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SOME COMPOSITION PROPERTIES ON TOTALLY REGULAR INTUITIONISTIC FUZZY GRAPHS

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Abstract
In this paper, the Composition of totally regular intuitionistic fuzzy graphs need not be a totally regular intuitionistic fuzzy graph is discussed. Also the conditions for the Composition of totally regular intuitionistic fuzzy graphs to be totally regular under some restrictions are obtained.

Keywords: Total degree of a vertex, Composition, Regular IFG, Totally regular IFG.

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1. INTRODUCTION

Intuitionistic fuzzy graph theory was introduced by Atanassov in [1]. In [6] Nagoor Gani and Shajitha Begum introduced degree, order and size in intuitionistic fuzzy graph. In [10] Radha and Vijaya introduced the totally regular property of the composition of some fuzzy graphs. Nagoor Gani and Sheik Mujibur Rahman introduced the total degree of a vertex in union, join Cartesian product and composition of some intuitionistic fuzzy graphs in [8]. In this paper, we discuss some totally regular properties of the composition of intuitionistic fuzzy graphs.

2. PRELIMINARIES

Definition 2.1: An intuitionistic fuzzy graph (IFG) is of the form $G = \langle V, E \rangle$ where
(i) $V = \{v_1, v_2, \dots, v_n\}$ such that $\mu_1 : V \to [0,1]$ and $\nu_1 : V \to [0,1]$ denotes the degree of membership and non-membership of the element $v_i \in V$ respectively and $0 \le \mu_1(v_i) + \nu_1(v_i) \le 1$, for every $v_i \in V$.
(ii)$E \subseteq V \times V$ where,$\mu_2 : V \times V \to [0,1]$ and $\nu_2 : V \times V \to [0,1]$ such that

$$\mu_2(v_i, v_j) \le \min\left(\mu_1(v_i), \mu_1(v_j)\right)$$

$$\nu_2(v_i, v_j) \le \max\left(\nu_1(v_i), \nu_1(v_j)\right)$$

and $0 \le \mu_2(v_i, v_j) + \nu_2(v_i, v_j) \le 1$, for every $(v_i, v_j) \in E$.
Here the triple $(v_i, \mu_{1i}, \nu_{1i})$ denotes the degree of membership and non-membership of the vertex v_i. The triple $(e_{ij}, \mu_{2ij}, \nu_{2ij})$ denotes the degree of membership and non-membership of the edge relation $e_{ij} = (v_i, v_j)$ on $V \times V$.

Definition 2.2: Let $G = \langle V, E\rangle$ be an IFG. Then the degree of a vertex v is defined by $d(v) = (d_\mu(v), d_\nu(v))$, where $d_\mu(v) = \sum_{u\neq v} \mu_2\,(v,u)$ and $d_\nu(v) = \sum_{u\neq v} \nu_2\,(v,u)$.

Definition 2.3: Let $G = \langle V, E\rangle$ be an IFG. If $(d_\mu(v), d_\nu(v)) = (k_1, k_2)$ for all $v \in V$ that is if each vertex has same membership degree k_1 and same non-membership degree k_2 then G is said to be a regular intuitionistic fuzzy graph.

Definition 2.4: Let $G = \langle V, E\rangle$ be an IFG. Then the total degree of a vertex $u \in V$ is defined by

$$td\,(u) = (td_\mu(u), td_\nu(u)) = \left(\sum_{u\neq v} \mu_2\,(u,v) + \mu_1(u), \sum_{u\neq v} \nu_2\,(u,v) + \nu_1(u)\right)$$
$$= (d_\mu(u) + \mu_1(u), d_\nu(u) + \nu_1(u))$$

If each vertex of G has same membership total degree k_1 and same non-membership total degree k_2, then G is said to be a totally regular intuitionistic fuzzy graph.

Lemma 2.5: Let $G_1: (V, E)$ and $G_2: (V', E')$ be two intuitionistic fuzzy graphs.
If $\mu_1 \ge \mu_2', \mu_1' \ge \mu_2, \nu_1 \ge \nu_2', \nu_1' \ge \nu_2$, then

(i) $td_{\mu G_1[G_2]}(u_1, u_2) = d_{\mu(G_2)}(u_2) + p_2 d_{\mu(G_1)}(u_1) + \mu_1(u_1) \wedge \mu_1'(u_2)$

$td_{\nu G_1[G_2]}(u_1, u_2) = d_{\nu(G_2)}(u_2) + p_2 d_{\nu(G_1)}(u_1) + \nu_1(u_1) \vee \nu_1'(u_2)$

(ii) $td_{\mu G_1[G_2]}(u_1, u_2) = td_{\mu(G_2)}(u_2) + p_2 td_{\mu(G_1)}(u_1) - (p_2 - 1)\mu_1(u_1) \vee \mu_1'(u_2)$

$td_{\nu G_1[G_2]}(u_1, u_2) = td_{\nu(G_2)}(u_2) + p_2 td_{\nu(G_1)}(u_1) - (p_2 - 1)\nu_1(u_1) \wedge \nu_1'(u_2)$.

Theorem 2.6: Let $G_1: (V, E)$ and $G_2: (V', E')$ be two intuitionistic fuzzy graphs. If $\mu_1' \le \mu_2,\ \mu_1' \le \mu_1$ and $\nu_1' \le \nu_2, \nu_1' \le \nu_1$ then

$td_{\mu G_1[G_2]}(u_1, u_2) = d_{\mu(G_2)}(u_2) + \mu_1'(u_2)[p_2 d_{G_1^*}(u_1) + 1]$

$td_{\nu G_1[G_2]}(u_1, u_2) = d_{\nu(G_2)}(u_2) + \nu_1'(u_2)[p_2 d_{G_1^*}(u_1) + 1]$.

Theorem 2.7: Let $G_1: (V, E)$ and $G_2: (V', E')$ be two intuitionistic fuzzy graphs.

(i) If $\mu_1 \le \mu_2',\ \mu_1 \le \mu_1'$ and $\nu_1 \le \nu_2', \nu_1 \le \nu_1'$ then

$td_{\mu G_1[G_2]}(u_1, u_2) = p_2 td_{\mu(G_1)}(u_1) + \mu_1(u_1)\left[d_{G_2^*}(u_2) - p_2 + 1\right]$

$td_{\nu G_1[G_2]}(u_1, u_2) = p_2 td_{\nu(G_1)}(u_1) + \nu_1(u_1)[d_{G_2^*}(u_2) - p_2 + 1]$

(ii) If $\mu_1' \le \mu_2, \mu_1' \le \mu_1$ and $\nu_1' \le \nu_2, \nu_1' \le \nu_1$ then

$td_{\mu G_1[G_2]}(u_1, u_2) = td_{\mu(G_2)}(u_2) + p_2\mu_1'(u_2) d_{G_1^*}(u_1)$

$td_{\nu G_1[G_2]}(u_1, u_2) = td_{\nu(G_2)}(u_2) + p_2\nu_1'(u_2) d_{G_1^*}(u_1)$.

Theorem 2.8: Let $G_1: (V, E)$ and $G_2: (V', E')$ be two intuitionistic fuzzy graphs. If $\mu_1 \le \mu_2', \nu_1 \le \nu_2'$, μ_1 and ν_1 are constant functions and $\mu_1 \wedge \mu_1',\ \nu_1 \vee \nu_1'$ are also constant functions then

$td_{\mu G_1[G_2]}(u_1, u_2) = c_1 d_{G_2^*}(u_2) + p_2 d_{\mu(G_1)}(u_1) + C$

$td_{\nu G_1[G_2]}(u_1, u_2) = c_2 d_{G_2^*}(u_2) + p_2 d_{\nu(G_1)}(u_1) + C$

Theorem 2.9: Let $G_1: (V, E)$ and $G_2: (V', E')$ be two intuitionistic fuzzy graphs. If $\mu_1 \leq \mu_2', \nu_1 \leq \nu_2',\ \mu_1\ and\ \nu_1$ are constant functions and $\mu_1 \wedge \mu_1', \nu_1 \vee \nu_1'$ are also constant functions then

$td_{\mu G_1[G_2]}(u_1, u_2) = p_2 td_{\mu(G_1)}(u_1) + c_1[d_{G_2^*}(u_2) - p_2] + C$

$td_{\nu G_1[G_2]}(u_1, u_2) = p_2 td_{\nu(G_1)}(u_1) + c_2\left[d_{G_2^*}(u_2) - p_2\right] + C$.

3. TOTALLY REGULAR PROPERTY OF COMPOSITION

Example 3.1: In the figure 1, G_1 is a totally regular intuitionistic fuzzy graph and G_2 is not a totally regular intuitionistic fuzzy graph. But $G_1[G_2]$ is a totally regular intuitionistic fuzzy graph.

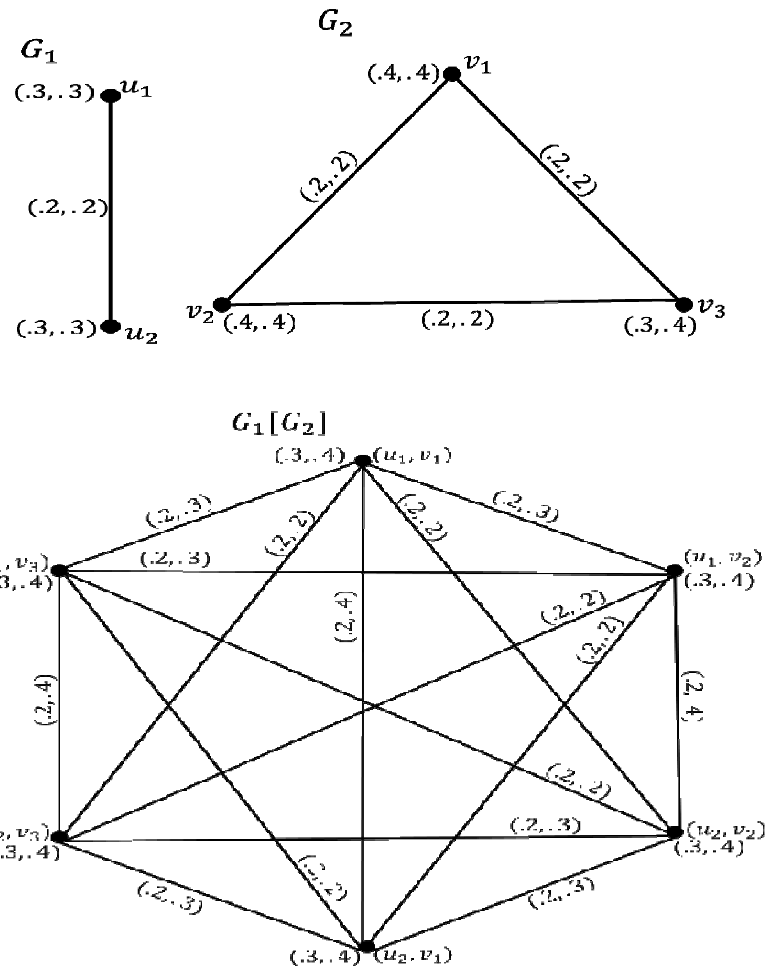


Figure 1:

Remark 3.2: The above example shows that the composition of totally regular intuitionistic fuzzy graphs need not be a totally regular intuitionistic fuzzy graph and if $G_1[G_2]$ is a totally regular intuitionistic fuzzy graph, both G_1 and G_2 need not be totally regular intuitionistic fuzzy graphs. In the following theorems, we obtain the conditions for the composition of two intuitionistic fuzzy graphs to be totally regular in some particular cases.

Theorem 3.3: Let $G_1:(V,E)$ and $G_2:(V',E')$ be two intuitionistic fuzzy graphs. If $\mu_1 \geq \mu_2', \mu_1' \geq \mu_2, \nu_1 \geq \nu_2', \nu_1' \geq \nu_2$ and $\mu_1 \wedge \mu_1', \nu_1 \vee \nu_1'$ are constant functions then $G_1[G_2]$ is a totally regular intuitionistic fuzzy graph if and only if G_1 and G_2 are regular intuitionistic fuzzy graphs.

Proof:

Let $\mu_1 \wedge \mu_1' = c_1,\ \nu_1 \vee \nu_1' = c_2$, forall $u \in V_1, v \in V_2$, where $c_1,\ c_2$ are constants. Suppose that G_1 and G_2 are regular intuitionistic fuzzy graphs of degrees $k_1 and\ k_2$ respectively. From lemma (2.5)

$td_{\mu G_1[G_2]}(u_1,u_2) = d_{\mu(G_2)}(u_2) + p_2 d_{\mu(G_1)}(u_1) + \mu_1(u_1) \wedge \mu_1'(u_2)$

$\Rightarrow\ \ td_{\mu G_1[G_2]}(u_1,u_2) = k_1 + p_2 k_2 + c_1$ (1)

$td_{\nu G_1[G_2]}(u_1,u_2) = d_{\nu(G_2)}(u_2) + p_2 d_{\nu(G_1)}(u_1) + \nu_1(u_1) \vee \nu_1'(u_2)$

$\Rightarrow\ \ td_{\nu G_1[G_2]}(u_1,u_2) = k_1 + p_2 k_2 + c_2$. (2)

Hence $G_1[G_2]$is a totally regular intuitionistic fuzzy graph. Conversely assume that $G_1[G_2]$is a totally regular intuitionistic fuzzy graph. Then for any two points $(u_1,u_2)$and (v_1,v_2)in $V_1 \times V_2$.

$td_{\mu G_1[G_2]}(u_1,u_2) = td_{\mu G_1[G_2]}(v_1,v_2)$ and $td_{\nu G_1[G_2]}(u_1,u_2) = td_{\nu G_1[G_2]}(v_1,v_2)$.

From (1),

$d_{\mu(G_2)}(u_2) + p_2 d_{\mu(G_1)}(u_1) + \mu_1(u_1) \wedge \mu_1'(u_2) = d_{\mu(G_2)}(v_2) + p_2 d_{\mu(G_1)}(v_1) + \mu_1(v_1) \wedge \mu_1'(v_2)$

$\Rightarrow d_{\mu(G_2)}(u_2) + p_2 d_{\mu(G_1)}(u_1) + c_1 = d_{\mu(G_2)}(v_2) + p_2 d_{\mu(G_1)}(v_1) + c_1$

$\Rightarrow d_{\mu(G_2)}(u_2) + p_2 d_{\mu(G_1)}(u_1) = d_{\mu(G_2)}(v_2) + p_2 d_{\mu(G_1)}(v_1)$ (3)

From (2),

$d_{\nu(G_2)}(u_2) + p_2 d_{\nu(G_1)}(u_1) + \nu_1(u_1) \vee \nu_1'(u_2) = d_{\nu(G_2)}(v_2) + p_2 d_{\nu(G_1)}(v_1) + \nu_1(v_1) \vee \nu_1'(v_2)$

$\Rightarrow d_{\nu(G_2)}(u_2) + p_2 d_{\nu(G_1)}(u_1) + c_2 = d_{\nu(G_2)}(v_2) + p_2 d_{\nu(G_1)}(v_1) + c_2$

$\Rightarrow d_{\nu(G_2)}(u_2) + p_2 d_{\nu(G_1)}(u_1) = d_{\nu(G_2)}(v_2) + p_2 d_{\nu(G_1)}(v_1)$(4)

Fix $u \in V_1$ and consider $(u,u_2) and\ (u,v_2)$in $V_1 \times V_2$where, $(u_1,u_2) \in V_2$ are arbitrary.

From (3),

$\Rightarrow d_{\mu(G_2)}(u_2) + p_2 d_{\mu(G_1)}(u) = d_{\mu(G_2)}(v_2) + p_2 d_{\mu(G_1)}(u)$

$\Rightarrow d_{\mu(G_2)}(u_2) = d_{\mu(G_2)}(v_2)$

From (4), $\Rightarrow d_{\nu(G_2)}(u_2) + p_2 d_{\nu(G_1)}(u) = d_{\nu(G_2)}(v_2) + p_2 d_{\nu(G_1)}(u)$

$\Rightarrow d_{\nu(G_2)}(u_2) = d_{\nu(G_2)}(v_2)$

This is true for all $(u_2,v_2) \in V_2$. Thus $G_2$is a regular intuitionistic fuzzy graph.

Fix $v \in V_2$and consider $(u_1,v)$and(u_2,v)in $V_1 \times V_2$where$(u_1,v_1) \in V_1$are arbitrary.

From (3),

$\Rightarrow d_{\mu(G_2)}(v) + p_2 d_{\mu(G_1)}(u_1) = d_{\mu(G_2)}(v) + p_2 d_{\mu(G_1)}(v_1)$

$\Rightarrow d_{\mu(G_1)}(u_1) = d_{\mu(G_1)}(v_1)$

From (4), $\Rightarrow d_{\nu(G_2)}(v) + p_2 d_{\nu(G_1)}(u_1) = d_{\nu(G_2)}(v) + p_2 d_{\nu(G_1)}(v_1)$

$\Rightarrow d_{\nu(G_1)}(u_1) = d_{\nu(G_1)}(v_1)$

This is true for all $(u_1,v_1) \in V_1$. Thus $G_1$is a regular intuitionistic fuzzy graph.

Theorem 3.4: Let $G_1:(V,E)$ and $G_2:(V',E')$ be two intuitionistic fuzzy graphs. If $\mu_1 \geq \mu_2'$,$\mu_1' \geq \mu_2$,$\nu_1 \geq \nu_2'$,$\nu_1' \geq \nu_2$ and $\mu_1 \vee \mu_1', \nu_1 \wedge \nu_1'$ are constant functions then $G_1[G_2]$ is a totally regular intuitionistic fuzzy graph if and only if G_1 and G_2 are totally regular intuitionistic fuzzy graphs.

Proof:

Proof is similar to the proof of theorem 3.3.

Theorem 3.5: Let $G_1:(V,E)$ and $G_2:(V',E')$ be two intuitionistic fuzzy graphs. If $\mu_1' \leq \mu_2$, $\nu_1' \leq \nu_2$ and μ_1', ν_1' are constant functions then $G_1[G_2]$ is a totally regular intuitionistic fuzzy graph if and only if G_2 is a regular intuitionistic fuzzy graph and $G_1^*$is a regular graph.

Proof:

We have $\mu_1' \leq \mu_2, \nu_1' \leq \nu_2$. Hence $\mu_1 \geq \mu_2'$, $\mu_1' \leq \mu_1$and $\nu_1 \geq \nu_2', \nu_1' \leq \nu_1$.Let $\mu_1'(u_2) = c_1$, $\nu_1'(u_2) = c_2$, forall $u \in V_1$, where $c_1, c_2$are constants. Suppose that $G_2$is a regular intuitionistic fuzzy graph of degree k_2 and $G_1$is a regular graph of degree r_1. By definition, for any $(u_1,u_2) \in V_1 \times V_2$,

from (2.6), $td_{\mu G_1[G_2]}(u_1,u_2) = d_{\mu(G_2)}(u_2) + \mu_1'(u_2)[p_2 d_{G_1^*}(u_1) + 1] = d_{\mu(G_2)}(u_2) + c_1[p_2 d_{G_1^*}(u_1) + 1]$

$$\Rightarrow td_{\mu G_1[G_2]}(u_1,u_2) = k_2 + c_1[p_2 r_1 + 1].$$

Also from (2.6) follows that, $td_{\nu G_1[G_2]}(u_1,u_2) = d_{\nu(G_2)}(u_2) + \nu_1'(u_2)[p_2 d_{G_1^*}(u_1) + 1]$

$$= d_{\nu(G_2)}(u_2) + c_2[p_2 d_{G_1^*}(u_1) + 1]$$

$$\Rightarrow td_{\nu G_1[G_2]}(u_1,u_2) = k_2 + c_2[p_2 r_1 + 1].$$

Hence $G_1[G_2]$is a totally regular intuitionistic fuzzy graph.

Conversely assume that $G_1[G_2]$is a totally regular intuitionistic fuzzy graph. Then for any two points (u_1,u_2) *and* (v_1,v_2) *in* $V_1 \times V_2$.

$td_{\mu G_1[G_2]}(u_1,u_2) = td_{\mu G_1[G_2]}(v_1,v_2)$

$\Rightarrow d_{\mu(G_2)}(u_2) + c_1[p_2 d_{G_1^*}(u_1) + 1] = d_{\mu(G_2)}(v_2) + c_1[p_2 d_{G_1^*}(v_1) + 1]$ (5)

$td_{\nu G_1[G_2]}(u_1,u_2) = td_{\nu G_1[G_2]}(v_1,v_2)$

$\Rightarrow d_{\nu(G_2)}(u_2) + c_2[p_2 d_{G_1^*}(u_1) + 1] = d_{\nu(G_2)}(v_2) + c_2[p_2 d_{G_1^*}(v_1) + 1]$ (6)

Fix $u \in V_1$and consider(u,u_2) and (u,v_2) in $V_1 \times V_2$where, $(u_1,u_2) \in V_2$are arbitrary.

From (5), $d_{\mu(G_2)}(u_2) + c_1[p_2 d_{G_1^*}(u) + 1] = d_{\mu(G_2)}(v_2) + c_1[p_2 d_{G_1^*}(u) + 1]$

$\Rightarrow d_{\mu(G_2)}(u_2) = d_{\mu(G_2)}(v_2)$.

Now (6) $\Rightarrow d_{\nu(G_2)}(u_2) + c_2[p_2 d_{G_1^*}(u) + 1] = d_{\nu(G_2)}(v_2) + c_2[p_2 d_{G_1^*}(u) + 1]$

$\Rightarrow d_{\nu(G_2)}(u_2) = d_{\nu(G_2)}(v_2)$.

This is true for all $(u_2,v_2) \in V_2$.Thus G_2 is a regular intuitionistic fuzzy graph.

Fix $v \in V_2$and consider(u_1,v) and (u_2,v)in $V_1 \times V_2$where$(u_1,v_1) \in V_1$ are arbitrary.

From (5), $d_{\mu(G_2)}(v) + c_1[p_2 d_{G_1^*}(u_1) + 1] = d_{\mu(G_2)}(v) + c_1[p_2 d_{G_1^*}(v_1) + 1]$

$\Rightarrow c_1[p_2 d_{G_1^*}(u_1) + 1] = c_1[p_2 d_{G_1^*}(v_1) + 1] \Rightarrow d_{G_1^*}(u_1) = d_{G_1^*}(v_1)$.

This is true for all $(u_1,v_1) \in V_1$. Thus $G_1^*$is a regular graph.

Theorem 3.6: Let $G_1:(V,E)$ and $G_2:(V',E')$ be two intuitionistic fuzzy graphs. If $\mu_1 \leq \mu_2'$, $\nu_1 \leq \nu_2'$ and μ_1, ν_1 are constant functions then $G_1[G_2]$ is a totally regular intuitionistic fuzzy graph if and only if G_2 is a regular intuitionistic fuzzy graph and G_2^* is a regular graph.

Proof:

Proof is similar to the proof of theorem 3.5.

Theorem 3.7: Let $G_1:(V,E)$ and $G_2:(V',E')$ be two intuitionistic fuzzy graphs. If $\mu_1' \leq \mu_2$, $\nu_1' \leq \nu_2$ and μ_1', ν_1' are constant functions then $G_1[G_2]$ is a totally regular intuitionistic fuzzy graph if and only if G_2 is a totally regular intuitionistic fuzzy graph and $G_1^*$is a regular graph.

Proof:

We have $\mu_1' \leq \mu_2$, $\nu_1' \leq \nu_2$. Hence $\mu_1 \geq \mu_2'$, $\mu_1' \leq \mu_1$ and $\nu_1 \geq \nu_2'$, $\nu_1' \leq \nu_1$.

Let$\mu_1'(u_2) = c_1$, $\nu_1'(u_2) = c_2$, forall $u \in V_1$, where c_1, c_2 are constants. Suppose that $G_2$is a totally regular intuitionistic fuzzy graph of degree $k_2$and G_1 is a regular graph of degreer_1. By definition, for any $(u_1,u_2) \in V_1 \times V_2$,

from (2.7) we get, $td_{\mu G_1[G_2]}(u_1,u_2) = td_{\mu(G_2)}(u_2) + p_2\mu_1'(u_2)d_{G_1^*}(u_1) = k_2 + c_1p_2r_1$

and from (2.7) follows that, $td_{\nu G_1[G_2]}(u_1,u_2) = td_{\nu(G_2)}(u_2) + p_2\nu_1'(u_2)d_{G_1^*}(u_1) = k_2 + c_2p_2r_1$

Hence $G_1[G_2]$is a totally regular intuitionistic fuzzy graph.

Conversely assume that $G_1[G_2]$is a totally regular intuitionistic fuzzy graph. Then for any two points (u_1,u_2) and (v_1,v_2) in $V_1 \times V_2$,

$td_{\mu G_1[G_2]}(u_1,u_2) = td_{\mu G_1[G_2]}(v_1,v_2)$

$\Rightarrow td_{\mu(G_2)}(u_2) + p_2\mu_1'(u_2)d_{G_1^*}(u_1) = td_{\mu(G_2)}(v_2) + p_2\mu_1'(v_2)d_{G_1^*}(v_1)$

$\Rightarrow td_{\mu(G_2)}(u_2) + c_1p_2d_{G_1^*}(u_1) = td_{\mu(G_2)}(v_2) + c_1p_2d_{G_1^*}(v_1)$ (7)

$$td_{\nu G_1[G_2]}(u_1,u_2) = td_{\nu G_1[G_2]}(v_1,v_2)$$

$\Rightarrow td_{\nu(G_2)}(u_2) + p_2\nu_1'(u_2)d_{G_1^*}(u_1) = td_{\nu(G_2)}(v_2) + p_2\nu_1'(v_2)d_{G_1^*}(v_1)$

$\Rightarrow td_{\nu(G_2)}(u_2) + c_2p_2d_{G_1^*}(u_1) = td_{\nu(G_2)}(v_2) + c_2p_2d_{G_1^*}(v_1)$ (8)

Fix $u \in V_1$ and consider $(u,u_2)$and(u,v_2) in $V_1 \times V_2$ where$(u_2,v_2) \in V_2$are arbitrary.

From (7), $td_{\mu(G_2)}(u_2) + c_1p_2d_{G_1^*}(u) = td_{\mu(G_2)}(v_2) + c_1p_2d_{G_1^*}(u)$

$\Rightarrow td_{\mu(G_2)}(u_2) = td_{\mu(G_2)}(v_2)$.

From (8), $td_{\nu(G_2)}(u_2) + c_2p_2d_{G_1^*}(u) = td_{\nu(G_2)}(v_2) + c_2p_2d_{G_1^*}(u)$

$\Rightarrow td_{\nu(G_2)}(u_2) = td_{\nu(G_2)}(v_2)$.

This is true for all$(u_2,v_2) \in V_2$.

Thus G_2 is a totally regular intuitionistic fuzzy graph.

Fix $v \in V_2$ and consider $(u_1,v)$and(u_2,v) in $V_1 \times V_2$where $(u_1,v_1) \in V_1$ are arbitrary.

From (7),$td_{\mu(G_2)}(v) + c_1p_2d_{G_1^*}(u_1) = td_{\mu(G_2)}(v) + c_1p_2d_{G_1^*}(u_1)$ $\Rightarrow d_{G_1^*}(u_1) = d_{G_1^*}(v_1)$.

This is true for all $(u_1,v_1) \in V_1$. Thus G_1^* is a regular graph.

Theorem 3.8: Let $G_1:(V,E)$ and $G_2:(V',E')$ be two intuitionistic fuzzy graphs. If $\mu_1 \leq \mu_2'$, $\nu_1 \leq \nu_2'$ and μ_1, ν_1 are constant functions then $G_1[G_2]$ is a totally regular intuitionistic fuzzy graph if and only if G_1 is a totally regular intuitionistic fuzzy graph and $G_2^*$is a regular graph.

Proof:

Proof is similar to the proof of theorem 3.7.

Theorem 3.9: Let $G_1:(V,E)$ and $G_2:(V',E')$ be two intuitionistic fuzzy graphs. If$\mu_1 \leq \mu_2'$, $\nu_1 \leq \nu_2'$ and μ_1, ν_1 are constant functions and $\mu_1 \wedge \mu_1', \nu_1 \vee \nu_1'$ are also constant functions then $G_1[G_2]$ is a totally regular intuitionistic fuzzy graph if and only if $G_2^*$is a regular graph and $G_1$is a regular intuitionistic fuzzy graph.

Proof:

Let $\mu_1 \wedge \mu_1' = c_1and\nu_1 \vee \nu_1' = c_2$, for all $u \in V_1$, $v \in V_2$, where c_1, $c_2$are constants. We have $\mu_1 \leq \mu_2'$, $\nu_1 \leq \nu_2'$. Hence $\mu_1' \geq \mu_2, \nu_1' \geq \nu_2$. Suppose that $G_1$is a regular intuitionistic fuzzy graph of degree $k_1$and $G_2$is a regular graph of degree r_2.

From (2.8), $td_{\mu G_1[G_2]}(u_1,u_2) = c_1 d_{G_2^*}(u_2) + p_2 d_{\mu(G_1)}(u_1) + C$

$$\Rightarrow td_{\mu G_1[G_2]}(u_1,u_2) = c_1 r_2 + p_2 k_1 + C \tag{9}$$

From (2.8), $td_{\nu G_1[G_2]}(u_1,u_2) = c_2 d_{G_2^*}(u_2) + p_2 d_{\nu(G_1)}(u_1) + C$

$$\Rightarrow td_{\nu G_1[G_2]}(u_1,u_2) = c_2 r_2 + p_2 k_1 + C \tag{10}$$

Hence $G_1[G_2]$ is a totally regular intuitionistic fuzzy graph.

Conversely assume that $G_1[G_2]$ is a totally regular intuitionistic fuzzy graph. Then for any two points(u_1,u_2) and (v_1,v_2) in $V_1 \times V_2$,

$td_{\mu G_1[G_2]}(u_1,u_2) = td_{\mu G_1[G_2]}(v_1,v_2)$

$\Rightarrow c_1 d_{G_2^*}(u_2) + p_2 d_{\mu(G_1)}(u_1) + C = c_1 d_{G_2^*}(v_2) + p_2 d_{\mu(G_1)}(v_1) + C$

$\Rightarrow c_1 d_{G_2^*}(u_2) + p_2 d_{\mu(G_1)}(u_1) = c_1 d_{G_2^*}(v_2) + p_2 d_{\mu(G_1)}(v_1)$ (11)

$td_{\nu G_1[G_2]}(u_1,u_2) = td_{\nu G_1[G_2]}(v_1,v_2)$

$\Rightarrow c_2 d_{G_2^*}(u_2) + p_2 d_{\nu(G_1)}(u_1) + C = c_2 d_{G_2^*}(v_2) + p_2 d_{\nu(G_1)}(v_1) + C$

$\Rightarrow c_2 d_{G_2^*}(u_2) + p_2 d_{\nu(G_1)}(u_1) = c_2 d_{G_2^*}(v_2) + p_2 d_{\nu(G_1)}(v_1)$ (12)

Fix $u \in V_1$ and consider$(u,u_2)$and(u,v_2) in $V_1 \times V_2$where $(u_1,v_2) \in V_2$ are arbitrary.

From (11),$c_1 d_{G_2^*}(u_2) + p_2 d_{\mu(G_1)}(u) = c_1 d_{G_2^*}(v_2) + p_2 d_{\mu(G_1)}(u) \Rightarrow d_{G_2^*}(u_2) = d_{G_2^*}(v_2)$.

This is true for all $(u_2,v_2) \in V_2$.Thus $G_2^*$is a regular graph.

Fix $v \in V_2$ and consider(u_1,v) and (u_2,v) in $V_1 \times V_2$where$(u_1,v_1) \in V_1$is arbitrary.

From (11), $c_1 d_{G_2^*}(v) + p_2 d_{\mu(G_1)}(u_1) = c_1 d_{G_2^*}(v) + p_2 d_{\mu(G_1)}(v_1) \Rightarrow d_{\mu(G_1)}(u_1) = d_{\mu(G_1)}(v_1)$

From (12), $c_2 d_{G_2^*}(v) + p_2 d_{\nu(G_1)}(u_1) = c_2 d_{G_2^*}(v) + p_2 d_{\nu(G_1)}(v_1) \Rightarrow d_{\nu(G_1)}(u_1) = d_{\nu(G_1)}(v_1)$

This is true for all $(u_1,v_1) \in V_1$. Thus $G_1$is a regular intuitionistic fuzzy graph.

Theorem 3.10: Let $G_1:(V,E)$ and $G_2:(V',E')$ be two intuitionistic fuzzy graphs. If $\mu_1' \leq \mu_2$, $\nu_1' \leq \nu_2$ and μ_1', ν_1' are constant functions and $\mu_1 \wedge \mu_1', \nu_1 \vee \nu_1'$ are also constant functions then $G_1[G_2]$ is a totally regular intuitionistic fuzzy graph if and only if $G_2$is a regular intuitionistic fuzzy graph and $G_1^*$is a regular graph.

Proof:

Proof is similar to the proof of theorem 3.9.

Theorem 3.11: Let $G_1:(V,E)$ and $G_2:(V',E')$ be two intuitionistic fuzzy graphs. If $\mu_1 \leq \mu_2'$, $\nu_1 \leq \nu_2'$ and μ_1, ν_1 are constant functions and $\mu_1 \wedge \mu_1', \nu_1 \vee \nu_1'$ are also constant functions then $G_1[G_2]$ is a totally regular intuitionistic fuzzy graph if and only if G_2^* is a regular graph and G_1 is a totally regular intuitionistic fuzzy graph.

Proof:

Let $\mu_1 \wedge \mu_1' = c_1$ and $\nu_1 \vee \nu_1' = c_2$, for all $u \in V_1$, $v \in V_2$, where c_1, c_2 are constants.

We have $\mu_1 \leq \mu_2'$, $\nu_1 \leq \nu_2'$. Hence $\mu_1' \geq \mu_2, \nu_1' \geq \nu_2$. Suppose that G_1 is a totally regular intuitionistic fuzzy graph of degree k_1 and G_2 is a regular graph of degree r_2.

From (2.9), $td_{\mu G_1[G_2]}(u_1,u_2) = p_2 td_{\mu(G_1)}(u_1) + c_1[d_{G_2^*}(u_2) - p_2] + C$

$\Rightarrow td_{\mu G_1[G_2]}(u_1,u_2) = p_2 k_1 + c_1[r_2 - p_2] + C$ (13)

From (2.9), $td_{\nu G_1[G_2]}(u_1,u_2) = p_2 td_{\nu(G_1)}(u_1) + c_2[d_{G_2^*}(u_2) - p_2] + C$

$\Rightarrow td_{\nu G_1[G_2]}(u_1,u_2) = p_2 k_1 + c_2[r_2 - p_2] + C$ (14)

Hence $G_1[G_2]$ is a totally regular intuitionistic fuzzy graph.

Conversely assume that $G_1[G_2]$ is a totally regular intuitionistic fuzzy graph then for any two points (u_1,u_2) and (v_1,v_2) in $V_1 \times V_2$

$td_{\mu G_1[G_2]}(u_1,u_2) = td_{\mu G_1[G_2]}(v_1,v_2)$

$\Rightarrow p_2 td_{\mu(G_1)}(u_1) + c_1\left[d_{G_2^*}(u_2) - p_2\right] + C = p_2 td_{\mu(G_1)}(v_1) + c_1\left[d_{G_2^*}(v_2) - p_2\right] + C$

$\Rightarrow p_2 td_{\mu(G_1)}(u_1) + c_1\left[d_{G_2^*}(u_2) - p_2\right] = p_2 td_{\mu(G_1)}(v_1) + c_1\left[d_{G_2^*}(v_2) - p_2\right]$ (15)

$td_{\nu G_1[G_2]}(u_1,u_2) = td_{\nu G_1[G_2]}(v_1,v_2)$

$\Rightarrow p_2 td_{\nu(G_1)}(u_1) + c_2\left[d_{G_2^*}(u_2) - p_2\right] + C = p_2 td_{\nu(G_1)}(v_1) + c_2\left[d_{G_2^*}(v_2) - p_2\right] + C$

$\Rightarrow p_2 td_{\nu(G_1)}(u_1) + c_2\left[d_{G_2^*}(u_2) - p_2\right] = p_2 td_{\nu(G_1)}(v_1) + c_2\left[d_{G_2^*}(v_2) - p_2\right]$ (16)

Fix $u \in V_1$ and consider (u,u_2) and (u,v_2) in $V_1 \times V_2$ where $(u_1,v_2) \in V_2$ is arbitrary.

From (15), $p_2 td_{\mu(G_1)}(u) + c_1\left[d_{G_2^*}(u_2) - p_2\right] = p_2 td_{\mu(G_1)}(u) + c_1\left[d_{G_2^*}(v_2) - p_2\right]$

$\Rightarrow c_1\left[d_{G_2^*}(u_2) - p_2\right] = c_1\left[d_{G_2^*}(v_2) - p_2\right] \Rightarrow d_{G_2^*}(u_2) = d_{G_2^*}(v_2)$.

This is true for all $(u_2,v_2) \in V_2$. Thus G_2^* is a regular graph.

Fix $v \in V_2$ and consider (u_1,v) and (u_2,v) I $V_1 \times V_2$ where $(u_1,v_1) \in V_1$ is arbitrary.

From (15), $p_2 td_{\mu(G_1)}(u_1) + c_1\left[d_{G_2^*}(v) - p_2\right] = p_2 td_{\mu(G_1)}(v_1) + c_1\left[d_{G_2^*}(v) - p_2\right]$

$\Rightarrow td_{\mu(G_1)}(u_1) = td_{\mu(G_1)}(v_1)$

From (16), $p_2 td_{\nu(G_1)}(u_1) + c_2\left[d_{G_2^*}(v) - p_2\right] = p_2 td_{\nu(G_1)}(v_1) + c_2\left[d_{G_2^*}(v) - p_2\right]$

$\Rightarrow td_{\nu(G_1)}(u_1) = td_{\nu(G_1)}(v_1)$.

This is true for all $(u_1,v_1) \in V_1$. Thus G_1 is a totally regular intuitionistic fuzzy graph.

Theorem 3.12: Let $G_1:(V,E)$ and $G_2:(V',E')$ be two intuitionistic fuzzy graphs. If $\mu_1' \leq \mu_2$, $\nu_1' \leq \nu_2$ and μ_1', ν_1' are constant functions and $\mu_1 \wedge \mu_1', \nu_1 \vee \nu_1'$ are also constant functions then $G_1[G_2]$ is a totally regular intuitionistic fuzzy graph if and only if G_1^* is a regular graph and G_2 is a totally regular intuitionistic fuzzy graph.

Proof:

Proof is similar to the proof of theorem 3.11.

Theorem 3.13: Let $G_1:(V,E)$ and $G_2:(V',E')$ be two intuitionistic fuzzy graphs such that $\mu_1 \leq \mu_2'$, $\nu_1 \leq \nu_2'$. If $G_1[G_2]$ is a totally regular intuitionistic fuzzy graph, G_1 is a regular intuitionistic fuzzy graph and G_2^* is a regular graph then μ_1, ν_1 are constant functions.

Proof:

Suppose that G_1 is a k- totally regular intuitionistic fuzzy graph, G_2^* is a r_2- regular graph. Since $\mu_1 \leq \mu_2', \mu_1 \leq \mu_1'$, and $\nu_1 \leq \nu_2', \nu_1 \leq \nu_1'$.

From (2.7) follows that, $td_{\mu G_1[G_2]}(u_1,u_2) = p_2 td_{\mu(G_1)}(u_1) + \mu_1(u_1)[d_{G_2^*}(u_2) - p_2 + 1]$

and, $td_{\nu G_1[G_2]}(u_1,u_2) = p_2 td_{\nu(G_1)}(u_1) + \nu_1(u_1)[d_{G_2^*}(u_2) - p_2 + 1]$

Since $G_1[G_2]$ is a totally regular intuitionistic fuzzy graph, for any two points (u_1,u_2) and (v_1,v_2) in $V_1 \times V_2$,

$td_{\mu G_1[G_2]}(u_1,u_2) = td_{\mu G_1[G_2]}(v_1,v_2)$

$p_2 td_{\mu(G_1)}(u_1) + \mu_1(u_1)\left[d_{G_2^*}(u_2) - p_2 + 1\right] = p_2 td_{\mu(G_1)}(v_1) + \mu_1(v_1)[d_{G_2^*}(u_2) - p_2 + 1]$

$p_2 k + \mu_1(u_1)[r_2 - p_2 + 1] = p_2 k + \mu_1(v_1)[r_2 - p_2 + 1]$

$\Rightarrow \mu_1(u_1)[r_2 - p_2 + 1] = \mu_1(v_1)[r_2 - p_2 + 1] \Rightarrow \mu_1(u_1) = \mu_1(v_1)$

$td_{\nu G_1[G_2]}(u_1,u_2) = td_{\nu G_1[G_2]}(v_1,v_2)$

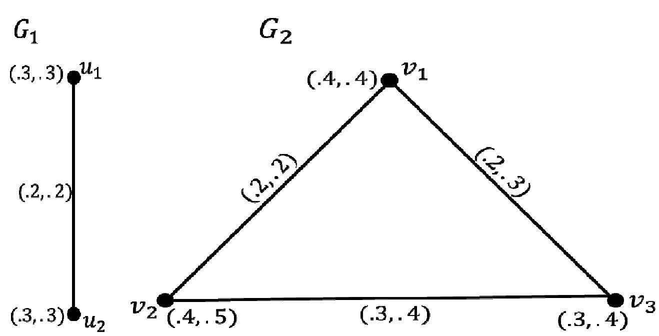
$p_2 td_{\nu(G_1)}(u_1) + \nu_1(u_1)\left[d_{G_2^*}(u_2) - p_2 + 1\right] = p_2 td_{\nu(G_1)}(v_1) + \nu_1(v_1)[d_{G_2^*}(u_2) - p_2 + 1]$

$p_2 k + \nu_1(u_1)[r_2 - p_2 + 1] = p_2 k + \nu_1(v_1)[r_2 - p_2 + 1]$

$\Rightarrow \nu_1(u_1)[r_2 - p_2 + 1] = \nu_1(v_1)[r_2 - p_2 + 1] \Rightarrow \nu_1(u_1) = \nu_1(v_1)$.

This is true for all $(u_1, v_1) \in V_1$ hence μ_1, ν_1 are constant functions.

Remark 3.14: Converse of the theorem 3.13 need not be true. For example, in figure 2 G_1 is both regular and totally regular intuitionistic fuzzy graph, G_2^* is regular but $G_1[G_2]$ is not a totally regular intuitionistic fuzzy graph. Here (μ_1, ν_1) are constant functions.



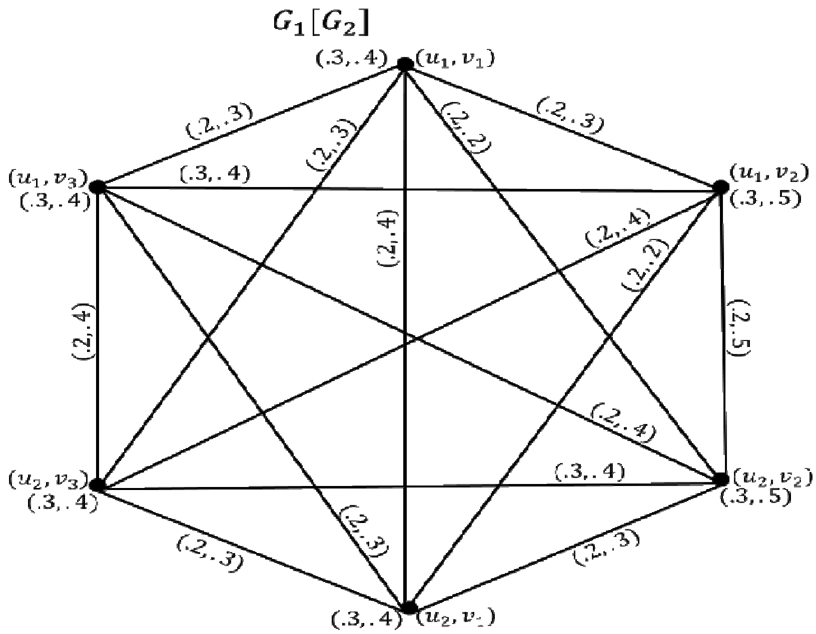


Figure 2:

4. CONCLUSION

In this paper, we have discussed that the composition of two totally regular intuitionistic fuzzy graphs need not be a totally regular intuitionistic fuzzy graph. We have also obtained the conditions for the composition of two intuitionistic fuzzy graphs to be totally regular in some particular cases.

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