Original Article

Bulletin of Pure and Applied Sciences.
Vol. 38E (Math & Stat.), No.1, 2019. P.117-134

Available online at
www.bpasjournals.com

Print version ISSN 0970 6577
Online version ISSN 2320 3226
DOI: 10.5958/2320-3226.2019.00011.0

JOULE HEATING AND THERMAL DIFFUSION EFFECT ON MHD FLUID FLOW PAST A VERTICAL POROUS PLATE EMBEDDED IN A POROUS MEDIUM

M. Obulesu[1,*], R. Siva Prasad[2]

Author Affiliation:
[1]Research Scholar, Department of Mathematics, Sri Krishnadevaraya University, Anantapur, Andhra Pradesh 515 003, India.
E-mail: mopuriobulesu1982@gmail.com
[2]Department of Mathematics, Sri Krishnadevaraya University, Anantapur, Andhra Pradesh 515 003, India.
E-mail: rsprasad_racharla@yahoo.co.in

***Corresponding Author:**
M. Obulesu, Research Scholar, Department of Mathematics, Sri Krishnadevaraya University, Anantapur, Andhra Pradesh 515 003, India.

E-mail: mopuriobulesu1982@gmail.com

Received on 05.08.2018 Revised on 05.02.2019 Accepted on 28.02.2019

Abstract: In this paper an attempt is made to study the Joule Heating and Thermal Diffusion effect on MHD fluid past a vertical porous plate embedded in porous media, under the influence of a uniform magnetic field applied normal to the surface. The governing equations are solved analytically using a regular perturbation technique. The expression for velocity, temperature and concentration fields are obtained with the aid of these, the expressions for the coefficient of skin friction, the rate of heat transfer in the form of Nusselt number and the rate of mass transfer in the form of Sherwood number are derived. Finally the effect of variation of physical parameters on the flow quantities are studied with the help of graphs and tables. It is observed that the velocity and concentration increase during a generative reaction and decrease in a destructive reaction. The same is observed to be true for the behavior of the fluid temperature. The presence of magnetic field and radiation diminishes the velocity and also the temperature.

Keywords: Joule effect; Soret effect; MHD; Viscous dissipation; Porous media; Heat source/ sink; Chemical reaction and Thermal radiation.

2010 AMS Mathematics Subject Classification: 76E99, 76S05, 76S99, 76W99.

1. INTRODUCTION

The study of magneto hydrodynamics (MHD) plays an important role in agriculture, engineering and petroleum industries. MHD has won practical applications, for instance, it may be used to deal with problems such as the cooling of nuclear reactors by liquid sodium and induction flow water which depends on the potential difference in the fluid direction perpendicular to the motion and goes to the magnetic field and also the study of the MHD of viscous conducting fluids is playing a significant role, owing to its practical interest and abundant applications, in astro-physical and geo-physical phenomenon. Astro-Physicists and geo-physicists realized the importance of MHD in stellar and planetary processes. The main impetus to the engineering approach to the electromagnetic fluid interaction studies has come from the principles of the magneto hydro dynamics, direct conversion generator, ion propulsion study of flow problems of electrically conducting fluid, particularly of ionized gases is currently receiving considerable interest. Such studies have been made for years in connection with astro-physical and geo-physical problems such as the Sun spot theory, motion of the interstellar gases etc. Recently, some engineering problems connected with the studies of the flow of an electrically conducting fluid in ionized gas called plasma are conducted. Many names have been used in referring to the study of plasma phenomena [1-10].

MHD double diffusive and chemically reactive flow through porous medium bounded by two vertical plates was studied by Ravikumar et al. [1]. MHD free convective flow through a porous medium past a vertical plate with ramped wall temperature was studied by Sinha et al. [2]. Effect of heat transfer on MHD blood flow through an inclined stenosed porous artery with variable viscosity and heat source was discussed by Tripathi et al. [3]. Steady MHD mixed convective flow in the presence of inclined magnetic field and thermal radiation with effects of chemical reaction and Soret embedded in a porous medium was studied by Sharmilaa et al.[4]. Rama Krishna Reddy and Raju [5] studied MHD free convective flow past a porous plate.

The Soret effect arises when the mass flux contains a term that depends on the temperature gradient. The major focus of our study is the effect on mixed convection flow of the addition of a second fluid. The influence of Soret effect on the flow of an electrically conducting fluid past a vertical plate in the presence of various physical parameters was investigated by Mohammad Ibrahim and Suneetha[6], Sharma et al. [7,8,9] and Satyanarayana and Sravanthi [10].

Flow through porous media has attracted considerable research activity in recent years because of its several important applications notably in the flow of oil through porous rock, the extraction of energy from the geothermal regions, the evaluation of capability of heat removal from particulate nuclear in a nuclear reactor, the filtration of solids from liquids, flow of liquids through ion exchange beds, drug permeation through human skin, chemical reactors for the economical separation or purification of mixture and so on. In understanding the circulation of blood in lungs, the effect of porosity has to include either in the medium or near the boundary. In lungs, the blood can be visualized as flowing between the opposing layers of capillary endothelium held apart by endothelium covered “past” the septal tissue. This capillary endothelium is covered in turn by thin layer (interstitial space) lining the alveoli. The blood space in lung is idealized into a two dimensional channel and the interstitial tissue space into porous medium. An endothelial layer between the two regions is permeable to water and certain other solutes, it can be considered as a permeable membrane of negligible thickness. The epithelial tissues between the air and vascular space are less permeable and thus it can be treated as an impermeable membrane. The irregular ports which keep apart the endothelial walls can be ignored for the time being.

Sharma et al. [11], considered unsteady natural convection flow past a vertical surface in a rotating porous medium with variable permeability. Mahapatra et al. [12] studied the effects of chemical reaction on free convection flow through a porous medium bounded by a vertical surface. Raju et al. [13] studied an unsteady MHD free convection oscillating couette flow through a porous medium with periodic wall temperature in the presence of chemical reaction and thermal radiation. Hydromagnetic forced flow between a rotating disc and a naturally permeable stationary porous disc saturated with fluid was studied by Sharma et al. [14]. Sarada et al.[15] discussed the effect of chemical reaction on an unsteady MHD free convection flow past an infinite vertical porous plate with variable suction.

Recently, the viscous as well as the Joule’s dissipation along with heat generation was taken into account in the energy equation. Ibrahim [16] studied the effects of Joule heating and viscous dissipation on steady MHD maragngoni convective flow over a flat surface in the presence of radiation. The combined effect of Joule’s and viscous dissipation on mixed convection MHD flow in a vertical channel was noticed by Abo-Eldahab and El-Aziz [17]. Heat source and chemical effects on MHD convection flow embedded in a porous medium with Soret, viscous and Joules dissipation, was studied by Mohammad Ibrahim et al.[18]. Veeresh et al. [19] studied Joule heating and thermal diffusion effects on MHD radiative and convective casson fluid flow past an oscillating semi-infinite vertical porous plate. Reddy et al. [20] considered Joule heating influence on MHD casson fluid over a vertical porous plate in the presence of thermal diffusion and chemical reaction. Some other relevant works pertinent ot this field are [21-26].

2. MATHEMATICAL FORMULATION

We consider a viscous, incompressible, electrically conducting and radiating fluid through a porous medium occupying a semi-infinite region of the space bounded by a vertical infinite surface. The x^* axis is taken along the surface in the upward direction and the y^* axis is normal to it. The uniform magnetic field of strength B_0 in the presence of radiation and Joule heating is imposed transversely in the direction of y^* axis. The properties of the fluid are assumed to be constant except for the density in the body force term. In addition a chemically reactive species is assumed to be emitted from the vertical surface into a hydrodynamic flow field. It diffuses into the fluid, where it under goes a homogenous chemical reaction. The reaction is assumed to take place entirely in the stream. Then the fully developed flow under the above assumptions through a highly porous medium is governed by the following set of equations:

$$\frac{\partial v^*}{\partial y^*}=0 \tag{1}$$

$$V^*\frac{\partial u^*}{\partial y^*}=\vartheta\frac{\partial^2 u^*}{\partial y^{*2}}+g\beta_T(T^*-T_\infty^*)+gB_C(C^*-C_\infty^*)-\frac{\sigma B_0^2}{\rho}u^*-\frac{\vartheta u^*}{K_p} \tag{2}$$

$$V^*\frac{\partial T^*}{\partial y^*}=\frac{K}{\rho C_p}\frac{\partial^2 T^*}{\partial y^{*2}}+\frac{\vartheta}{C_p}\left(\frac{\partial u^*}{\partial y^*}\right)^2-\frac{1}{\rho C_p}\frac{\partial q_r^*}{\partial y^*}-\frac{Q_1}{\rho C_p}(T^*-T_\infty^*)+\frac{\sigma B_0^2}{\rho C_p}u^{*2} \tag{3}$$

$$V^*\frac{\partial C^*}{\partial y^*}=D\frac{\partial^2 C^*}{\partial y^{*2}}-K_C(C^*-C_\infty^*)+D_1\frac{\partial^2 T^*}{\partial y^{*2}} \tag{4}$$

The relevant boundary conditions are given as follows

$$\begin{aligned} &u^*=0, \quad T^*=T_w, \quad C^*=C_w \quad \text{at} \quad y=0 \\ &u^*\to 0, \quad T^*\to T_\infty, \quad C^*\to C_\infty \quad \text{as} \quad y\to\infty \end{aligned} \tag{5}$$

Eq.(1) gives that v^* = Constant = - V_0 (6)

In the optically thick limit, the fluid does not absorb its own emitted radiation in which there is no self-absorption, but it does absorb radiation emitted by the boundaries. Cogley et al. [21] showed that in the optically thick limit for a non-gray gas near equilibrium as given below.

$$\frac{\partial q_r}{\partial y^*}=4(T^*-T_\infty^*)\int_0^\infty K_{\lambda w}\frac{de_{b\lambda}}{dT^*}d\lambda=4I_1(T^*-T_\infty^*) \tag{7}$$

On introducing the following non-dimensional quantities,

$$u=\frac{u^*}{v_0}, y=\frac{v_0 y^*}{\vartheta}, \theta=\frac{T^*-T_\infty^*}{T_w-T_\infty^*}, \phi=\frac{C^*-C_\infty^*}{C_w-C_\infty^*}, \Pr=\frac{\mu C_p}{K}, Sc=\frac{\vartheta}{D}, M=\frac{\sigma B_0^2\vartheta}{\rho v_0^2},$$

$$Gr=\frac{\vartheta g\beta_T(T_w-T_\infty^*)}{v_0^3}, Gm=\frac{\vartheta g\beta_C(C_w-C_\infty^*)}{v_0^3}, E=\frac{v_0^2}{C_P(T_w-T_\infty^*)}, K=\frac{v_0^2K_P}{\vartheta^2}$$

$$c=\frac{\vartheta K_C}{v_0^2}, F=\frac{4I_1\vartheta^2}{Kv_0^2}, Q=\frac{Q_1v^2}{Kv_0^2}, S_0=\frac{D_1(T_w-T_\infty)}{\vartheta(C_w-C_\infty)} \tag{8}$$

where M is the magnetic parameter, E is the Eckert Number, K is the Permeability parameter, K_0 is the chemical reaction parameter, F is the radiation parameter, Q is the heat source parameter and S_0 is the Soret parameter. The non-dimensional form of the governing equations (2) – (4) reduces to

$$u''+u'=-\mathrm{Gr}\,\theta-\mathrm{Gm}\,\varphi+\mathrm{M_1}u \tag{9}$$

$$\theta''+\Pr\theta'-(F+Q)\,\theta=-\Pr u'^2-\Pr M u^2 \tag{10}$$

$$\phi''+Sc\phi'-\mathrm{ScK_0}\phi=-\mathrm{S_0}\,\mathrm{Sc}\,\theta'' \tag{11}$$

where $M_1 = M + 1 / K$.

The corresponding boundary conditions are given by

$$\begin{aligned} &u=0, \quad \theta=1, \quad \phi=1 \quad \text{at} \quad y=0 \\ &u\to 0, \quad \theta\to 0, \quad \phi\to 0 \quad \text{at} \quad y\to\infty \end{aligned} \tag{12}$$

3. SOLUTION OF THE PROBLEM

In order to solve the coupled nonlinear system of Eqs. (9)–(11) with the boundary conditions (12), the following simple perturbation is used. The governing equations (9)–(11) are expanded in powers of Eckert number E (<<1).

$$u = u_0 + Eu_1 + O(E^2),\ \theta = \theta_0 + E\theta_1 + O(E^2), \qquad (13)$$

$$\phi = \phi_0 + E\,\phi_1 + O(E^2)$$

Substituting equations (13) into equation (9)–(11) and equating the coefficients of the terms with the same powers of E, and neglecting the terms of higher order, the following equations are obtained.

Zero order terms:

$$u_0'' + u_0' = -\mathrm{Gr}\,\theta_0 - \mathrm{Gm}\,\phi_0 + \mathrm{M_1 u_0} \qquad (14)$$

$$\theta_0'' + \mathrm{Pr}\,\theta_0' - (\mathrm{F}+\mathrm{Q})\,\theta_0 = 0 \qquad (15)$$

$$\phi_0'' + Sc\,\phi_0' - \mathrm{ScK_0}\phi_0 = -\mathrm{S_0}\,\mathrm{Sc}\,\theta_0'' \qquad (16)$$

First order terms :

$$u_1'' + u_1' = -\mathrm{Gr}\,\theta_1 - \mathrm{Gm}\,\phi_1 + \mathrm{M_1 u_1} \qquad (17)$$

$$\theta_1'' + \mathrm{Pr}\,\theta_1' - (\mathrm{F+Q})\ \theta_1 = -\,\mathrm{Pru}_0'^{2} - \mathrm{Pr\,M}u_0^2 \qquad (18)$$

$$\phi_1'' + Sc\phi_1' - ScK_0\phi_1 = -S_0 Sc\theta_1'' \qquad (19)$$

The corresponding boundary conditions are

$$u_0 = 0,\ u_1 = 0,\quad \theta_0 = 1,\ \theta_1 = 0,\quad \phi_0 = 1,\ \phi_1 = 0 \qquad \text{at } y = 0$$

$$u_0 \to 0,\ u_1 \to 0,\quad \theta_0 \to 0,\ \theta_1 \to 0,\quad \phi_0 \to 0,\ \phi_1 \to 0 \qquad \text{as } y \to \infty \qquad (20)$$

Solving equations (14) – (19) under the boundary conditions (20), the following solutions are obtained

$$\theta_0 = \exp(-m_1 y), \qquad (21)$$

$$\phi_0 = A_2 \exp(-m_2 y) - A_1 \exp(-m_1 y) \qquad (22)$$

$$u_0 = A_5 \exp(-m_3 y) - A_4 \exp(-m_2 y) + A_3 \exp(-m_1 y) \qquad (23)$$

$$\begin{aligned}\theta_1 &= A_{18}\exp(-2m_1 y) + A_{19}\exp(-2m_2 y) + A_{20}\exp(-2m_3 y) + A_{21}\exp(-\delta_1 y) + \\ &\quad A_{22}\exp(-\delta_2 y) + A_{23}\exp(-\delta_3 y) + A_{24}\exp(-m_1 y)\end{aligned} \qquad (24)$$

$$\begin{aligned}\varphi_1 &= A_{25}\exp(-m_1 y) + A_{26}\exp(-2m_1 y) + A_{27}\exp(-2m_2 y) + A_{28}\exp(-2m_3 y) \\ &\quad + A_{29}\exp(-\delta_1 y) + A_{30}\exp(-\delta_2 y) + A_{31}\exp(-\delta_3 y) + A_{32}\exp(-m_2 y)\end{aligned} \qquad (25)$$

$$\begin{aligned}u_1 &= A_{33}\exp(-m_1 y) + A_{34}\exp(-m_2 y) + A_{35}\exp(-2m_1 y) + A_{36}\exp(-2m_2 y) \\ &\quad + A_{37}\exp(-2m_3 y) + A_{38}\exp(-\delta_1 y) + A_{39}\exp(-\delta_2 y) + A_{40}\exp(-\delta_3 y) \\ &\quad + A_{41}\exp(-m_3 y)\end{aligned} \qquad (26)$$

Substituting the equations (21) – (26) into the equation (13) we obtain the velocity temperature and concentration field

$$\begin{aligned}u &= A_5\exp(-m_3 y) - A_4\exp(-m_2 y) + A_3\exp(-m_1 y) \\ &\quad + E[A_{33}\exp(-m_1 y) + A_{34}\exp(-m_2 y) + A_{35}\exp(-2m_1 y) + A_{36}\exp(-2m_2 y) \\ &\qquad + A_{37}\exp(-2m_3 y) + A_{38}\exp(-\delta_1 y) + A_{39}\exp(-\delta_2 y) + A_{40}\exp(-\delta_3 y) \\ &\qquad + A_{41}\exp(-m_3 y)]\end{aligned} \qquad (27)$$

$$\theta = \exp(-m_1 y) + E[A_{18}\exp(-2m_1 y) + A_{19}\exp(-2m_2 y) + A_{20}\exp(-2m_3 y) + A_{21}\exp(-\delta_1 y) + A_{22}\exp(-\delta_2 y) + A_{23}\exp(-\delta_3 y) + A_{24}\exp(-m_1 y)] \tag{28}$$

$$\phi = A_2\exp(-m_2 y) - A_1\exp(-m_1 y) + E[A_{25}\exp(-m_1 y) + A_{26}\exp(-2m_1 y) + A_{27}\exp(-2m_2 y) + A_{28}\exp(-2m_3 y) + A_{29}\exp(-\delta_1 y) + A_{30}\exp(-\delta_2 y) + A_{31}\exp(-\delta_3 y) + A_{32}\exp(-m_2 y)] \tag{29}$$

Skin Friction:

The non-dimensional skin friction at the surface is given by

$$\tau = \left(\frac{\partial u}{\partial y}\right)_{y=0}$$

$$\tau = [-m_3 A_5 + m_2 A_4 - m_1 A_3] + E[m_1 A_{33} - m_2 A_{34} - 2m_1 A_{35} - 2m_2 A_{36} - 2m_3 A_{37} - \delta_1 A_{38} - \delta_2 A_{39} - \delta_3 A_{40} - m_3 A_{41}] \tag{30}$$

Nusselt Number :

The rate of heat transfer in terms of the Nusselt number is given by

$$Nu = -\left(\frac{\partial \theta}{\partial y}\right)_{y=0}$$

$$Nu = m_1 + E[m_1 A_{24} + 2m_1 A_{18} + 2m_2 A_{19} + 2m_3 A_{20} + \delta_1 A_{21} + \delta_2 A_{22} + \delta_3 A_{23}] \tag{31}$$

Sherwood Number :

The rate of mass transfer on the wall in terms of the Sherwood number is given by

$$Sh = -\left(\frac{\partial \phi}{\partial y}\right)_{y=0}$$

$$Sh = [m_2 A_2 - m_1 A_1] + E[m_1 A_{25} + 2m_1 A_{26} + 2m_2 A_{27} + 2m_3 A_{28} + \delta_1 A_{29} + \delta_2 A_{30} + \delta_3 A_{31} + m_2 A_{32}] \tag{32}$$

4. RESULTS AND DISCUSSION

In order to get a physical insight into the problem numerical calculations are carried out for the Velocity, Temperature and Concentration profiles and the following discussion is set out.

Throughout the computations we employ, S_0 =1, Sc=0.22, Pr=0.71, Gr=5,G m=5, K=0.5 K_0 =0.5, M=1, F=1,Q=0.5,E=0.02.

In order to reveal the effects of various parameters on the dimensionless velocity fields, temperature field, concentration field, skin friction, Nusselt number and Sherwood number and the effect of the various physical parameters such as the Grashof number (Gr), the modified Grashof number (Gm), magnetic parameter (M), Permeability parameter (K), Prandtl number (Pr), Heat Sink (Q), Radiation Parameter (F), Soret parameter (S_0), Eckert number (E), Schmidt number (Sc) and the Chemical reaction parameter (K_0) on velocity, temperature and concentration, through the Figures 1-21 and study these by choosing arbitrary values. The influence of these parameters on skin friction, Nusselt number and Sherwood number is also shown in Tables 1 – 3.

Figs. 1 – 11 demonstrate the variations of the fluid velocity under the effects of different parameters. Fig.1 represents the velocity profiles for different values of Grashof number (Gr). From this figure it is noticed that the velocity increases with increase in Gr. In Fig.2 the effect of modified Grashof number (Gm) on the velocity is presented. As Gm increases, velocity also increases. In Fig. 3, velocity profiles are displayed with the variation in magnetic parameter (M). From this figure it is noticed that the velocity gets reduced by the increase of magnetic parameter (M). Fig. 4 depicts the variations in velocity profiles for different values of permeability parameter (K), from where it is noticed that th velocity increases as K increases. Fig.5 depicts the variations in velocity profiles for different values of the radiation parameter F which shows that velocity decreases as F increases. In Fig.6 the effect of Heat source parameter Q on velocity is presented. As Q increases the velocity also decreases. In Fig.7 the effect of Soret parameter S_0 on velocity is presented. As S_0 increases the velocity also increases. In Fig. 8 the velocity profiles are displayed with the variation in Schmidt number (Sc). From this figure it is noticed that the velocity gets reduced by the increase of Schmidt number (Sc). The similar effects are noticed from Figs. 9-10, in the presence of Prandtl number (Pr) and Chemical reaction parameter (K_0) which also decreass the fluid velocity. Fig.11 depicts the variations in velocity profiles for different values of Eckert number (E) from where it is seen that the velocity increases as the Eckert number increases.

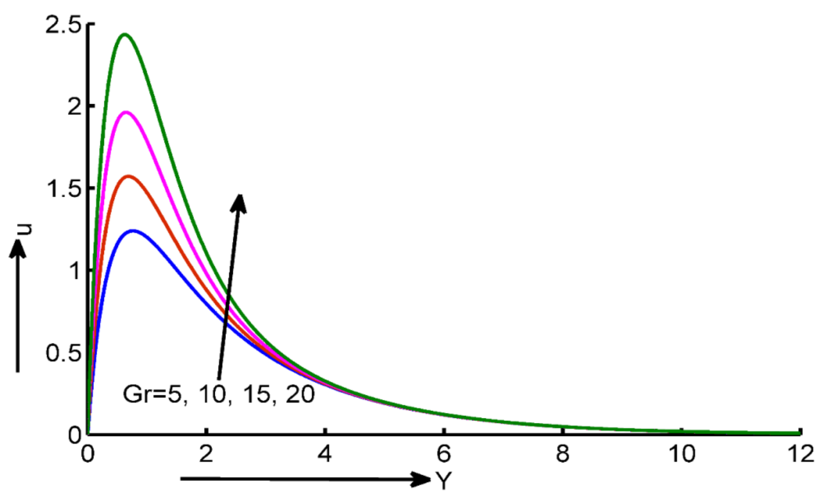

Figure 1: Effect of Grashof number on velocity

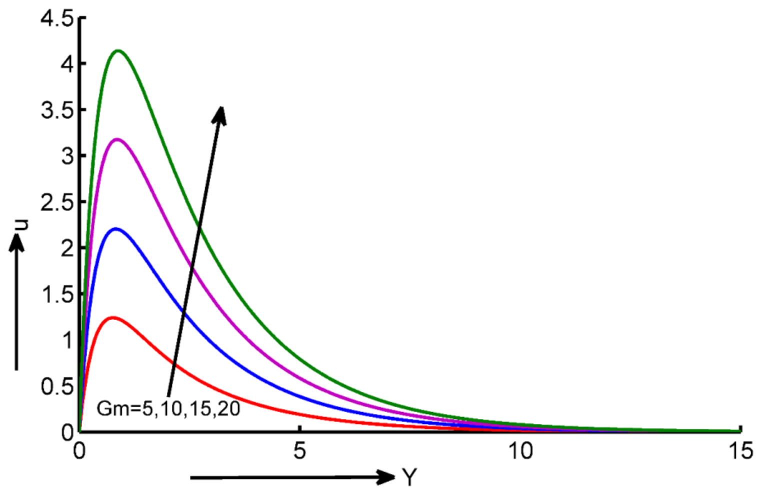

Figure 2: Effect of modified Grashof number on velocity

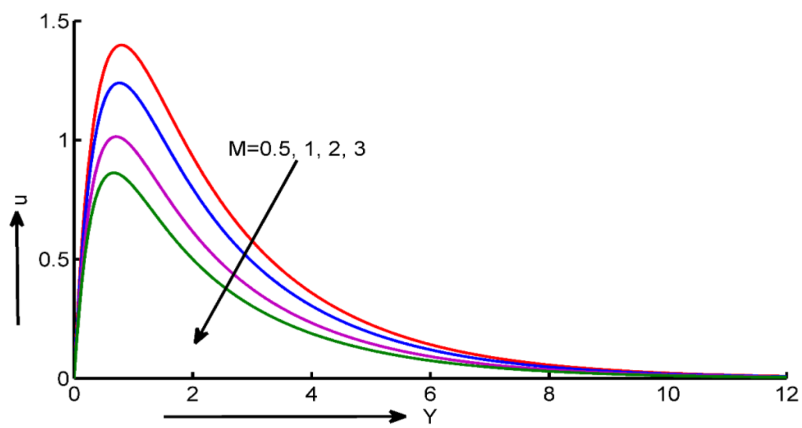

Figure 3: Effect of Magnetic Parameter on Velocity

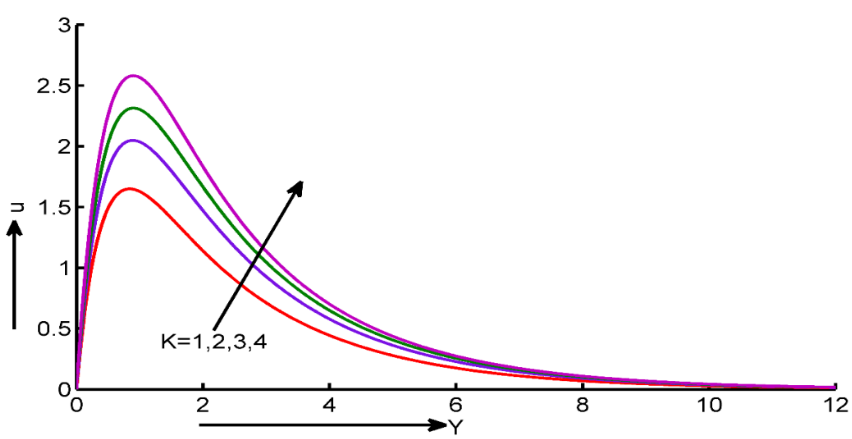

Figure 4: Effect of Permeability Parameter on Velocity

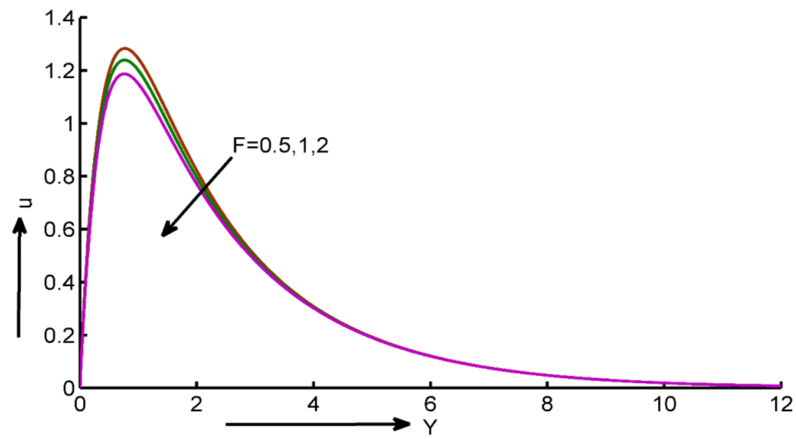

Figure 5: Effect of Radiation Parameter on Velocity

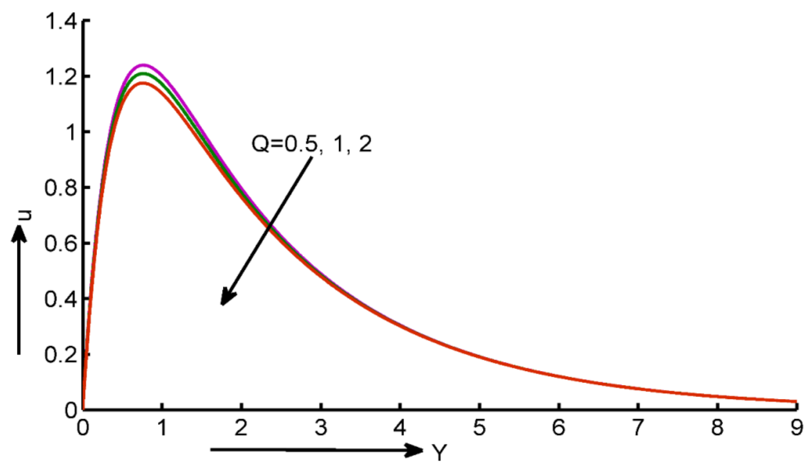

Figure 6: Effect of Heat Source Parameter on Velocity

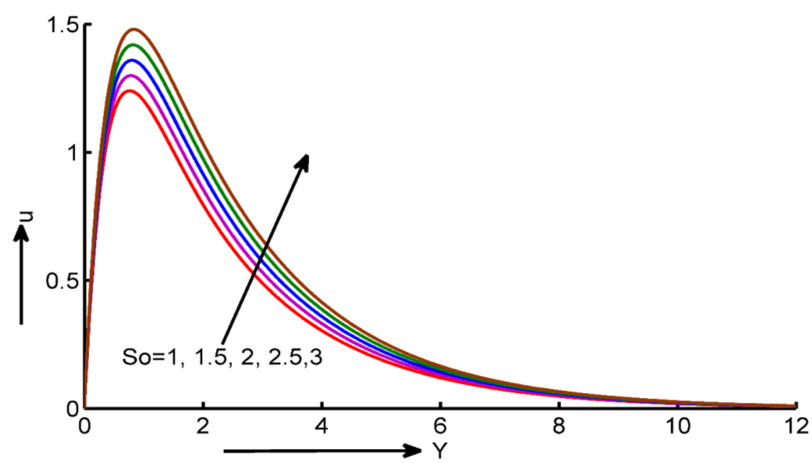

Figure 7: Effect of Soret Parameter on Velocity

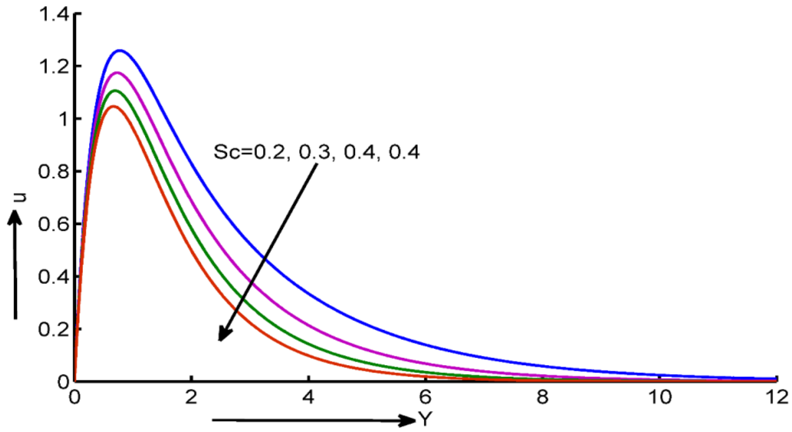

Figure 8: Effect of Schmidt Number on Velocity

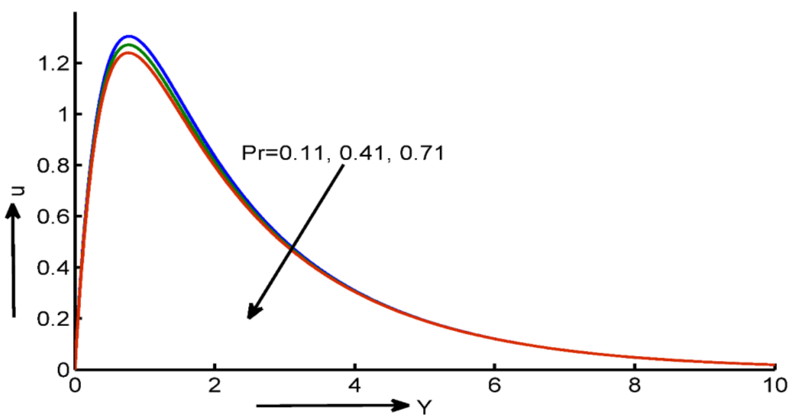

Figure 9: Effect of Prandtl Number on Velocity

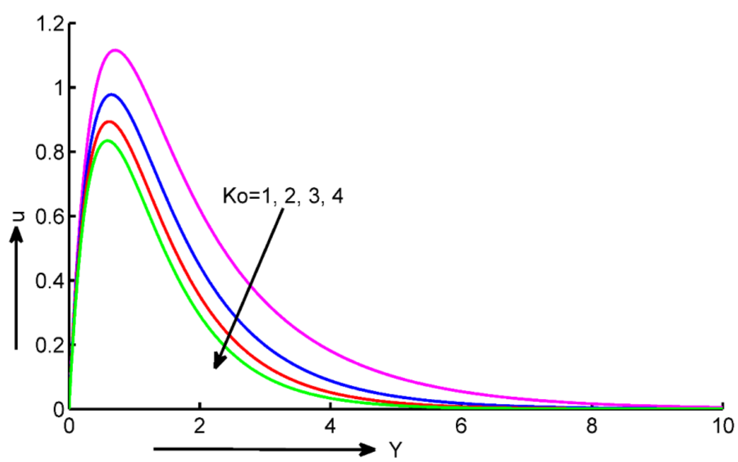

Figure10: Effect of chemical reaction parameter on velocity

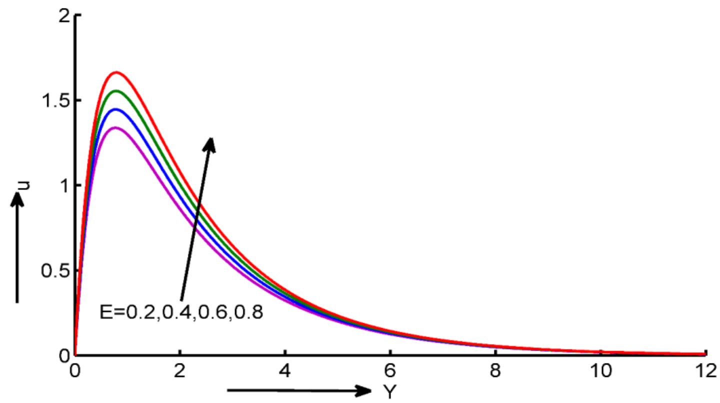

Figure 11: Effect of Eckert Number on Velocity

Figures 12–18 display the variations of the fluid temperature under the effects of different parameters. From Figs. 12 – 14 it is clear that temperature decreases with the increase in Prandtl number (Pr) Radiation Parameter (F) and Heat Sink (Q). In Fig. 15 the effect of Eckert number (E) is shown on the temperature profile. From this figure it is observed that the temperature increases with an increase in E. Fig.16 depicts the variations in temperature profile for different values of Magnetic parameter (M). From this figure it is noticed that the temperature decreases when M increases. In Fig. 17 the effect of Soret parameter (S_0) is shown on the temperature profile. From this figure it is observed that temperature increases with an increase in S_0. A similar effect is noticed from Fig. 18 we see that as the Grashof number (Gr) increases this also decreases the fluid temperature.

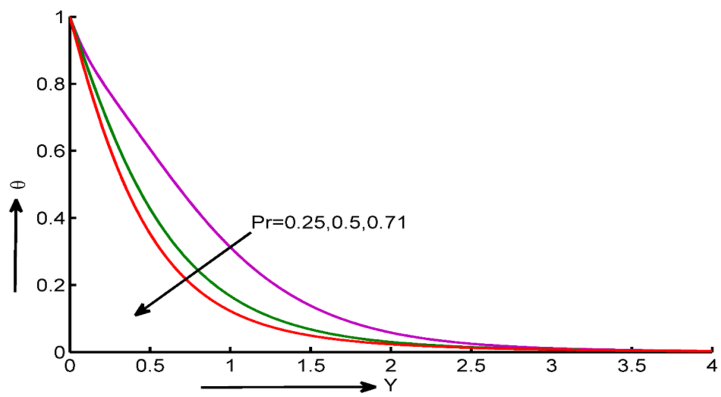

Figure 12: Effect of Prandtl Number on Temperature

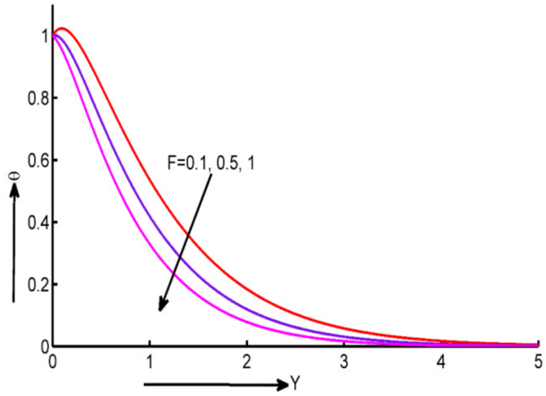

Figure 13: Effect of Radiation Parameter on Temperature

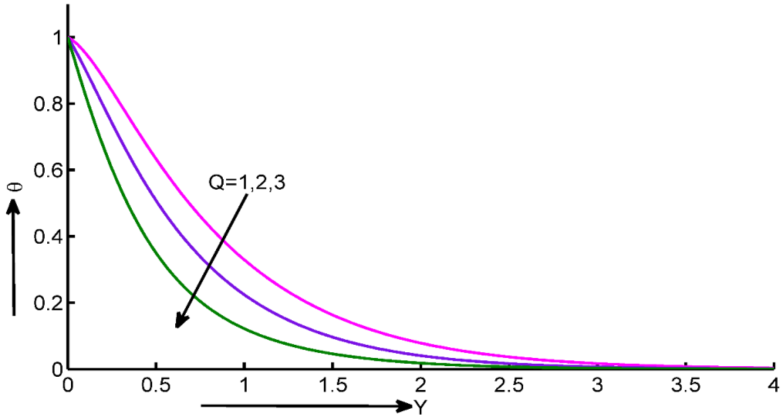

Figure 14: Effect of Heat Source Parameter on Temperature

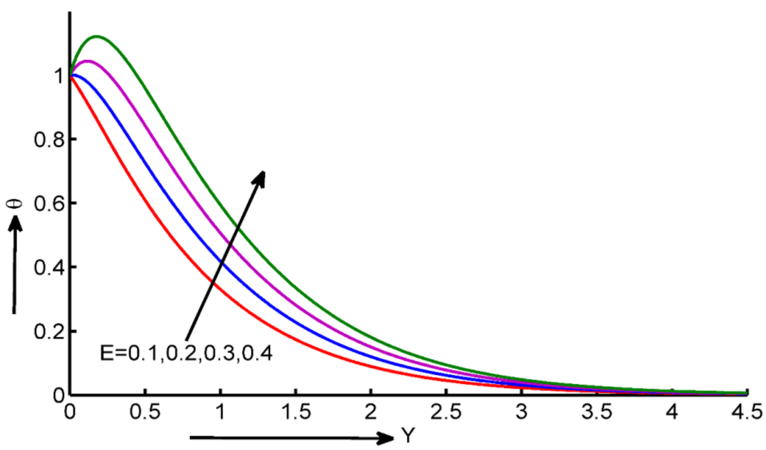

Figure 15: Effect of Eckert Number on Temperature

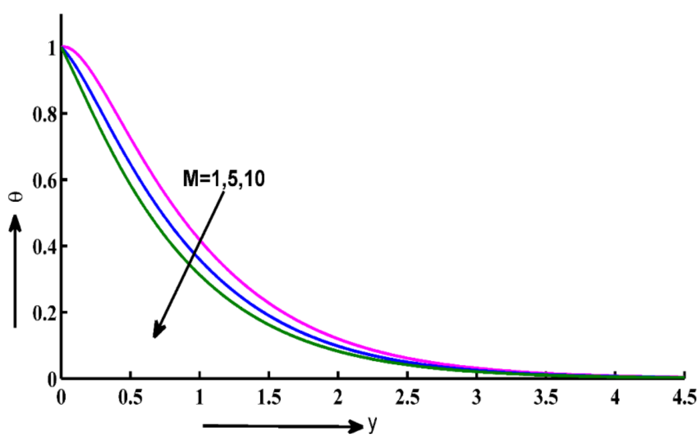

Figure 16: Effect of Magnetic Parameter on Temperature

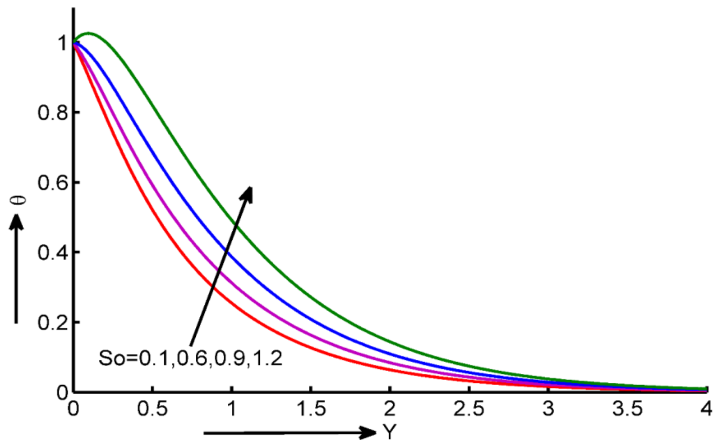

Figure 17: Effect of Soret Parameter on Temperature

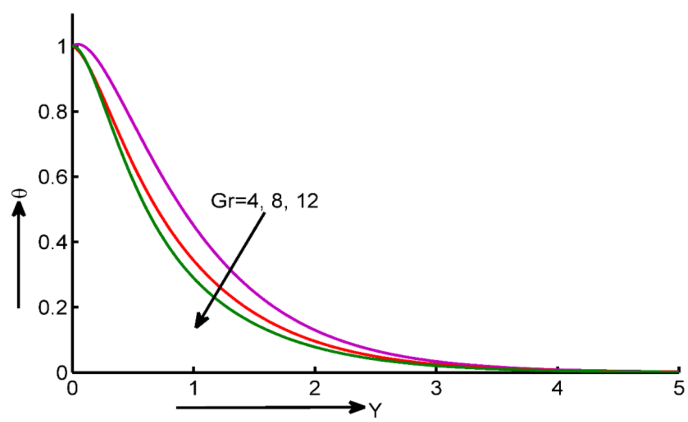

Figure 18: Effect of Grashof Number on Temperature

To analyze the effect of Schmidt parameter (Sc) on the concentration profile we see from the Fig.19 that the concentration field decreases when Sc increases. Fig. 20 depicts the variations in concentration profile for different values of the Soret parameter (S_0). From this figure it is noticed that the concentration increases when S_0 increases. The effect of the concentration profiles for different values of Chemical reaction parameter (K_0) is shown in Fig. 20 which leads us to the conclusion that the concentration decreases with an increase in the Chemical reaction parameter (K_0).

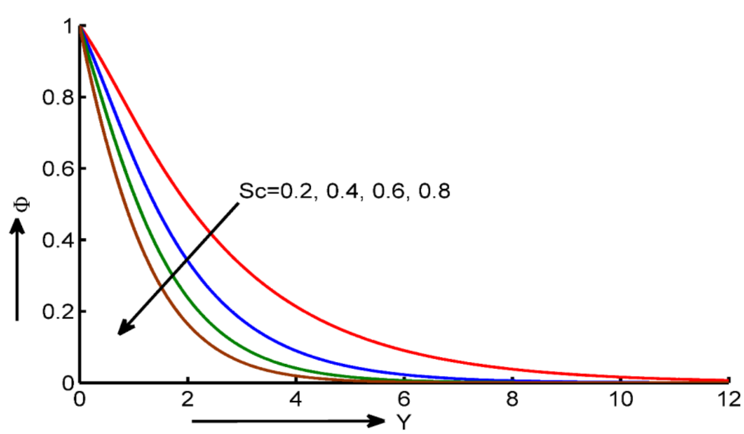

Figure 19: Effect of Schmidt Number on Concentration

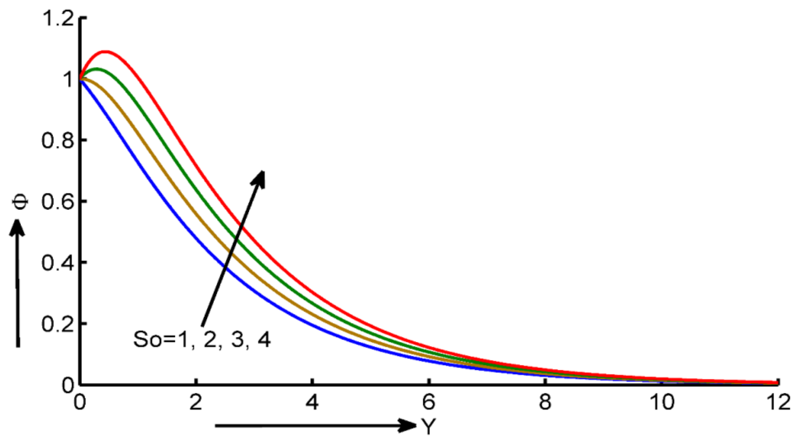

Figure 20: Effect of Soret Parameter on Concentration

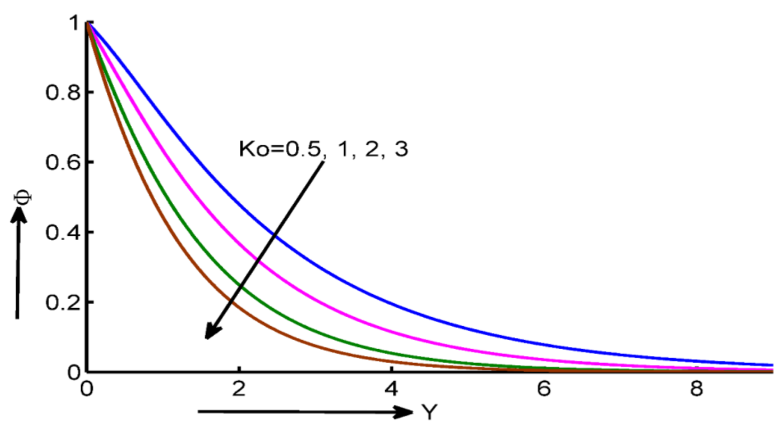

Figure 21: Effect of Chemical Reaction Parameteron Concentration

Table – 1 exhibits the numerical values of skin-friction for various values of the Grashof number (Gr), the modified Grashof number (Gm), the Magnetic parameter (M) and the Permeability parameter (K). From Table 1 we observe that the skin-friction increases with an increase in the Grashof number (Gr), the modified Grashof number (Gm) and the Permeability parameter (K) whereas it decreases under the influence of the magnetic parameter.

Table 1: Variations in theSkin Friction

Gr	Gm	K	M	τ
5	5	0.5	1	6.3456
7	5	0.5	1	7.0958
9	5	0.5	1	7.8539
5	6	0.5	1	7.1362
5	7	0.5	1	7.9275
5	8	0.5	1	8.7194
5	5	0.1	1	3.5684
5	5	0.2	1	4.6393
5	5	0.3	1	5.3658
5	5	0.5	0.5	6.8879
5	5	0.5	0.7	6.6541
5	5	0.5	0.9	6.4434
5	5	0.5	1.5	5.9186

Table – 2 demonstrates the numerical values of the Nusselt number (Nu) for different values of the Prandtl number (Pr), the Radiation parameter (F), the Heat source parameter (Q) and the Magnetic parameter (M). Table 2 clearly shows that the Nusselt number increases with an increase in the Prandtl number, the Radiation parameter, the Magnetic parameter and the Heat source parameter.

Table 2: Variations in the Nusselt Number

Pr	F	Q	M	Nu
0.3	0.5	0.5	1	1.1325
0.7	0.5	0.5	1	1.3533
0.71	0.5	0.5	1	1.3592
7.1	0.5	0.5	1	6.7953
0.71	0.5	0.5	1	1.3592
0.71	0.7	0.5	1	1.4507
0.71	0.9	0.5	1	1.5350
0.71	1.2	0.5	1	1.6506
0.71	0.5	0.6	1	1.4060
0.71	0.5	0.7	1	1.4507
0.71	0.5	0.8	1	1.4937
0.71	0.5	0.9	1	1.5350
0.71	0.5	0.5	2	1.3574
0.71	0.5	0.5	3	1.3628
0.71	0.5	0.5	4	1.3689
0.71	0.5	0.5	5	1.3743

Table – 3 shows the numerical values of the Sherwood number (Sh) for the distinct values of the Schmidt number (Sc), the Chemical reaction parameter (K_0) and the Soret number(S_0). The direct conclusion following from Table – 3 id that the Sherwood number enhances with rising values of the Schmidt number and the Chemical reaction parameter whereas it decrease under the influence of the Soret number(S_0).

Table 3: Variations in the Sherwood Number

Sc	K_0	S_0	Sh
0.22	0.5	1	0.2577
0.60	0.5	1	0.6161
0.78	0.5	1	0.8933
0.22	1	1	0.4143
0.22	1.5	1	0.5367
0.22	2	1	0.6409
0.22	0.5	0.5	0.3583
0.22	0.5	1	0.2577
0.22	0.5	1.5	0.1579

5. CONCLUSIONS

In this paper we discuss the effect of Joule heating and thermal diffusion on MHD fluid flow past a vertical porous plate embedded in porous media. From the aforesaid analysis of the problem the following conclusions are made:

1. The velocity increases with an increase in the Grashof number and as well as the modified Grashof number, the Permeability parameter, the Soret parameter and the Eckert number of the porous medium while, it decreases in the presence of the magnetic parameter, the Radiation parameter, the Heat source parameter, the Schmidt number, the Prandtl number and the chemical reaction parameter.
2. The temperature increases in the presence of the Eckert number and the Soret parameter while it decreases in the presence of the magnetic parameter, the heat sink, the Prandtl number, the Grashof number and the Radiation parameter.
3. The concentration decreases with an increase in the Schmidt number and the chemical reaction parameter while it increases in the presence of the Soret parameter.

4. A significant increase in seen in the skin friction for the Grashof number, the modified Grashof number and the permeability parameter while a decrease in it is seen in the presence of the magnetic parameter.
5. The rate of heat transfer increases with the Prandtl number, the heat sink , the Magnetic paremeter and the radiation parameter.
6. The rate of mass transfer increases with the Schmidt number and the Chemical reaction parameter while a decrease is seen in the presence of the Soret parameter.

APPENDIX

$$M_1 = M + \frac{1}{K} \qquad M_2 = F + Q$$

$$m_1 = \frac{\Pr + \sqrt{\Pr^2 + 4M_2}}{2} \qquad m_2 = \frac{Sc + \sqrt{Sc^2 + 4K_0 Sc}}{2} \qquad m_3 = \frac{1 + \sqrt{1 + 4M_1}}{2}$$

$$A_1 = \frac{m_1^2 S_0 Sc}{m_1^2 - Scm_1 - K_0 Sc} \qquad A_2 = 1 + A_1 \qquad A_3 = \frac{A_1 Gm - Gr}{m_1^2 - m_1 - M_1}$$

$$A_4 = \frac{A_2 Gm}{m_2^2 - m_2 - M_1} \qquad A_5 = A_4 - A_3$$

$$\lambda_1 = m_1 A_3 \qquad \lambda_2 = m_2 A_4 \qquad \lambda_3 = m_3 A_5$$

$$A_6 = \frac{-\Pr \lambda_1^2}{4m_1^2 - 2\Pr m_1 - M_2} \qquad A_7 = \frac{-\Pr \lambda_2^2}{4m_2^2 - 2\Pr m_2 - M_2} \qquad A_8 = \frac{-\Pr \lambda_3^2}{4m_3^2 - 2\Pr m_3 - M_2}$$

$$\delta_1 = m_1 + m_2 \qquad \delta_2 = m_2 + m_3 \qquad \delta_3 = m_3 + m_1$$

$$A_9 = \frac{2\lambda_1 \lambda_2 \Pr}{\delta_1^2 - \Pr \delta_1 - M_2} \qquad A_{10} = \frac{2\lambda_2 \lambda_3 \Pr}{\delta_2^2 - \Pr \delta_2 - M_2} \qquad A_{11} = \frac{-2\lambda_1 \lambda_3 \Pr}{\delta_3^2 - \Pr \delta_3 - M_2}$$

$$A_{12} = \frac{-\Pr M A_3^2}{4m_1^2 - 2\Pr m_1 - M_2} \qquad A_{13} = \frac{-\Pr M A_4^2}{4m_2^2 - 2\Pr m_2 - M_2} \qquad A_{14} = \frac{-\Pr M A_5^2}{4m_3^2 - 2\Pr m_3 - M_2}$$

$$A_{15} = \frac{2A_3 A_4 \Pr M}{\delta_1^2 - \Pr \delta_1 - M_2} \qquad A_{16} = \frac{2A_4 A_5 \Pr M}{\delta_2^2 - \Pr \delta_2 - M_2} \qquad A_{17} = \frac{-2A_5 A_3 \Pr M}{\delta_3^2 - \Pr \delta_3 - M_2}$$

$$A_{18} = A_6 + A_{12} \qquad A_{19} = A_7 + A_{13} \qquad A_{20} = A_8 + A_{14}$$

$$A_{21} = A_9 + A_{15} \qquad A_{22} = A_{10} + A_{16} \qquad A_{23} = A_{11} + A_{17}$$

$$A_{24} = -(A_{18} + A_{19} + A_{20} + A_{21} + A_{22} + A_{23})$$

$$A_{25} = \frac{-m_1^2 A_{24} S_0 Sc}{m_1^2 - Scm_1 - K_0 Sc} \qquad A_{26} = \frac{-4m_1^2 A_{18} S_0 Sc}{4m_1^2 - 2Scm_1 - K_0 Sc} \qquad A_{27} = \frac{-4m_2^2 A_{19} S_0 Sc}{4m_2^2 - 2Scm_2 - K_0 Sc}$$

$$A_{28} = \frac{-4m_3^2 A_{20} S_0 Sc}{4m_3^2 - 2Scm_3 - K_0 Sc} \qquad A_{29} = \frac{-\delta_1^2 A_{21} S_0 Sc}{\delta_1^2 - Sc\delta_1 - ScK_0} \qquad A_{30} = \frac{-\delta_2^2 A_{22} S_0 Sc}{\delta_2^2 - Sc\delta_2 - ScK_0}$$

$$A_{31} = \frac{-\delta_3^2 A_{23} S_0 Sc}{\delta_3^2 - Sc\delta_3 - ScK_0} \qquad A_{32} = -(A_{25} + A_{26} + A_{27} + A_{28} + A_{29} + A_{30} + A_{31})$$

$$A_{33} = \frac{-A_{24} Gr - A_{25} Gm}{m_1^2 - m_1 - M_1} \qquad A_{34} = \frac{-A_{32} Gm}{m_2^2 - m_2 - M_1} \qquad A_{35} = \frac{-A_{18} Gr - A_{26} Gm}{4m_1^2 - 2m_1 - M_1}$$

$$A_{36} = \frac{-A_{19}Gr - A_{27}Gm}{4m_2^2 - 2m_2 - M_1} \qquad A_{37} = \frac{-A_{20}Gr - A_{28}Gm}{4m_3^2 - 2m_3 - M_1} \qquad A_{38} = \frac{-A_{21}Gr - A_{29}Gm}{\delta_1^2 - \delta_1 - M_1}$$

$$A_{39} = \frac{-A_{22}Gr - A_{30}Gm}{\delta_2^2 - \delta_2 - M_1} \qquad A_{40} = \frac{-A_{23}Gr - A_{31}Gm}{\delta_3^2 - \delta_3 - M_1}$$

$$A_{41} = -(A_{33} + A_{34} + A_{35} + A_{36} + A_{37} + A_{38} + A_{39} + A_{40})$$

Nomenclature

C	nondimensional fluid concentration
C*	concentration
C_α	fluid concentration far away from the wall
C_p	specific heat at a constant pressure
D	mass diffusivity
E	Eckert number
$e_{b\lambda}$	Planck function
F	radiation parameter
Gm	mass Grashof number
Gr	thermal Grashof number
g	acceleration due to gravity
k	nondimensional permeability coefficient of a porous medium
k_0	nondimensional rate of chemical reaction
k_c	rate of chemical reaction
k_p	permeability of porous medium
$K_{\lambda w}$	absorption coefficient
M	magnetic parameter
Nu	Nusselt number
Pr	Prandtl number
q_r	radiative flux
Sc	Schmidt number
T_∞	fluid temperature far away from the wall
T*	temperature
u*, v*	velocity components
u	nondimensional velocity
v_0	suction velocity

Greek symbols

K	thermal conductivity
ν	kinematic viscosity
σ	electrical conductivity
μ	dynamic viscosity
β_T	coefficient of volume expansion
β_e	coefficient of volume expansion with concentration
ρ	fluid density
τ	non-dimensional skin friction
θ	nondimensional temperature

Subscripts and super scripts

W	wall
∞	far away from the wall
Prime (')	denotes differentiation with respect to y

REFERENCES

[1]. Ravikumar V., Raju M.C., Raju G.S.S., Chamkha A.J. (2013). MHD double diffusive and chemically reactive flow through porous medium bounded by two vertical plates, *Int. J. Energy Technology*, Vol. 5, No. 4, 1-8.

[2]. Sinha A., Ahmed N., Agarwalla S. (2017). MHD free convective flow through a porous medium past a vertical plate with ramped wall temperature, *Applied Mathematical Sciences*, Vol.11, No.10, 963-974.

[3]. Tripatihi B., Sharma B.K. (2018). Effect of heat transfer on MHD blood flow through an inclined stenosed porous artery with variable viscosity and heat source, *Romanian J. Biophys.*, Vol.28, No. 3, 89-102.

[4]. Sharmilaa K., Kannathal K. (2018). Steady MHD Mixed Convective flow in presence of inclined magnetic filed and thermal radiation with effects of chemical reaction and Soret embedded in a porous medium, *IJSRSET*, Vol 4, No. 1, .

[5]. Rama Krishna Reddy P., Raju M.C. (2018). MHD free convective flow past a porous plate, *International Journal of Pure and Applied* Mathematics, Vol. 118, No. 5, 507-529.

[6]. Mohammed Ibrahim S. and Suneetha K. (2015). Chemical reaction and Soret effects on unsteady MHD flow of a viscoelastic fluid past an impulsively started infinite vertical plate with heat source/sink, *Int. J. Math and Comp. Sci.*, Vol. 1, No. 1, 5-15.

[7]. Sharma M., Sharma B.K., Gaur R.K. and Tripathi B. (2018). Soret and Dufour effects in biomagnetic fluid of blood flow through a tapered porous stenosed artery, *J. Nanofluids*, (Article in Press).

[8]. Sharma B.K., Tailor V. and Goyal M. (2017). Heat source and Soret effects on magneto-micropolar fluid flow with variable permeability and chemical reaction, *Global Journal of Pure and Applied Mathematics*, 13(9), 5195-5212.

[9]. Sharma B.K., Gupta S., Krishna V.V. and Bhargavi R.J. (2014). Soret and Dufour effects on an unsteady MHD mixed convective flow past an infinite vertical plate with Ohmic dissipation and heat source, *Afrika Matematika*, 25, 799-825.

[10]. Satya Narayana P.V., Sravanthi C.S. (2012). Simultaneous effects of Soret and Ohmic heating on MHD free convective heat and mass transfer flow of micropolar fluid with porous medium, *Elixir Appl. Math.*, 48, 9379-9386.

[11]. Sharma, P.K., Sharma, B. K. and Chaudhary, R.C. (2008). Unsteady natural convection flow past a vertical surface in a rotating porous medium with variable permeability, *Bull. Cal. Math. Soc.*, 100, 491-500.

[12]. Mahapatra N., Dash G.C., Panda S., Acharya M. (2010). Effects of chemical reaction on free convection flow through a porous medium bounded by a vertical surface, *Int. Eng. Phys. Thermo Phys.*, Vol.83, No.1, .

[13]. Raju, M.C., Varma, S.V.K. and Reddy T.S. (2011). Unsteady MHD free convection oscillating couette flow through a porous medium with periodic wall temperature in presence of chemical reaction and thermal radiation, *International Journal of Science and Advances in Technology*, Vol. 1(10), 51-58.

[14]. Sharma, B.K., Chand, T. and Chaudhary, R.C.(2012): Hydromagnetic forced flow between a rotating disc and a naturally permeable stationary porous disc saturated with fluid, *Journal of Petroleum and Gas Engineering*, 3(1), 8-15. DOI: 10.5897/JPGE11.012

[15]. Sarada, K., Shanker, B. (2013). The effect of chemical reaction on an unsteady MHD free convection flow past an infinite vertical porous plate with variable suction, *IJMER*, Vol.3(2), 725-735.

[16]. Ibrahim Mohammed (2013). Effects of mass transfer, radiation, Joule heating and viscous dissipation on steady MHD marangoni convection flow over a flat surface with suction and injection, *Int. J. Eng. Math.*, 9 903818.

[17]. Abo-Eldahab, E.M., El Aziz, M.A. (2005). Viscous dissipation and Joule heating effects on MHD-free convection from a vertical plate with power-law variation in surface temperature in the presence of Hall and ios-slips currents, *Appl. Math. Model*, 29, 579–595.

[18]. Ibrahim, S. Mohammed and Suneetha, K. (2016). Heat source and chemical effects on MHD convection flow embedded in a porous medium with Soret, viscous and Joule's dissipation, *Aim Shams Engg. Journal*, Vol. 7, 811-818.

[19]. Veeresh, C., Varma, S.V.K., Vijaya Kumar, A.G., Umamaheswar, M. and Raju M.C. (2017). Joule heating and thermal diffusion effects on MHD radiative and convective casson fluid flow past an oscillating semi-infinite vertical porous plate, *Frontiers in Heat and Mass Transfer (FHMT)*, 8, 1 , .

[20]. Harinath Reddy, S., Chandra Reddy, P., Raju, M.C. and Keshava Reddy E. (2018). Joule heating influence on MHD casson fluid over a vertical porous plate in the presence of thermal diffusion and chemical reaction, *International Journal of Research in Advent Technology*, Vol.6, No.11, November 2018 E-ISSN: 2321-9637.

[21]. Cogley, A.C., Vincenti, W.G. and Gill, Se (1968). Differential approximation for radiative transfer in a non-gray gas near equilibrium, *AIAAJ,* Vol.6, 551-553.

[22]. Sidda Reddy, K., Chandra Reddy, P. and Raju G.S.S. (2018). Thermal diffusion and Joule heating effects on MHD radiating fluid embedded in porous medium, *International Journal for Research in Engineering Application & Management (IJREAM)*, 4(4), 206-212.

[23]. Harinath Reddy, S., Raju, M.C. and Chandra Reddy, P. (2018). Joule Heating and Radiation Absorption Effects on MHD Convective and Chemically Reactive Flow past a Porous Plate, *Bulletin of Pure and Applied Sciences, Section –E – (Math. & Stat.)* , Vol. 37E, No. 1, 117-136.

[24]. Alim, M.A., Alam, M.D. and Mamun, A. (2007). Joule heating effect on the coupling of conduction with magneto hydrodynamic free convection flow from a vertical plate, *Nonlinear Analysis Modelling and Control*, 12(3), 307–316.

[25]. Palani, G. and Kim, K.Y. (2011). Joule heating and viscous dissipation effects on MHD flow past a semi-infinite inclined plate with variable surface temperature, *Journal of Engineering Thermo Physics*, 20, 501-517.

[26]. Awasthi, Bhavtosh (2017). Joule Heating Effect in Presence of Thermal Radiation on MHD Convective Flow Past Over A Vertical Surface in A Porous Medium, *International Journal of Engineering and Science Invention (IJESI)*, Vol. 6, No. 9, 30-40.