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AN APPROXIMATE SOLUTION FOR SOLVING THE SYSTEM OF FREDHOLM INTEGRAL EQUATIONS OF THE SECOND KIND

Fawziah M. Al-saar[1,*], Kirtiwant P. Ghadle[2]

Author Affiliation:
[1]Department of Mathematics, Dr. Babasaheb Ambedkar Marathwada University, Aurangabad, Maharashtra 431004, India.
[1]Department of Mathematics, Faculty of Education and Languages, Amran University, Amran, Yemen.
[2]Department of Mathematics, Dr. Babasaheb Ambedkar Marathwada University, Aurangabad, Maharashtra 431004, India.

***Corresponding Author:**
Fawziah Mohammad Al-saar, Department of Mathematics, Dr. Babasaheb Ambedkar Marathwada University, Aurangabad, Maharashtra 431004, India.

E-mail: ombraah20016@gmail.com

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Abstract: In this article, we make a comparison between the Adomian decomposition method (ADM) and the modified Adomian decomposition method (MADM) to approximate the solution of the system of Fredholm integral equations of the second kind. The MADM is shown here to result from the ADM by a simple modification of the latter. A numerical example is given to demonstrate the efficiency and applicability of the methods developed by us here and a comparison between the results of applying the ADM and the MADM is also made for revealing that these new techniques employed here by us are very powerful and effective.

Key words and phrases: System of Fredholm integral equations, Adomian decomposition method, modified Adomian decomposition method.

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1. INTRODUCTION

We consider the system of second kind Fredholm integral equations given by

$$Y(x) = F(x) + \int_a^b K(x,t)\, y(t)\, dt, \tag{1.1}$$

where

$$Y(x) = \left(y_1(x), \ldots, y_n(x)\right)^T,$$

$$F(x) = \left(f_1(x), \ldots, f_n(x)\right)^T,$$

$$K(x,t) = k_{i,j}(x,t), \quad i = 1, \ldots, n \quad j = 1, \ldots, n.$$

Here the functions F and K are known and Y is the unknown function. Numerical solutions of linear Fredholm integral equations of the second kind are earlier considered by many authors. Hamoud et al. [7] studied fuzzy Volterra-Fredholm integral equations by using some iterative methods. Masouri et al. [9] studied Chebyshev cardinal functions to solve a second kind Fredholm integral equations system. Katani and Shahmorad [8] implemented block by block method for studying the system of Volterra integral equations. Hamoud and Ghadle [3] used the combined modified Laplace method with Adomian decomposition method with to solve the nonlinear Volterra-Fredholm integro-differential equations. They also used analytical techniques in [6] to find the approximate solutions of fractional Volterra-Fredholm integro-differential equations. Sulaiman [10] employed modification in successive approximation method to solve a second kind Fredholm integral equation system with the symmetric kernel. Al-Saar et al. [1] introduced the approximate solutions of Fredholm integral equations by Adomian decomposition method and its modification. Hamoud and Ghadle [4] utilized the modified Adomian decomposition method for solving fuzzy Volterra-Fredholm integral equations. Hamoud and Ghadle in [5] surveyed recently developed reliable methods for solving Volterra-Fredholm integral and integro-differential equations. Babolian et al. [2] also applied the decomposition method to study the systems of Fredholm integral equations of the second kind.

In this article, we consider (ADM) and its modification methods for solving the linear system of Fredholm integral equations of the second kind. The organization of this paper is as follows. A description of the Adomian decomposition method and its modification is provided in section 2. Section 3 presents our numerical example to illustrate the computational efficiency of these methods. Section 4 discusses the comparison results which we got them from the aforesaid numerical computations. Finally, the conclusion is summarized in section 5 of the paper.

2. ADOMIAN DECOMPOSITION METHOD (ADM) AND ITS MODIFICATION (MADM)

In this section, we consider a system of linear Fredholm integral equations of the second kind of the form:

$$y_i(x) = f_i(x) + \int_a^b \sum_{j=1}^{n} k_{i,j}(x,t)\, y_j(t)\, dt, \quad i = 1,2,\ldots,n \tag{2.1}$$

for solving (2.1) by using ADM [11], we put

$$y_i(x) = \sum_{m=1}^{\infty} y_{i,m}(t), \quad i = 1,2,\ldots,n. \tag{2.2}$$

substituting (2.2) into (2.1), we get

$$\sum_{j=1}^{n} y_j(x) = f_i(x) + \sum_{j=1}^{n} \int_a^b k_{i,j}(x,t) y_j(t)\, dt, \quad i = 1,2,\ldots,n. \tag{2.3}$$

We construct the recursion scheme of the ADM as :

$$y_{i,0}(x) = f_i(x), \quad i = 1,2,\ldots,n,$$

$$y_{i,k+1}(x) = \int_a^b \sum_{j=1}^{n} k_{i,j}(x,t)\, y_{j,k}(t) dt, \quad i = 1,2,\ldots,n, \quad k = 0,1,\ldots \tag{2.4}$$

Observe here that the convergence question of this technique has been formally proven and justified by Vahidi and Mokhtari [12] .

In practice, all terms of the series $y_i(x) = \sum_{m=0}^{\infty} y_{i,m}(t)$ can not be determined and so the better approximations of the solution $y_i(x)$ can be obtained by evaluating more terms of the truncated series as follows:

$$\varphi_{i,k}(x) = \sum_{m=0}^{k-1} y_{i,m}(x), \quad \text{where } \lim_{k\to 1} \varphi_{i,k}(x) = y_i(x). \tag{2.5}$$

Considering (2.5), we obtain:

$$\varphi_{i,k}(x) = f_i(x) + \int_0^x \sum_{j=1}^{n} k_{i,j}(x,t)\, y_{j,m}(t)\, dt, \quad i = 1,\ldots,n, \quad m = 0,1,2,\ldots$$

In fact (2.4) is exactly the same as the well known successive approximations method for solving the system of linear Fredholm integral equations defining as:

$$y_{i,m+1}(x)=f_i(x)+\int_0^x \sum_{j=1}^{n} k_{i,j}(x,t)\, y_{j,m}(t)\,dt, \quad i=1,\ldots,n, \quad m=0,1,2,\ldots$$

The initial approximations for the successive approximations method is usually zero function. In other words, if the initial approximations in this method is selected $y_0(x)$, then the Adomian decomposition method and the successive approximations method are exactly the same.

Now we derive a simple modification of ADM (i.e., MADM) to successfully employ it for increasing the rate of convergence of the ADM for the solution of systems of linear Fredholm integral equations of the second kind in (2.1). The first term of $y_i(x)$ in the ADM is defined by the known function $f_i(x)$. We define the first terms in the modification technique of (2.2) as follows

$$y_{1,0}(x)=f_1(x)$$

$$y_{i,0}(x)=f_i(x)++\int_a^b \sum_{j=1}^{i-1} k_{i,j}(x,t)\, y_{j,0}(t)\,dt, \quad i=2,\ldots,n. \tag{2.6}$$

Since, the values of $y_{i,0}(x)$ for $i=2,\ldots,n$ are modified by previous values and the other terms of (2.2) are defined as follows

$$y_{1,k+1}(x)=\int_a^b \sum_{j=1}^{n} k_{i,j}(x,t)\, y_{j,k}(t)\,dt,$$

$$y_{i,k+1}(x)=\int_a^b \left(\sum_{j=1}^{i-1} k_{i,j}(x,t)\, y_{j,k+1}(t)+\sum_{j=i}^{n} k_{i,j}(x,t)\, y_{j,k}(t) \right) dt, \tag{2.7}$$

$$i=2,\ldots,n; \quad k=0,1,\ldots$$

Now we use our method with $n=2$ for further illustration to consider Fredholm integral equation system of the second kind

$$y_1(x)=f_1(x)+\int_0^1 \left(k_{1,1}(x,t)\, y_1(t)+k_{1,2}(x,t)\, y_2(t)\right) dt,$$

$$y_2(x)=f_2(x)+\int_0^1 \left(k_{2,1}(x,t)\, y_1(t)+k_{2,2}(x,t)\, y_2(t)\right) dt, \tag{2.8}$$

For solving (2.8) by the ADM, we have

$$y_1(x)=\sum_{m=0}^{\infty} y_{1,m}(x),$$

$$y_2(x)=\sum_{m=0}^{\infty} y_{2,m}(x), \tag{2.9}$$

Substituting (2.8) into (2.9), we obtain

$$\sum_{m=0}^{\infty} y_{1,m}(x)=f_1(x)+\int_0^1 \left(k_{1,1}(x,t)\sum_{m=0}^{\infty} y_{1,m}(t)+k_{1,2}(x,t)\sum_{m=0}^{\infty} y_{2,m}(t) \right) dt,$$

$$\sum_{m=0}^{\infty} y_{2,m}(x)=f_2(x)+\int_0^1 \left(k_{2,1}(x,t)\sum_{m=0}^{\infty} y_{1,m}(t)+k_{2,2}(x,t)\sum_{m=0}^{\infty} y_{2,m}(t) \right) dt, \tag{2.10}$$

The MADM recursion scheme is defined as

$$y_{1,0}(x)=f_1(x),$$

$$y_{2,0}(x)=f_2(x)+\int_0^b \left(k_{2,1}(x,t)\, y_{1,0}(t)\right) dt, \tag{2.11}$$

and

$$y_{1,1}(x)=\int_0^1 \left(k_{1,1}(x,t)\, y_{1,0}(t)+k_{1,2}(x,t)\, y_{2,0}(t)\right) dt,$$

$$y_{2,1}(x)=\int_0^1\left(k_{2,1}(x,t)\,y_{1,1}(t)+k_{2,2}(x,t)\,y_{2,0}(t)\right)dt, \tag{2.12}$$

the values of $y_{2,0}(x)$ and $y_{2,1}(x)$ are modified by previous values and the other terms of (2.8) are defined as follows

$$y_{1,k+1}(x)=\int_0^1\left(k_{1,1}(x,t)\,y_{1,k}(t)+k_{1,2}(x,t)\,y_{2,k}(t)\right)dt,$$

$$y_{2,k+1}(x)=\int_0^1\left(k_{2,1}(x,t)\,y_{1,k+1}(t)+k_{2,2}(x,t)\,y_{2,k}(t)\right)dt, \quad k=1,2, \tag{2.13}$$

(2.13) is used to calculate each $y_{2,k+1}$ for $k=1,2,\ldots$.

3. NUMERICAL RESULTS

A numerical example is given in this section to certify the convergence and the error of the presented methods.

Example: Consider the linear system of Fredholm integral equation of the second kind [10]

$$y_1(x)=1-\frac{x^2}{6}+\int_0^x x^2t^2y_2(t)\,dt, \tag{3.1}$$

$$y_2(x)=x^3-\frac{x}{2}+\int_0^1 xty_1(t)\,dt,$$

which have the exact solutions $y_1(x)=1,\ y_2(x)=x^3$.

a) Applying the ADM we have

$$y_{1,0}(x)=1-\frac{x^2}{6},$$

$$y_{2,0}(x)=x^3-\frac{x}{2},$$

and

$$y_{1,m+1}(x)=\int_0^1 x^2t^2y_{2,m}(t)\,dt,$$

$$y_{2,m+1}(x)=\int_0^1 xty_{1,m}(t)\,dt, \quad m=0,1,\ldots$$

b) Applying the MADM we have

$$y_{1,0}(x)=1-\frac{x^2}{6},$$

$$y_{2,0}(x)=x^3-\frac{x}{2}+\int_0^1 xty_{1,0}(t)\,dt,$$

and

$$y_{1,m+1}(x)=\int_0^1 x^2t^2y_{2,m}(t)\,dt,$$

$$y_{2,m+1}(x)=\int_0^1 xty_{1,m+1}(t)\,dt, \quad m=0,1,\ldots$$

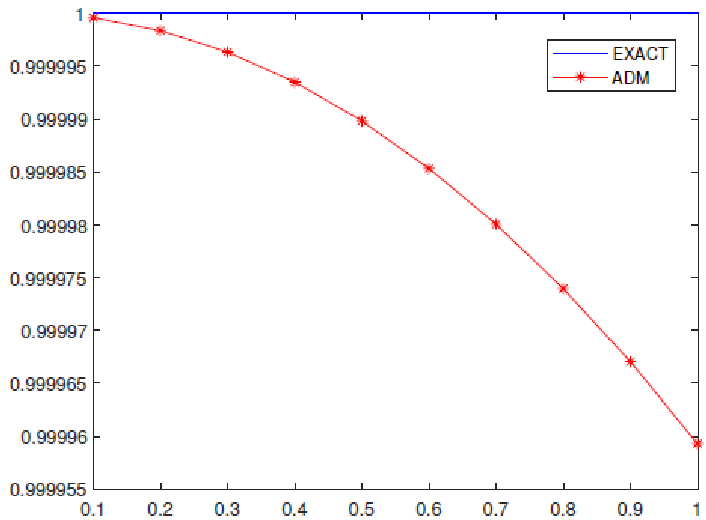

Figure 1: Comparison between exact and approximate solutions for $y_1(x)$.

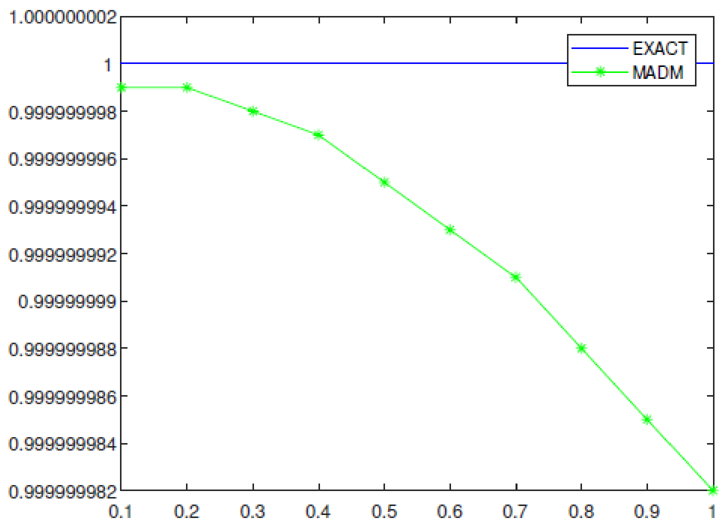

Figure 2: Comparison between exact and approximate solutions for $y_1(x)$.

4. COMPARISON OF RESULTS

The approximated solution with seven terms is given in Table 1 and Table 2. The tables also show the absolute error between the exact and approximate solutions. Fig. 1 and Fig. 3 show the comparison between the exact solution and the approximate solution obtained by ADM and Fig. 2 and Fig. 4 show the comparison between the exact solution and the approximate solution obtained by the MADM. It is seen from the figures that the solution obtained by the proposed methods is nearly identical with the exact solution. In this example, the simplicity and accuracy of the proposed methods are illustrated by computing the absolute error $E_7(x)$ for the example. The accuracy of the results can be improved by introducing more terms of the approximate solutions.

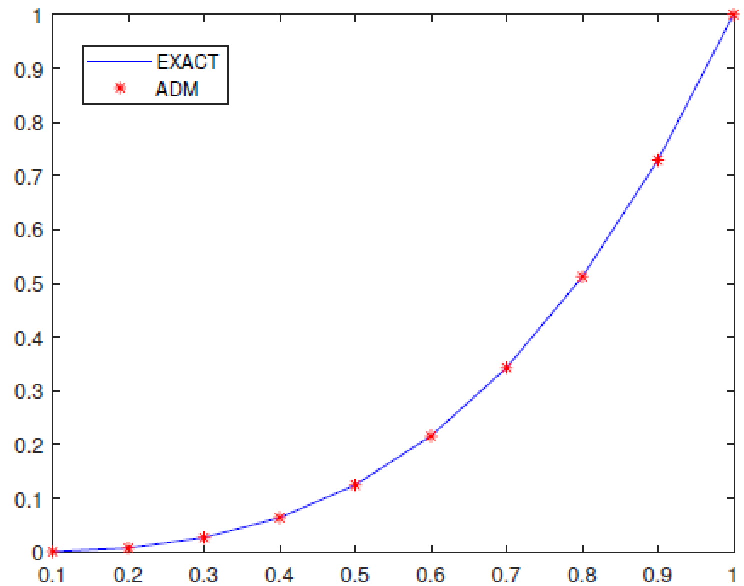

Figure 3: Comparison between the exact and approximate solutions for $y_2(x)$.

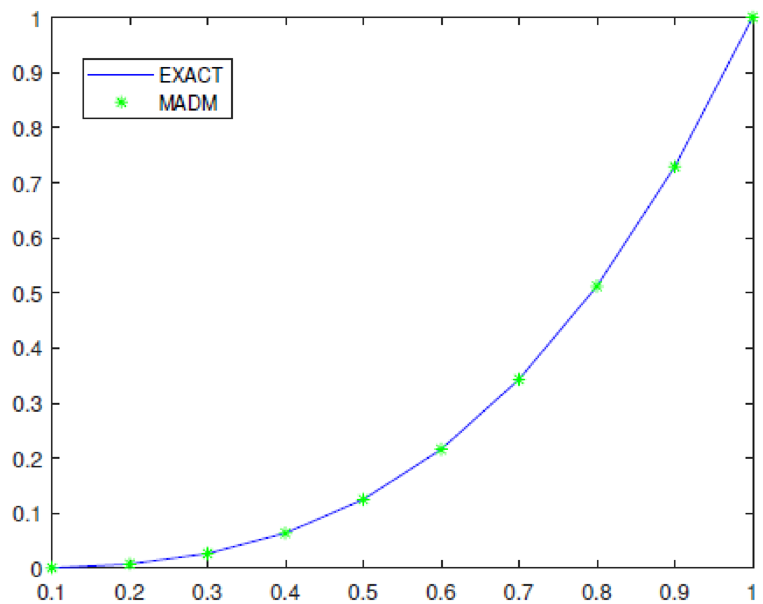

Figure 4: Comparison between the exact and approximate solutions for $y_2(x)$.

In Table 1 and Table 2, ADM and MADM solutions are compared with the exact solution of the Fredholm integral (3.1). Sulaiman [10] used the successive approximation method and its modification to solve (3.1) with the absolute error $E_5(x)$. The comparison of our proposed methods with the methods adopted by Sulaiman [10] show the efficiency and accuracy of our methods.

Table 1: Comparison between the exact solutions and approximate solutions of $y_1(x)$.

x	$y_{Exact}(x)$	$y_{ADM}(x)$	$y_{MADM}(x)$	$E_7 y_{ADM}(x)$	$E_7 y_{MADM}(x)$
0.1	1	0.999999593	0.999999999	0.407×10^{-6}	7.0666×10^{-10}
0.2	1	0.999998372	0.999999999	1.628×10^{-6}	7.0666×10^{-10}
0.3	1	0.999996337	0.999999998	3.663×10^{-6}	2×10^{-9}
0.4	1	0.999993489	0.999999997	6.511×10^{-6}	3×10^{-9}
0.5	1	0.999989827	0.999999995	10.173×10^{-6}	5×10^{-9}
0.6	1	0.99998535	0.999999993	14.56×10^{-6}	7×10^{-9}
0.7	1	0.99998006	0.999999991	19.94×10^{-6}	8×10^{-9}
0.8	1	0.999973957	0.999999988	26.043×10^{-6}	12×10^{-9}
0.9	1	0.999967039	0.999999985	32.961×10^{-6}	15×10^{-9}
1.0	1	0.999959308	0.999999982	40.692×10^{-6}	18×10^{-9}

Table 2: Comparison between the exact solutions and approximate solutions of $y_2(x)$.

x	$y_{Exact}(x)$	$y_{ADM}(x)$	$y_{MADM}(x)$	$E_7 y_{ADM}(x)$	$E_7 y_{MADM}(x)$
0.1	0.001	0.000987792	0.000999999	12.208×10^{-6}	1×10^{-9}
0.2	0.008	0.007975585	0.007999999	24.415×10^{-6}	1×10^{-9}
0.3	0.027	0.026963378	0.026999998	36.622×10^{-6}	2×10^{-9}
0.4	0.064	0.063951171	0.063999998	48.829×10^{-6}	2×10^{-9}
0.5	0.125	0.124938964	0.124999997	61.036×10^{-6}	3×10^{-9}
0.6	0.216	0.215926756	0.215999997	73.244×10^{-6}	3×10^{-9}
0.7	0.343	0.342914549	0.342999996	85.451×10^{-6}	4×10^{-9}
0.8	0.512	0.511902342	0.511999996	97.658×10^{-6}	4×10^{-9}
0.9	0.729	0.728890135	0.728999995	109.865×10^{-6}	5×10^{-9}
1.0	1.000	0.999877928	0.999999995	122.072×10^{-6}	5×10^{-9}

5. CONCLUSION

In this paper a modified form of ADM is successfully employed for solving the system of linear Fredholm integral equations of the second kind. The modified method is better than the ADM in the sense of its applicability and accuracy. It is observed that the methods developed here are very powerful and robust techniques for finding an approximate solution of the various types of integral and integro-differential equation systems. The numerical results presented above prove the efficacy and the better accuracy of these methods for solving the system of Fredholm integral equations of the second kind over the other methods currently available in the literature for this purpose.

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