$.

Definition 3.3: The minimum of the fuzzy positive eccentricities and maximum of the fuzzy negative eccentricities of all the nodes is called the bipolar fuzzy radius of the graph G. It is denoted as $r_{bf}(G)$. Thus $r_{bf}^P(G) = \min_{u\in V(G)} e_{bf}^P(u)$ and $r_{bf}^N(G) = \max_{u\in V(G)} e_{bf}^N(u)$.

Definition 3.4: The maximum of the fuzzy positive eccentricities and minimum of the fuzzy negative eccentricities of all nodes is called the bipolar fuzzy diameter of the graph G. It is denoted as $D_{bf}(G)$. That is, $D_{bf}^P(G) = \max_{u\in V(G)} e_{bf}^P(u)$ and $D_{bf}^N(G) = \min_{u\in V(G)} e_{bf}^N(u)$.
In example 2.1, $\left(e_{bf}^P(a), e_{bf}^N(a)\right) = (0.4, -0.2)$, $\left(e_{bf}^P(b), e_{bf}^N(b)\right) = (0.4, -0.2)$, $\left(e_{bf}^P(c), e_{bf}^N(c)\right) = (0.4, -0.2)$, $\left(e_{bf}^P(d), e_{bf}^N(d)\right) = (0.4, -0.2)$.

Definition 3.5: Nodes with minimum fuzzy positive eccentricity and nodes with maximum fuzzy negative eccentricity are called bipolar fuzzy central nodes or bipolar fuzzy radial nodes.

Definition 3.6: Nodes with maximum fuzzy positive eccentricity and nodes with minimum fuzzy negative eccentricity are called bipolar fuzzy diametral nodes or bipolar fuzzy peripheral nodes.
In example 3.1, $\left(r_{bf}^P(G), r_{bf}^N(G)\right) = (0.4, -0.2)$ and $\left(D_{bf}^P(G), D_{bf}^N(G)\right) = (0.4, -0.2)$. Here all the nodes are both bipolar fuzzy central as well as bipolar fuzzy diametral. Now consider the following example (Figure.1).

Example 3.2:

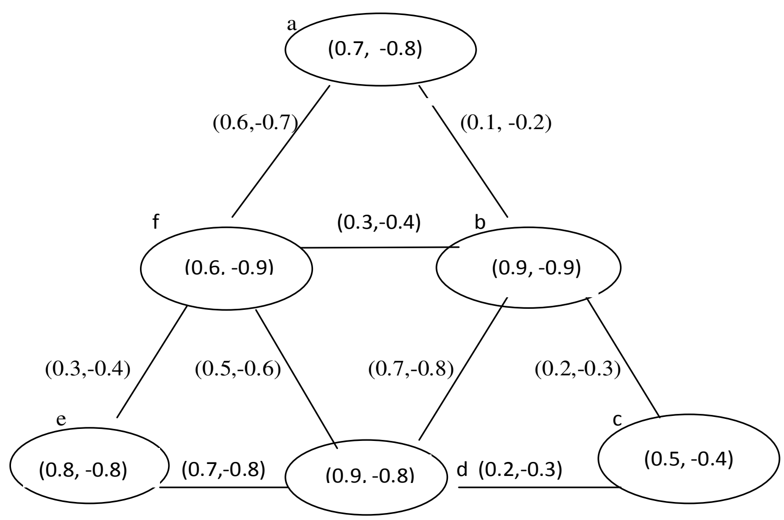

Figure 1: Bipolar fuzzy eccentricity and center

$\left(d_{bf}^P(a,b), d_{bf}^N(a,b)\right) = (0.1, -0.2)$, $\left(d_{bf}^P(a,c), d_{bf}^N(a,c)\right) = (0.2, -0.4)$,
$\left(d_{bf}^P(a,d), d_{bf}^N(a,d)\right) = (0.2, -0.4)$, $\left(d_{bf}^P(a,e), d_{bf}^N(a,e)\right) = (0.3, -0.6)$,
$\left(d_{bf}^P(a,f), d_{bf}^N(a,f)\right) = (0.2, -0.4)$, $\left(d_{bf}^P(b,c), d_{bf}^N(b,c)\right) = (0.2, -0.3)$
$\left(d_{bf}^P(b,d), d_{bf}^N(b,d)\right) = (0.3, -0.6)$, $\left(d_{bf}^P(b,e), d_{bf}^N(b,e)\right) = (0.3, -0.6)$,
$\left(d_{bf}^P(b,f), d_{bf}^N(b,f)\right) = (0.2, -0.4)$, $\left(d_{bf}^P(c,d), d_{bf}^N(c,d)\right) = (0.2, -0.3)$,
$\left(d_{bf}^P(c,e), d_{bf}^N(c,e)\right) = (0.4, -0.6)$, $\left(d_{bf}^P(c,f), d_{bf}^N(c,f)\right) = (0.3, -0.6)$,

$\left(d_{bf}^P(d,e), d_{bf}^N(d,e)\right) = (0.4, -0.8)$,$\left(d_{bf}^P(d,f), d_{bf}^N(d,f)\right) = (0.3, -0.6)$
&$\left(d_{bf}^P(e,f), d_{bf}^N(e,f)\right) = (0.3, -0.4)$.
Here, $\left(e_{bf}^P(a), e_{bf}^N(a)\right) = (0.3, -0.6)$, $\left(e_{bf}^P(b), e_{bf}^N(b)\right) = (0.3, -0.6)$, $\left(e_{bf}^P(c), e_{bf}^N(c)\right) = (0.4, -0.6)$, $\left(e_{bf}^P(d), e_{bf}^N(d)\right) = (0.4, -0.8)$, $\left(e_{bf}^P(e), e_{bf}^N(e)\right) = (0.4, -0.8)$, $\left(e_{bf}^P(f), e_{bf}^N(f)\right) = (0.3, -0.6)$ and $\left(r_{bf}^P(G), r_{bf}^N(G)\right) = (0.3, -0.6)$, $\left(D_{bf}^P(G), D_{bf}^N(G)\right) = (0.4, -0.8)$.

Note that *a*, *b* and *f* are not fuzzy positive eccentric nodes of any other nodes and *a*, *b*, *c* and *f* are not fuzzy negative eccentric nodes of any other nodes.

Remark 3.1: The bipolar fuzzy center of a bipolar fuzzy graph need not be the same as the center of its underlying graph.

In this section, we shall discuss the properties of self centered bipolar fuzzy graphs with respect to the new distance.

Definition 3.7: A bipolar fuzzy graph *G* is called bipolar fuzzy self centered, if it is isomorphic with its bipolar fuzzy center.

Example 3.3: A bipolar fuzzy graph and its underlying graph are shown below.

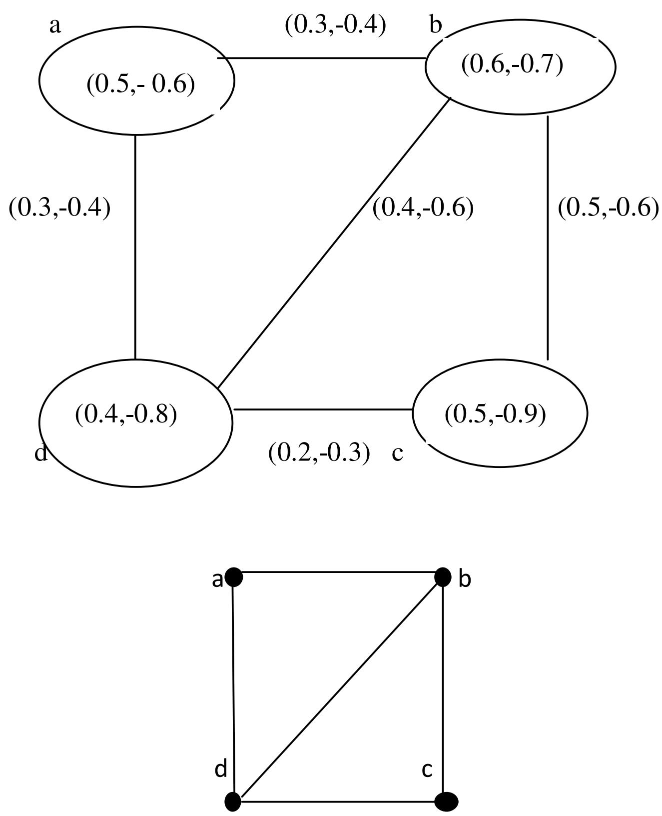

Figure 2: Bipolar fuzzy graph and its underlying graphs

$\left(d_{bf}^P(a,b), d_{bf}^N(a,b)\right) = (0.3, -0.4)$,$\left(d_{bf}^P(a,c), d_{bf}^N(a,c)\right) = (0.4, -0.6)$,
$\left(d_{bf}^P(a,d), d_{bf}^N(a,d)\right) = (0.3, -0.4)$,$\left(d_{bf}^P(b,c), d_{bf}^N(b,c)\right) = (0.4, -0.6)$,
$\left(d_{bf}^P(b,d), d_{bf}^N(b,d)\right) = (0.4, -0.6)$,$\left(d_{bf}^P(c,d), d_{bf}^N(c,d)\right) = (0.2, -0.3)$ and

$\left(e_{bf}^{P}(a), e_{bf}^{N}(a)\right) = (0.4, -0.6)$, $\left(e_{bf}^{P}(b), e_{bf}^{N}(b)\right) = (0.4, -0.6)$,
$\left(e_{bf}^{P}(c), e_{bf}^{N}(c)\right) = (0.4, -0.6)$, $\left(e_{bf}^{P}(d), e_{bf}^{N}(d)\right) = (0.4, -0.6)$

Also note that $d(a,b) = 1$, $d(a,c) = 2$, $d(a,d) = 1$, $d(b,c) = 1$, $d(b,d) = 1$, $d(c,d) = 1$ and $e(a) = 2$, $e(b) = 1$, $e(c) = 2$, $e(d) = 1$.

This bipolar fuzzy graph is bipolar fuzzy self centered but the underlying graph is not self centered.

In the following theorem we present a necessary condition for a bipolar fuzzy graph *G* to be bipolar fuzzy self centered.

Theorem 3.1: If a connected bipolar fuzzy graph *G* is bipolar fuzzy self centered, then each node of *G* is bipolar fuzzy eccentric.

Proof. Assume that the bipolar fuzzy graph *G* is bipolar fuzzy self centered. We have to prove that each node of *G* is bipolar fuzzy eccentric. Let $(\mu_A^P(u), \mu_A^N(u))$be any arbitrary nodes of *G*.

By the definition of a bipolar fuzzy eccentric node, $\left(e_{bf}^{P}(u), e_{bf}^{N}(u)\right) = \left(d_{bf}^{P}(u,v), d_{bf}^{N}(u,v)\right)$.

But since *G* is bipolar fuzzy self centered, $\left(e_{bf}^{P}(u), e_{bf}^{N}(u)\right) = \left(e_{bf}^{P}(v), e_{bf}^{N}(v)\right)$ and hence

$\left(e_{bf}^{P}(v), e_{bf}^{N}(v)\right) = \left(d_{bf}^{P}(u,v), d_{bf}^{N}(u,v)\right) = \left(d_{bf}^{P}(v,u), d_{bf}^{N}(v,u)\right)$.

Thus $(\mu_A^P(u), \mu_A^N(u))$ is a bipolar fuzzy eccentric node of $(\mu_A^P(v), \mu_A^N(v))$.

Hence every node of *G* is bipolar fuzzy eccentric.

The next result is also a necessary condition for bipolar fuzzy self centered graphs.

Theorem 3.2: If a connected bipolar fuzzy graph *G* is bipolar fuzzy self centered then for every pair of nodes $\left(\mu_A^P(u), \mu_A^N(u)\right), \left(\mu_A^P(v), \mu_A^N(v)\right)$ such that whenever $\left(\mu_A^P(u), \mu_A^N(u)\right)$ is a bipolar fuzzy eccentric node of $\left(\mu_A^P(v), \mu_A^N(v)\right)$ then $\left(\mu_A^P(v), \mu_A^N(v)\right)$ should be one of the bipolar fuzzy eccentric nodes of $\left(\mu_A^P(u), \mu_A^N(u)\right)$.

Proof: Assume that *G* is bipolar fuzzy self centered also assume that $(\mu_A^P(u), \mu_A^N(u))$ is a bipolar fuzzy eccentric nodes of $(\mu_A^P(v), \mu_A^N(v))$. This means $\left(e_{bf}^{P}(v), e_{bf}^{N}(v)\right) = \left(d_{bf}^{P}(v,u), d_{bf}^{N}(v,u)\right)$ since *G* is bipolar fuzzy self centered, all nodes will be having the same bipolar fuzzy eccentricity. Therefore $\left(e_{bf}^{P}(v), e_{bf}^{N}(v)\right) = \left(e_{bf}^{P}(u), e_{bf}^{N}(u)\right)$.

From the above two relations it follows that,

$$\left(e_{bf}^{P}(u), e_{bf}^{N}(u)\right) = \left(d_{bf}^{P}(v,u), d_{bf}^{N}(v,u)\right) = \left(d_{bf}^{P}(u,v), d_{bf}^{N}(u,v)\right).$$

That is, $(\mu_A^P(v), \mu_A^N(v))$ is a bipolar fuzzy eccentric node of $(\mu_A^P(u), \mu_A^N(u))$.

This result is not sufficient, as we are not able to prove that the bipolar fuzzy eccentricity of a third node is equal to the common bipolar fuzzy eccentricity of the selected pair of nodes.

4. THE BIPOLAR FUZZY DISTANCE MATRIX

In this section, we present an easy check for a bipolar fuzzy graph *G* to find whether it is bipolar fuzzy self centered or not.

Definition 4.1: Let *G*: (*A*, *B*) be a connected bipolar fuzzy graph with *n* nodes. The bipolar fuzzy distance matrix $\left(D_{bf}^{P}, D_{bf}^{N}\right) = (d_{i,j}^{P}, d_{i,j}^{N})$ is a square matrix of order *n* and is defined by $d_{i,j}^{P} = d_{bf}^{P}\left(v_i, v_j\right)$ and $d_{i,j}^{N} = d_{bf}^{N}(v_i, v_j)$. Note that the bipolar fuzzy distance matrix is a symmetric matrix.

Definition 4.2: The max-max and min-min composition of a square matrix with itself is again a square matrix of the same order whose $(i,j)^{th}$ entry is given by

$d_{i,j}^{P} = \max\{max\left(d_{i1}, d_{1j}\right), max\left(d_{i2}, d_{2j}\right), \ldots, max\left(d_{in}, d_{nj}\right)\}$ and

$d_{i,j}^{N} = \min\{\min\left(d_{i1}, d_{1j}\right), \min\left(d_{i2}, d_{2j}\right), \ldots\ldots \min\left(d_{in}, d_{nj}\right)\}$.

Example 4.1:

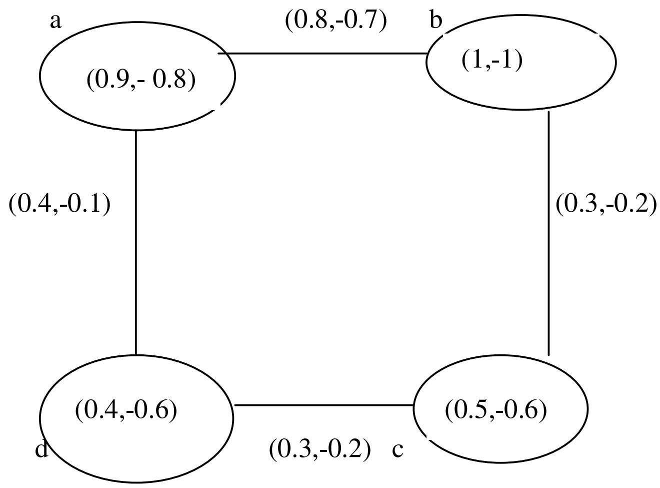

Figure 3: Bipolar Fuzzy distance matrix

The bipolar fuzzy distance matrix and its max-max and min-min composition are given below.

$$\left(D_{bf}^{P}, D_{bf}^{N}\right) = \begin{pmatrix} (0,0) & (0.8,-0.3) & (0.6,-0.2) & (0.4,-0.1) \\ (0.8,-0.3) & (0,0) & (0.3,-0.2) & (0.6,-0.2) \\ (0.6,-0.2) & (0.3-0.2) & (0,0) & (0.3,-0.2) \\ (0.4,-0.1) & (0.6,-0.2) & (0.3,-0.2) & (0,0) \end{pmatrix},$$

$$\left(\left(D_{bf}^{P} \circ D_{bf}^{P}\right), \left(D_{bf}^{N} \circ D_{bfF}^{N}\right)\right) = \begin{pmatrix} (0.8,-0.3) & (0.8,-0.3) & (0.8,-0.3) & (0.8,-0.3) \\ (0.8,-0.3) & (0.8,-0.3) & (0.8,-0.3) & (0.8,-0.3) \\ (0.8,-0.3) & (0.8-0.3) & (0.6,-0.2) & (0.6,-0.2) \\ (0.8,-0.3) & (0.8,-0.3) & (0.6,-0.2) & (0.6,-0.2) \end{pmatrix}$$

Next we have a theorem regarding the bipolar fuzzy eccentricities of nodes using the max-max and min-min composition of the bipolar fuzzy distance matrices.

Theorem 4.1. Let G: (A, B) be a connected bipolar fuzzy graph. The diagonal elements of the max-max and min-min composition of the bipolar fuzzy distance matrix of G with itself are the bipolar fuzzy eccentricities of the nodes.

Proof: Let $\left(D_{bf}^{P}, D_{bf}^{N}\right) = \left(d_{i,j}^{P}, d_{i,j}^{N}\right)$ be the bipolar fuzzy distance matrix of G. Then $\left(d_{i,j}^{P}, d_{i,j}^{N}\right) = \left(d_{bf}^{P}\left(v_i, v_j\right), d_{bf}^{N}\left(v_i, v_j\right)\right)$. In the max-max composition,

$\left(D_{bf}^{P} \circ D_{bf}^{P}\right)$ the i^{th} diagonal entry $d_{i,i}^{P} = \max\{\max(d_{i,1}^{P}, d_{1,i}^{P}), \max(d_{i,2}^{P}, d_{2,i}^{P}), \dots, \max(d_{i,n}^{P}, d_{n,i}^{P})\}$

$$= \max\{d_{bf}^{P}(v_i, v_1), d_{bf}^{P}(v_i, v_2), \dots, d_{bf}^{P}(v_i, v_n)\} = e_{bf}^{P}(v_i)$$

And, in the min-min composition,

$\left(D_{bf}^{N} \circ D_{bf}^{N}\right)$ the i^{th} diagonal entry

$$d_{i,i}^{N} = \min\{\min(d_{i,1}^{N}, d_{1,i}^{N}), \min(d_{i,2}^{N}, d_{2,i}^{N}), \dots, \min(d_{i,n}^{N}, d_{n,i}^{N})\}$$

$$= \min\{d_{bf}^{N}(v_i, v_1), d_{bf}^{N}(v_i, v_2), \dots, d_{bf}^{N}(v_i, v_n)\} = e_{bf}^{N}(v_i).$$

This completes the proof of the theorem.

Theorem 4.2: A connected bipolar fuzzy graph *G*: (*A*,*B*) is bipolar fuzzy self centered if and only if all the entries in the principal diagonal of the max-max and min-min composition of the bipolar fuzzy distance matrix with itself are the same.

Proof: As proved in the Theorem 4.1, the principal diagonal entries in the max-max and min-min composition of the bipolar fuzzy distance matrix with itself are the bipolar fuzzy eccentricities of the nodes. If they are same, this means $\left(e_{bf}^{P}(u),\ e_{bf}^{N}(u)\right)$ is the same for all *u* in *G*. Then *G* is bipolar fuzzy self centered. Hence the proof is completed.

We illustrate the above theorem in the following examples.

Example 4.2:

Consider the bipolar fuzzy graph in Figure 4.

The bipolar fuzzy distance matrix and the max-max, min-min composition are given below.

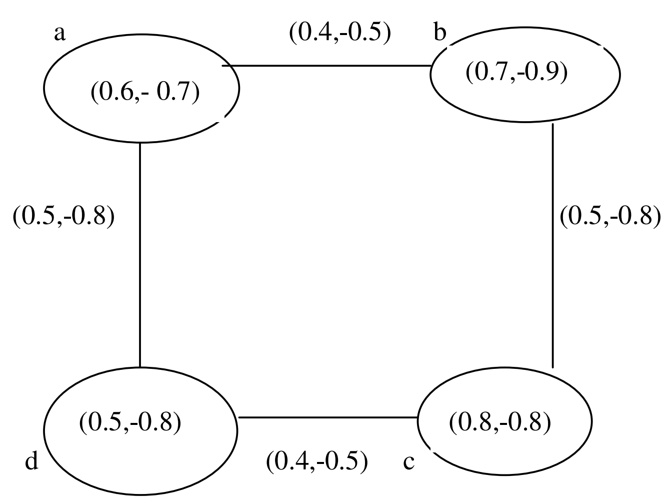

Figure 4: A Bipolar Fuzzy Self Centered graph

$$\left(D_{bf}^{P}, D_{bf}^{N}\right) = \begin{pmatrix} (0,0) & (0.4,-0.5) & (0.8,-1) & (0.5,-0.8) \\ (0.4,-0.5) & (0,0) & (0.5,-0.8) & (0.8,-1) \\ (0.8,-1) & (0.5,-0.8) & (0,0) & (0.4,-0.5) \\ (0.5,-0.8) & (0.8,-1) & (0.4,-0.5) & (0,0) \end{pmatrix},$$

$$\left(\left(D_{bf}^{P} \circ D_{bf}^{P}\right), \left(D_{bf}^{N} \circ D_{bf}^{N}\right)\right) = \begin{pmatrix} (0.8,-1) & (0.8,-1) & (0.8,-1) & (0.8,-1) \\ (0.8,-1) & (0.8,-1) & (0.8,-1) & (0.8,-1) \\ (0.8,-1) & (0.8-1) & (0.8,-1) & (0.8,-1) \\ (0.8,-1) & (0.8,-1) & (0.8,-1) & (0.8,-1) \end{pmatrix}$$

Thus, the bipolar fuzzy graph is self centered.

Example 4.3:

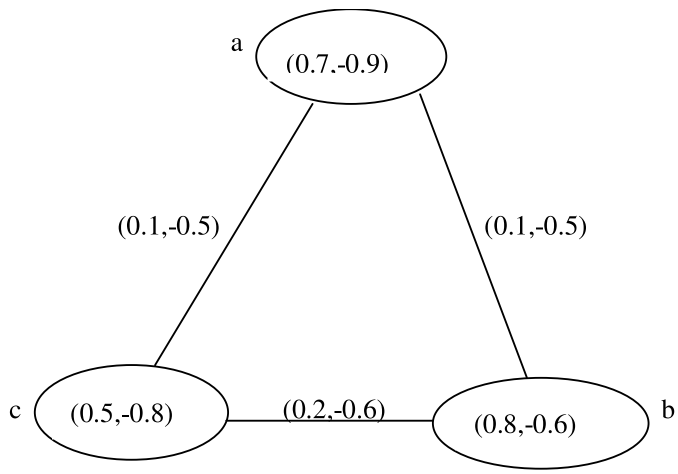

Figure 5: A Bipolar Fuzzy graph not Self Centered.

Consider the bipolar fuzzy graph shown in Figure 5.

The bipolar fuzzy distance matrix, the max-max and min-min composition are given below.

$$\left(D_{bf}^{P}, D_{bf}^{N}\right) = \begin{pmatrix} (0,0) & (0.1,-0.5) & (0.1,-0.5) \\ (0.1,-0.5) & (0,0) & (0.2,-0.6) \\ (0.1,-0.5) & (0.2,-0.6) & (0,0) \end{pmatrix}$$

$$\left((D_{bf}^{P} \circ D_{bf}^{P}),(D_{bf}^{N} \circ D_{bf}^{N})\right) = \begin{pmatrix} (0.1,-0.5) & (0.2,-0.6) & (0.2,-0.6) \\ (0.2,-0.6) & (0.2,-0.6) & (0.2,-0.6) \\ (0.2,-0.6) & (0.2,-0.6) & (0.2,-0.6) \end{pmatrix}$$

Clearly all the diagonal elements in the composition are not same, and hence the bipolar fuzzy graph is not self centered.

5. CONCLUSION

In this paper, a genuine effort is made to generalize the concept of distance. The concept of bipolar fuzzy distance is relevant as it represents the net flow between a given pair of nodes of a bipolar fuzzy graph. The concept of bipolar fuzzy center, bipolar fuzzy self centered graph and the complement of a bipolar fuzzy graph are also introduced. A characterization of bipolar fuzzy self centered graph is obtained.

REFERENCES

[1]. Akram. M. (2011). Bipolar fuzzy graphs, *Information Sciences*, DOI 10.1016/j.ins 2011.07.037,2011.

[2]. Bhattacharya, P. and Suraweera, F. (1991). An Algorithm to compute the max-min powers and a property of fuzzy graphs, *Pattern Recognition Lett.*, 12, 413-420.

[3]. Bhattacharya, P. (1987). Some Remarks on fuzzy graphs, *Pattern Recognition Lett.*, 6, 297-302.

[4]. Bhutani, K.R. and Rosenfeld, A. (2003). Strong arcs in fuzzy graphs, *Information Sciences*, 152, 319-322.

[5]. Bhutani, K.R. and Rosenfeld, A . (2003). Fuzzy end nodes in fuzzy graphs, *Information Sciences*, 152, 323-326.

[6]. Buckley, Fred and Harary, Frank (1990). Distance in graphs, Addision- Wesley, Redwood City, CA.

[7]. Mathew, S. and Sunitha, M.S. (2010). Node connectivity and arc connectivity of a fuzzy graph, *Information Sciences*, 180 (4), 519-531.

[8]. Sunitha, M.S. and Vijayakumar, A. (1999). A characterization of fuzzy trees, *Information Sciences*, 113, 293-300.

[9]. Sunitha, M.S. and Vijayakumar, A. (2005). Blocks in fuzzy graphs, *The Journal of Fuzzy Mathematics*, 13(1), 13-23.

[10]. Sunitha, M.S. and Vijayakumar, A. (2002). Complement of a fuzzy graph, *Indian Journal of Pure and Applied Mathematics*, 33(9), 1451-1464.

[11]. Linda, J.P. and Sunitha, M.S. (2014). Fuzzy detour g-centre in fuzzy graphs, *Annals of Fuzzy Mathematics and Informatics*, 7(2), 219-228.

[12]. Nagoor Gani, A. and Basheer Ahamed, M. (2003). Order and size in fuzzy graph, *Bulletin of Pure and Applied Sciences*, 22E (1), 145-148.

[13]. Yahya Mohamed S. and Suashini, N. (2017). Connectivity in Bipolar Fuzzy Graphs and its Complement, *Annals of Pure and Applied Mathematics*, Vol.15, No.1, 89-95.

[14]. Yahya Mohamed S. and Subashini, N. (2018). Structures of Edge Regular Bipolar Fuzzy Graphs, *International Journal of Mathematics Trends and Technology*, Vol. 58, Number 3, 215-220.

[15]. Yahya Mohamed S. and Subashini, N. (2018) Bipolar Fuzzy Graphs Based on Eccentricity Nodes, *Journal of Applies Science and Computations*, Vol. 5, Issue.11, 153-158 (Nov.2018).

[16]. Zadeh, L.A. (1965). Fuzzy sets, *Information and Control*, 8, 338-353.

[17]. Zhang, W.R. Bipolar fuzzy sets, *Proceedings of FUZZY- IEEE*, 835-840.