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New types of graphs on cordial labeling *

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Abstract Let f be a function from the vertices of a graph G to $\{0, 1\}$ and for each edge xy assign the label $|f(x) - f(y)|$. f is called as a *cordial labeling* of G if the number of vertices labeled 0 and the number of vertices labeled 1 differ by at most 1 and the number of edges labeled 0 and the number of edges labeled 1 also differ at most by 1. In this paper we prove that star glued with subdivided shell graph and super subdivision of circular ladder graph admit cordial labeling.

Key words Shell graph, subdivided shell graph, star graph, super subdivision, circular ladder graph.

2020 Mathematics Subject Classification 05C78.

1 Introduction

Graph labeling is an assignment of labels, traditionally represented by integers to vertices or edges or both of a graph. The majority of graph labeling approaches may be traced back to one developed by Rosa [6] in 1967. Cahit [1] discovered cordial labeling in 1987 and it was discovered to be the lesser variant of graceful and harmonious labeling. Let f be a function from the vertices of a graph G to $\{0, 1\}$ and for each edge xy assign the label $|f(x) - f(y)|$. f is called a *cordial labeling* [3] of G if the number of vertices labeled 0 and the number of vertices labeled 1 differ by at most 1 and the number of edges labeled 0 and the number of edges labeled 1 also differ at most by 1. In [2] Cahit established the following: every tree is cordial, $K_{m,n}$ is cordial for all m and n. All fans are cordial. Ho et al. [4] examined that a unicyclic graph is cordial unless it is C_{4k+2} and that the generalized Petersen graph $P(n, k)$ is cordial if and only if $n \not\equiv 2(\text{mod } 4)$. Liu and Zhu [5] proved that a 3-regular graph of order$n$ is cordial if and only if $n \not\equiv 4$ (mod 8). The labeled graph is incredibly useful in coding theory, astronomy, communication networks, circuit design and crystallographic research. Cordial labeling is found to be useful in DNA code word design problems and in noisy communication channels.

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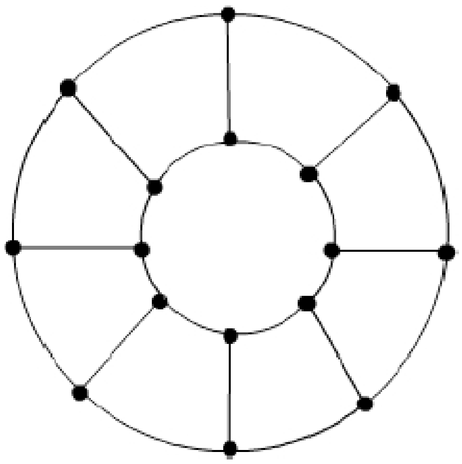

Fig. 1: A circular ladder graph.

In this paper, we prove that star glued with subdivided shell graph and super subdivision of circular ladder graph admits cordial labeling.

2 Preliminary definitions

In this section we state a few definitions which are relevant for proving the main results.

Definition 2.1. The shell graph is defined as a cycle C_n with $(n-3)$ chords sharing a common end point called the apex. A shell graph is denoted by $C_{n,n-3}$.

Definition 2.2. The subdivided shell graph is a shell graph in which the edges in the path of the shell are subdivided.

Definition 2.3. A Star graph denoted as $k_{1,n}$ is a tree with one vertex adjacent to every other vertex as n number of pendant vertices are connected to one vertex.

Definition 2.4. A circular ladder graph is defined as a cartesian product $C_n \times K_2$ where K_2 is a complete graph on two vertices and C_n is the cycle on n vertices (see Fig. 1).

Definition 2.5. A graph H is said to be a super subdivision of G if every edge uv of G is replaced by $K_{2,m}$ (m vary for each edge) by identifying u and v with two vertices in $K_{2,m}$ that form the partite set with exactly two members.

3 Main results

In this section we prove the following two theorems.

Theorem 3.1. *A star glued with subdivided shell graph is cordial.*

Proof. Let $G = (SSG)^*$ be the star glued with subdivided shell graph (SSG).The glued graph G is obtained by attaching the apex of the subdivided shell to each pendant vertex of the star. Let $r_1^1, r_2^1, \ldots, r_n^1$ be the vertices of the first copy of the subdivided shell graph. Let $r_1^2, r_2^2, \ldots, r_n^2$ be the vertices of the second copy of the subdivided shell graph. In general, $r_1^m, r_2^m, \ldots, r_n^m$ be the m^{th} copy of the subdivided shell graph taken in the clockwise direction. Let r_i^j be the apex of the subdivided shell graph where $j = 1, 2, 3, \ldots, m$. Fix the central vertex of the star as $k = 0$. The graph G is shown in Fig. 2.
The vertex set and the edge set are defined as $V(G) = mn+1$, $E(G) = \frac{3mn}{2} - m$. When $n \equiv 0 (\text{mod } 4)$, we define the vertex labeling as below:

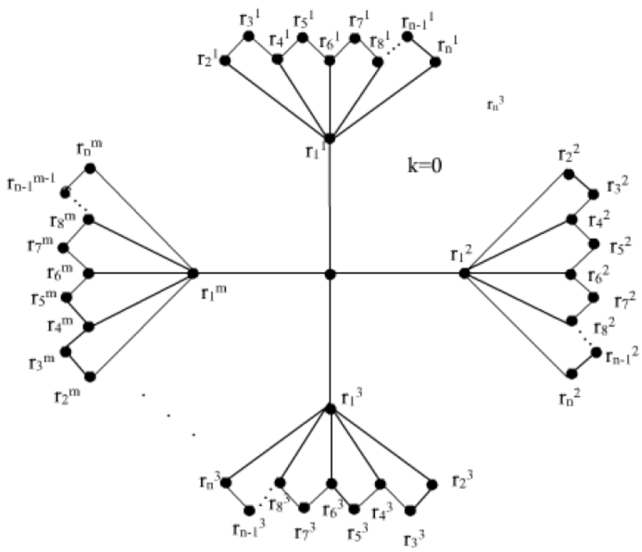

Fig. 2: The graph G with the star glued with subdivided shell graphs.

Case 1: If $j \equiv 1(\text{mod } 2)$

$$f(r_i^j) = \begin{cases} 0, & \text{if } i \equiv 1, 2(\text{mod } 4) \\ 1, & \text{if } i \equiv 0, 3(\text{mod } 4) \end{cases}$$

Case 2: If $j \equiv 0(\text{mod } 2)$

$$f(r_i^j) = \begin{cases} 0, & \text{if } i \equiv 0, 3(\text{mod } 4) \\ 1, & \text{if } i \equiv 1, 2(\text{mod } 4) \end{cases}$$

The number of vertices labeled with 0 and 1 are defined as follows:

$$V_f(0) = \left\lfloor \frac{mn}{2} \right\rfloor + 1,$$

$$V_f(1) = \left\lfloor \frac{mn}{2} \right\rfloor .$$

The number of edges labeled with 0 and 1 ares defined as follows:

Case (a): When m is odd,

$$e_f(0) = \frac{3mn}{4} - \left\lfloor \frac{m}{2} \right\rfloor ,$$

$$e_f(1) = \frac{3mn}{4} - \left\lfloor \frac{m}{2} \right\rfloor - 1.$$

Case (b): When m is even,

$$e_f(0) = \frac{3mn}{4} - \left\lfloor \frac{m}{2} \right\rfloor = e_f(1) .$$

From the above labeling pattern we can observe that $|V_f(0) - V_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. Therefore, the star glued with the subdivided shell graph admits cordial labeling. The illustrations for the cordial labeling of the graph G when m is odd and m is even are shown respectively in Fig. 3 and Fig. 4. □

Theorem 3.2. *The super subdivision of a circular ladder graph is cordial.*

Proof. Let $G = SSD(Cl_n)$ be the super subdivision of a circular ladder graph. The graph G is obtained by super subdivision of CL_n by a complete bipartite graph $K_{2,t}$ and the description is as follows: First label the vertices of CL_n and then label the attached complete bipartite graph $K_{2,t}$. The

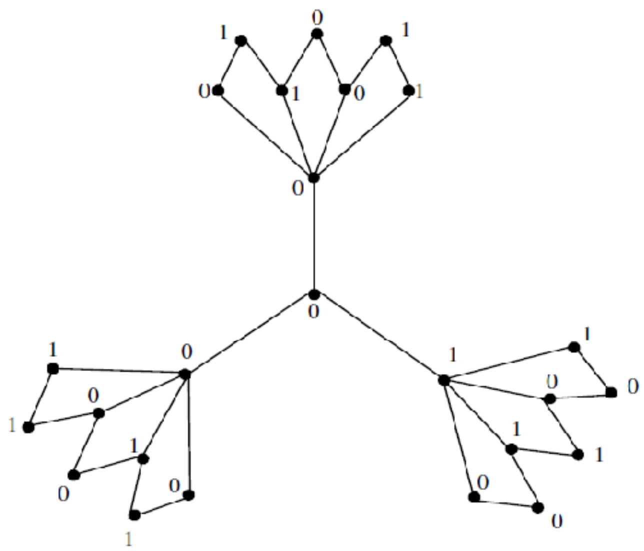

Fig. 3: The graph G when n is even and m is odd.

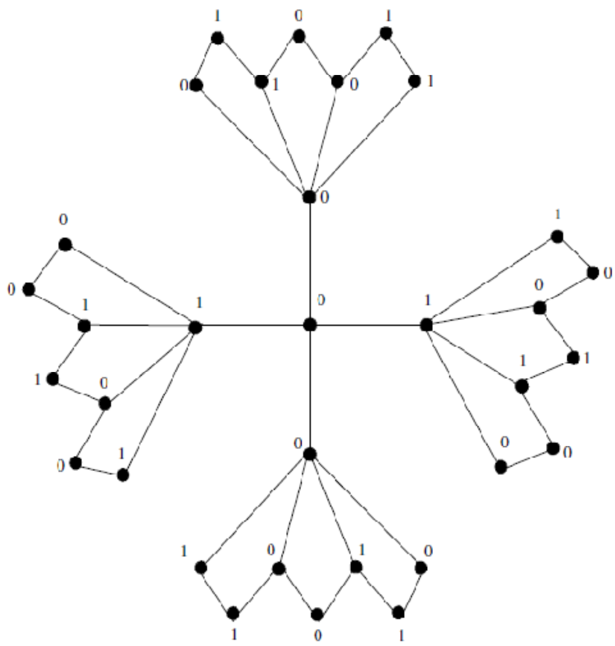

Fig. 4: The graph G when n is even and m is even.

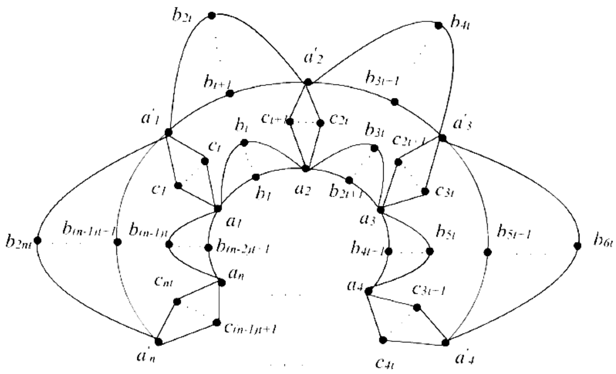

Fig. 5: The super subdivision of Cl_n.

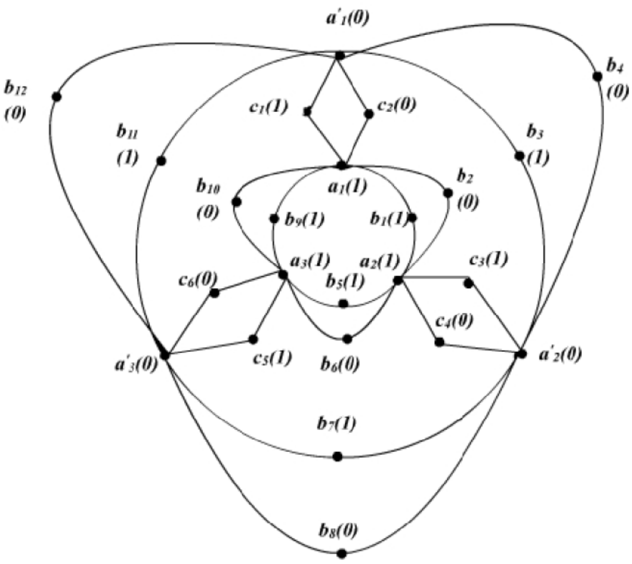

Fig. 6: The super subdivision of Cl_n when n is odd and t is even.

vertices of the inner cycle of CL_n are labeled as $a_1, a_2, a_3, \ldots, a_n$ and the corresponding vertices of the outer cycle of CL_n are labeled as $a_1^1, a_2^1, a_3^1, \ldots, a_n^1$. The vertices of super subdivision of the cycle are labeled as $b_1, b_2, \ldots, b_t, b_{t+1}, b_{t+2}, \ldots, b_{2t}, b_{2t+1}, \ldots, b_{2nt}$ and the vertices of the subdivided ladder are labeled as $c_1, c_2, \ldots, c_t, c_{t+1}, c_{t+2}, \ldots, c_{2t}, c_{2t+1}, \ldots, c_{nt}$. The vertex set is defined as $V(G) = 3tn + 2n$. The edge set is defined as $E(G) = 6tn$.

Define the vertex labeling as follows:

Case 1: When n is odd and t is even:

$$f(a_i) = 1,$$

$$f\left(a_i^{'}\right) = 0,$$

$$f(b_j) = \begin{cases} 0, & \text{if } j \equiv 0, 2 (\text{mod } 4), \\ 1, & \text{if } j \equiv 1, 3 (\text{mod } 4), \end{cases}$$

$$f(c_k) = \begin{cases} 0, & \text{if } k \equiv 0, 2 (\text{mod } 4), \\ 1, & \text{if } k \equiv 1, 3 (\text{mod } 4). \end{cases}$$

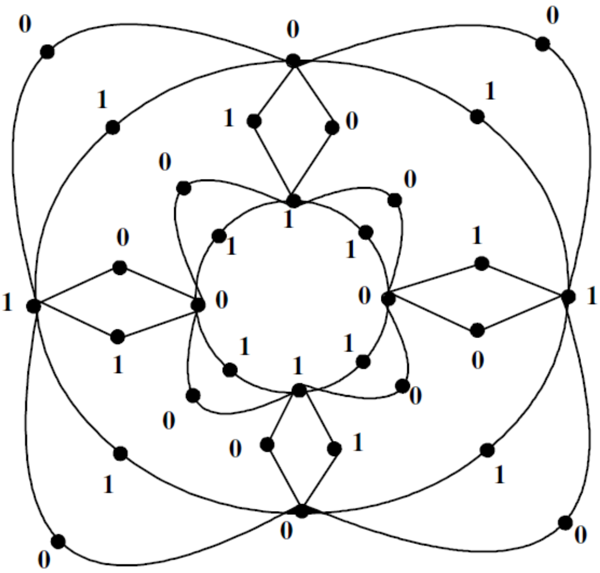

Fig. 7: The super subdivision of Cl_n when n is even and t is even.

Case 2: When n is even and t is even:

$$f(a_i) = \begin{cases} 1, & \text{if } i \equiv 1,3(\text{mod } 4), \\ 0, & \text{if } i \equiv 2,0(\text{mod } 4), \end{cases}$$

$$f(a_i^{'}) = \begin{cases} 0, & \text{if } i \equiv 1,3(\text{mod } 4), \\ 1, & \text{if } i \equiv 2,0(\text{mod } 4), \end{cases}$$

$$f(b_j) = \begin{cases} 0, & \text{if } j \equiv 0,2(\text{mod } 4), \\ 1, & \text{if } j \equiv 1,3(\text{mod } 4), \end{cases}$$

$$f(c_k) = \begin{cases} 0, & \text{if } k \equiv 0,2(\text{mod } 4), \\ 1, & \text{if } k \equiv 1,3(\text{mod4}). \end{cases}$$

Case 3: When n is even and t is odd:

$$f(a_i) = \begin{cases} 1, & \text{if } i \equiv 1,3(\text{mod } 4), \\ 0, & \text{if } i \equiv 2,0(\text{mod } 4), \end{cases}$$

$$f(a_i^{'}) = \begin{cases} 0, & \text{if } i \equiv 1,3(\text{mod } 4), \\ 1, & \text{if } i \equiv 2,0(\text{mod } 4), \end{cases}$$

$$f(b_j) = \begin{cases} 0, & \text{if } j \equiv 0,2(\text{mod } 4), \\ 1, & \text{if } j \equiv 1,3(\text{mod } 4), \end{cases}$$

$$f(c_k) = \begin{cases} 0, & \text{if } k \equiv 0,2(\text{mod } 4), \\ 1, & \text{if } k \equiv 1,3(\text{mod } 4). \end{cases}$$

The number of vertices labeled with 0 and 1 are defined as follows:

$$V_f\,(0) = \left\lfloor \frac{3tn+2n}{2} \right\rfloor,$$

$$V_f\,(1) = \left\lfloor \frac{3tn+2n}{2} \right\rfloor.$$

The number of edges labeled with 0 and 1 are defined as follows:

$$e_f\,(0) = \left\lfloor \frac{6tn}{2} \right\rfloor,$$

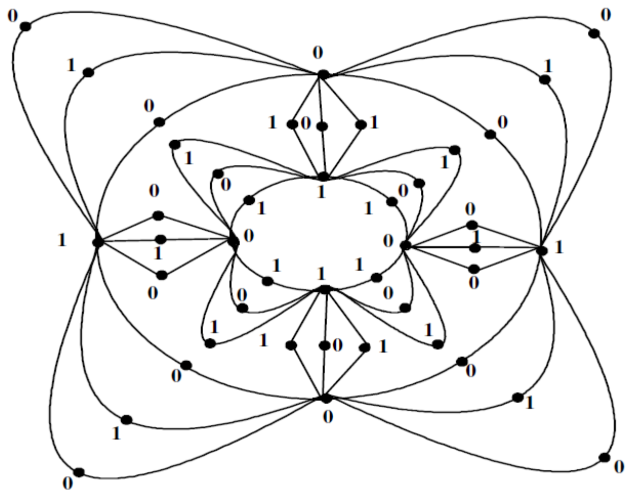

Fig. 8: The super subdivision of Cl_n when n is even and t is odd.

$$e_f(1) = \left\lfloor \frac{6tn}{2} \right\rfloor.$$

From the above labeling pattern, $|V_f(0) - V_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. Therefore, the super subdivision of circular ladder graph admits cordial labeling. □

An illustration for the cordial labeling of $SSD(CL_3)$ for the case when n is odd and t is even is shown in Fig. 6, while an illustration for the cordial labeling of $SSD(CL_4)$ for the case when n is even and t is even is shown in Fig. 7. Lastly, an illustration for the cordial labeling of $SSD(CL_4)$ for the case when n is even and t is odd is shown in Fig. 8.

4 Conclusion

In this paper we have proved that the star glued with the subdivided shell graph and the super subdivision of a circular ladder graph admit cordial labeling.

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