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Fuzzy stability of cubic (ρ_1, ρ_2)-functional inequality *

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Abstract In this paper, we introduce and ratify the generalized Hyers-Ulam stability of cubic (ρ_1, ρ_2)-functional inequality in fuzzy normed space using the fixed point method.

Key words Generalized Hyers-Ulam stability(H-US), Fuzzy Normed space, Cubic (ρ_1, ρ_2)-functional inequality, Cubic (ρ_1, ρ_2)-functional equation.

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1 Introduction

Few years ago, Glányi [8] proclaimed that any f gratifies the Jordan-von Neumann functional equation

$$2f(x) + 2f(y) = f(xy) + f(xy^{-1})$$

if f gratifies the functional inequality

$$||2f(x) + 2f(y) - f(xy^{-1})|| = ||f(xy)|| \tag{1.1}$$

Glányi [9] and Fechner [5] ratified the H-US of the functional inequality (1.1). Park, Cho and Han [18] investigated and ratified the H-US of the Cauchy additive functional inequality

$$||f(x) + f(y) + f(z)|| = ||f(x + y + z)|| \tag{1.2}$$

and the Cauchy-Jensen additive functional inequality

$$||f(x) + f(y) + 2f(z)|| = ||2f(\frac{x+y}{2} + z)||. \tag{1.3}$$

and they also ratified the H-US of (1.2) and (1.3) in Banach spaces. The Hyers-Ulam Stability is consequence of study of Ulam's [1] problem regarding stability of group homomorphism by various mathematicians namely Hyers' [10], Aoki [2], Th.M. Rassias [19], Găvruta [7] under various adaptations. Park [16,17] introduced additive ρ-functional inequalities and ratified their H-US in Banach spaces and

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non-Archimedean Banach spaces. In this paper, we introduce and ratify generalized H-US of cubic (ρ_1, ρ_2)-functional inequality

$$F(F_1(x,y,z),t) = \min(F(\rho_1 F_2(x,y,z),t), F(\rho_2 F_3(x,y,z),t)) \tag{1.4}$$

where

$$\begin{aligned} F_1(x,y,z) &= f(kx+(k+1)y+(k+2)z) - f(kx-(k+1)y-(k+2)z) \\ &-2f((k+1)y+(k+2)z) - f(kx+(k+1)y) - f(kx+(k+2)z) \\ &+f(kx-(k+1)y) + f(kx-(k+2)z) + 2f((k+1)y) + 2f((k+2)z) \\ F_2(x,y,z) &= 8f((kx+(k+1)y+(k+2)z)/2) - f(kx-(k+1)y-(k+2)z) \\ &-2f((k+1)y+(k+2)z) - f(kx+(k+1)y) - f(kx+(k+2)z) \\ &+f(kx-(k+1)y) + f(kx-(k+2)z) + 2f((k+1)y) + 2f((k+2)z) \\ 2(F_3(x,y,z)) &= 8f((kx+(k+1)y+(k+2)z)/2) - f(kx-(k+1)y-(k+2)z) \\ &-2f((k+1)y+(k+2)z) - f(kx+(k+1)y) - f(kx+(k+2)z) \\ &+8f((kx-(k+1)y)/2) + f(kx-(k+2)z) + 2f((k+1)y) + 2f((k+2)z) \end{aligned}$$

in fuzzy normed space, where k is a positive integer greater than one; $\rho_1, \rho_2 \in \mathbb{R}\backslash\{0\}$ with $\{\frac{1}{2\rho_1}+\frac{1}{\rho_3}\}<1$.

2 Preliminaries

Katsaras [11] defined the concept of fuzzy norm on a linear space in 1984. Since then, various mathematicians [3, 6, 12, 20] have defined fuzzy norms on a vector space from various points of view.

Definition 2.1. ([3, 15]): If X is a real vector space, then a function $F : X \times \mathbb{R} \to [0,1]$ is a fuzzy norm on X if for every $a, b \in X$ and all $r, m, n \in \mathbb{R}$, we have:

FN1. $F(a,n) = 0$ for $n = 0$,

FN2. $a = 0$ iff $F(a,n) = 1$ for all $n>0$,

FN3. $F(ra,n) = F(a, \frac{n}{|r|})$ if $r \neq 0$,

FN4. $F(a+b, m+n) = \min\{F(a,m), F(b,n)\}$,

FN5. $\lim_{n\to\infty} F(a,n) = 1$, where $F(a,.)$ is a non-decreasing function on $\mathbb{R}$,

FN6. $F(a,.)$ is continuous on $\mathbb{R}$, for $a \neq 0$,

then the pair (X, F) is called a *fuzzy normed vector space.*

Definition 2.2. ([3, 15]):

1. Let (X, F) be a fuzzy normed vector space. A sequence $\{a_n\}$ in X is said to be *convergent* if $\exists$ an $a \in X$ such that $\lim_{n\to\infty} F(a_n - a, r) = 1$ for all $r>0$, where $F-\lim_{n\to\infty} a_n = a$ (a is the limit of the sequence $\{a_n\}$).
2. Let (X, F) be a fuzzy normed vector space. A sequence $\{a_n\}$ in X is said to be a *Cauchy sequence* if for each $\varepsilon>0$ and each $r>0$ there exists an $n_0 \in \mathbb{N}$ such that for all $n \geq n_0$ and all $m>0$,we have $F(a_{n+m} - a_n, r) > 1-\varepsilon$.
3. The fuzzy norm is said to be *complete* if every Cauchy sequence is convergent and the fuzzy normed vector space is called a *fuzzy Banach space.*
4. A mapping $f{:}X \to Y$ where X and Y are fuzzy normed vector spaces is continuous at a point $a_0 \in X$ if for each sequence $\{a_n\}$ converging to $a_0 \in X$, the sequence $\{f(a_n)\}$ converges to $f(a_0)$. If $f{:}X \to Y$ is continuous at each $a \in X$, then $f{:}X \to Y$ is said to be *continuous* on X.

Definition 2.3. ([13]): Let X be a set. A function $d : X \times X \to [0, \infty)$ is called a generalized metric on X if d gratifies the following conditions:

1. $d(x,y) = 0$ if and only if $x{=}y$ for all $x, y \in X$,
2. $d(x,y) = d(y,x)$ for all $x, y \in X$,
3. $d(x,z) \leq d(x,y) + d(y,z)$ for all $x, y, z \in X$.

Theorem 2.4. *([4]): Let (X, d) be a complete generalized metric space and $G{:}X \to X$ a strictly contractive mapping with Lipschitz constant $L{<}1$. Then, for all $x \in X$, either $d(G^n x, G^{n+1}x) = \infty$, for all $n \in \{0, \mathbb{Z}^+\}$ or there exists a positive integer n_0 such that*

1. *$d(G^n x, G^{n+1}x) < \infty$ for all $n \geq n_0$,*
2. *the sequence $\{G^n x\}$ converges to a fixed point y^* of G,*
3. *y^* is the unique fixed point of G in the set $Y{=}\{y \in X{:}d(G^{n_0}x, y) < \infty\}$,*
4. *$d(y, y^*) \leq (1/(1{-}L))d(y, Gy)$ for all $y \in Y$.*

Throughout the paper, suppose that ρ_1 and ρ_2 are fixed nonzero real numbers with $\{\frac{1}{2\rho_1}{+}\frac{1}{\rho_2}\}{<}1$ and k is a positive integer greater than one. Also X and Y be real fuzzy normed space and fuzzy Banach space respectively with norm $F(., t)$.

3 Cubic ρ_1, ρ_2- functional inequality

Lemma 3.1. *Let $f{:}X \to Y$ be a mapping with $f(0) = 0$ and gratifies (1.4) for all $x, y, z \in X$ and all $t{>}0$. Then f is cubic.*

Proof. Suppose that function f gratifies (1.4). By letting $y = z = 0$ in (1.4), we get

$$1 \leq \min(F(\rho_1(8f(\frac{kx}{2}){-}f(kx)),t), (\rho_2(8f(\frac{kx}{2}){-}f(kx)),t))$$

$$\leq F((\rho_1{+}\rho_2)(8f(\frac{kx}{2}){-}f(kx)), 2t) = F((8f(\frac{x}{2}){-}f(x)), \frac{2t}{(\rho_1{+}\rho_2)k})$$

Therefore,

$$f(\frac{x}{2}) = \frac{1}{8}f(x) \tag{3.1}$$

Now from (1.4) and (3.1) we get

$$F(F_1(x, y, z),t) \leq \min(F(\rho_1 F_1(x, y, z),t), F(\frac{\rho_2}{2}F_1(x, y, z),t))$$

$$= \min(F(\frac{1}{2}F_1(x, y, z), \frac{t}{2|\rho_1|}), F(\frac{1}{2}F_1(x, y, z), \frac{t}{|\rho_2|}))$$

$$\leq F(F_1(x, y, z), (\frac{1}{2|\rho_1|}{+}\frac{1}{|\rho_2|})t)$$

i.e.,

$$F(F_1(x, y, z),t) \geq F(F_1(x, y, z), \frac{t}{\zeta})$$

where, $\zeta{=}\{\frac{1}{2\rho_1}{+}\frac{1}{\rho_2}\}$. Putting $\frac{t}{|\zeta|^{n-1}}$ instead of t, we get

$$F(F_1, \frac{t}{|\zeta|^{n-1}}) \geq F(F_1, \frac{t}{|\zeta|^n})$$

Thus, for all $n \in Z^+$ we have, $F(F_1, t)) \geq F(F_1, \frac{t}{|\zeta|^n})$.
Since $\zeta{<}1$, therefore by taking limit $n \to \infty$ and using (**FN5**) from the Definition 2.1, we get $F(F_1(x, y, z),t) = 1$ for all $x, y, z \in X$, and hence $F_1(x, y, z) = 0$. So, $f{:}X \to Y$ is cubic. □

Theorem 3.2. *Let $\Psi{:}X^3 \to [0,\infty)$ be a function such that*

$$\Psi(x, y, z) \leq \frac{L}{8}\Psi(2x, 2y, 2z)$$

for some $L{<}1$ and for all x,y,z $\in X$. Let $f{:}X \to Y$ be a mapping with $f(0) = 0$ and satisfying

$$\min(F(F_1(x, y, z),t), \frac{t}{t{+}\Psi(x, y, z)}) \leq \min(F(\rho_1 F_2(x, y, z),t), F(\rho_2 F_3(x, y, z),t)) \tag{3.2}$$

where,

$$F_1(x,y,z) = f(kx+(k+1)y+(k+2)z) - f(kx-(k+1)y-(k+2)z)$$
$$-2f((k+1)y+(k+2)z) - f(kx+(k+1)y) - f(kx+(k+2)z)$$
$$+f(kx-(k+1)y) + f(kx-(k+2)z) + 2f((k+1)y) + 2f((k+2)z)$$
$$F_2(x,y,z) = 8f((kx+(k+1)y+(k+2)z)/2) - f(kx-(k+1)y-(k+2)z)$$
$$-2f((k+1)y+(k+2)z) - f(kx+(k+1)y) - f(kx+(k+2)z)$$
$$+f(kx-(k+1)y) + f(kx-(k+2)z) + 2f((k+1)y) + 2f((k+2)z)$$
$$2(F_3(x,y,z)) = 8f((kx+(k+1)y+(k+2)z)/2) - f(kx-(k+1)y-(k+2)z)$$
$$-2f((k+1)y+(k+2)z) - f(kx+(k+1)y) - f(kx+(k+2)z)$$
$$+8f((kx-(k+1)y)/2) + f(kx-(k+2)z) + 2f((k+1)y) + 2f((k+2)z)$$

for all $x,y,z \in X$ and all $t>0$. Then $C(x) = F-\lim_{n\to\infty} 8^n f(\frac{x}{2^n})$ exists for all $x \in X$ and defines a cubic mapping $C{:}X \to Y$ such that

$$F(f(x)-C(x),t) \geq \frac{(2-2L)kt}{(2-2L)kt+\eta\Psi(x,0,0)} \tag{3.3}$$

for all $x \in X$, $t>0$, where, $\eta=\frac{1}{|\rho_1|}+\frac{1}{|\rho_2|}$.

Proof. Let $y=z=0$ in (3.2), we get

$$\frac{t}{t+\Psi(x,0,0)} \leq \min(F(\rho_1(8f(\frac{kx}{2})-f(kx)),t), F(\rho_2(8f(\frac{kx}{2})-f(kx)),t)$$
$$\leq \min(F((8f(\frac{kx}{2})-f(kx)),\frac{t}{|\rho_1|}), F(\rho_2(8f(\frac{kx}{2})-f(kx)),\frac{t}{|\rho_2|})$$
$$\leq F(8f(\frac{kx}{2})-f(kx),(\frac{1}{|\rho_1|}+\frac{1}{|\rho_2|})\frac{t}{2})$$

i.e.

$$F(f(x)-8f(\frac{x}{2}),\frac{\eta t}{2k}) \geq \frac{t}{t+\Psi(x,0,0)} \tag{3.4}$$

Now let us consider the set $S=\{g{:}X \to Y\}$ and a generalized metric on S, such that

$$d(g,h) = \inf(\varepsilon \in R^+{:}F(g(x)-h(x),\varepsilon t) = \frac{t}{t+\Psi(x,0,0)}, \text{ for all } x \in X, \text{ for all } t>0),$$

where $\inf(\Psi) = +\infty$. Next, using the Lemma 2.1 ([14]) we can say that (S,d) is complete. Now, let us consider a linear mapping $A{:}S \to S$ such that

$$Ag(x) = 8g(\frac{x}{2})$$

for all $x \in X$. Let $g,h \in S$ with $d(g,h) = \gamma$. Then

$$F(g(x)-h(x),\gamma t) \geq \frac{t}{t+\Psi(x,0,0)}$$

for all $x \in X, t>0$. Therefore,

$$F(Ag(x)-Ah(x),L\gamma t) = F(8g(x/2)-8h(x/2),L\gamma t) = F(g(x/2)-h(x/2),(L\gamma t/8)$$

$$\geq \frac{\frac{Lt}{8}}{\frac{Lt}{8}+\Psi(x/2,0,0)} \geq \frac{\frac{Lt}{8}}{\frac{Lt}{8}+\frac{L}{8}\Psi(x,0,0)} = \frac{t}{t+\Psi(x,0,0)}$$

for all $x \in X, t>0$. Hence $d(Ag,Ah) = L\gamma$, i.e. $d(Ag,Ah) = Ld(g,h)$ for all $g,h \in S$. Also using (3.4), we can say that

$$d(f,Af) \leq \frac{\eta}{2k}.$$

Now, by Theorem 2.4, there exists a mapping $C : X \to Y$ such that:

1. C is a fixed point of A, i.e.,

$$C(x) = 8C(\frac{x}{2}) \tag{3.5}$$

for all $x \in X$. Since the mapping C is a unique fixed point of A in the set

$$T= (g \in S{:}d(f,g) < \infty),$$

thus C is a unique mapping satisfying (3.5) such that there exists an $\varepsilon \in (0,\infty)$ satisfying

$$F(f(x){-}C(x),\varepsilon t) \geq \frac{t}{t{+}\Psi(x,0,0)}$$

for all $x \in X$.

2. $d(A^n f, C) \to 0$ as $n \to \infty$. This implies that $C(x) = F{-}\lim_{n\to\infty} 8^n f(\frac{x}{2^n})\ \forall\ x \in X$.
3. $d(f,C) \leq \frac{1}{1-L} d(f,Af)$, which implies that $d(f,C) \leq \frac{\eta}{2k-2kL}$. And thus inequality (3.3) is ratified.

$$\min(F(8^n F_1(\frac{x}{2^n},\frac{y}{2^n},\frac{z}{2^n}),8^n t),\frac{t}{t{+}\Psi(\frac{x}{2^n},\frac{y}{2^n},\frac{z}{2^n})})$$

$$\leq \min(F(8^n \rho_1 F_2(\frac{x}{2^n},\frac{y}{2^n},\frac{z}{2^n}),8^n t),F(8^n \rho_2 F_3(\frac{x}{2^n},\frac{y}{2^n},\frac{z}{2^n}),8^n t))$$

for all $x, y \in X$, all $t{>}0$ and all $n \in N$. So,

$$\begin{aligned}&\min(F(8^n F_1(\frac{x}{2^n},\frac{y}{2^n},\frac{z}{2^n}),t), \frac{t/8^n}{(t/8^n)+(L^n/8^n)\Psi(x,y,z)})\\ &\leqslant \min(F(8^n \rho_1 F_2(\frac{x}{2^n},\frac{y}{2^n},\frac{z}{2^n}),t), F(8^n \rho_2 F_3(\frac{x}{2^n},\frac{y}{2^n},\frac{z}{2^n}),t))\end{aligned} \tag{3.6}$$

Since $\lim_{n\to\infty} \frac{t/8^n}{(t/8^n)+(L^n/8^n)\Psi(x,y,z)} = 1$ for all $x, y \in X$ for all $t{>}0$, therefore by Lemma 3.1 the mapping $C{:}X \to Y$ is cubic.

□

Corollary 3.3. *Let $\varsigma \geq 0$ and p be a real number with $p{>}3$.Let X be a normed vector space with norm $||.||$ and (Y,N) be a fuzzy normed vector space. Let $f{:}X \to Y$ be a mapping with $f(0) = 0$ and*

$$\min\left(F\left(F_1\left(x,y,z\right),t\right), \frac{t}{t{+}\varsigma(||x||^p{+}||y||^p)}\right) = \min(F(\rho_1 F_2(x,y,z),t),F(\rho_2 F_3(x,y,z),t)) \tag{3.7}$$

where $F_1(x,y,z,),F_2(x,y,z,),F_3(x,y,z,)$ are as defined earlier for all $x,y,z \in X$ and all $t{>}0$. Then $C(x) = F{-}\lim_{n\to\infty} 8^n f(\frac{x}{2^n})$ exists for all $x \in X$ and a cubic mapping $C{:}X \to Y$ such that

$$F(f(x){-}C(x),t) \geq \frac{2(k^p{-}k^3)kt}{2(k^p{-}k^3)kt{+}\eta\varsigma||kx||^p} \tag{3.8}$$

for all $x \in X$, $t{>}0$, where $\eta{=}\frac{1}{|\rho_1|}{+}\frac{1}{|\rho_2|}$.

Proof. The proof follows from the Theorem 3.2 by taking $\Psi(x,y,z) = \varsigma(||x||^p{+}||y||^p{+}||z||^p)$ for all $x,y,z \in X$ and $L{=}\ |k|^{3-p}$ and we get desired result. □

Theorem 3.4. *Let $\Psi{:}X^3 \to [0,\infty)$ be a function such that*

$$\Psi(x,y,z) \leq 8L\Psi(\frac{x}{2},\frac{y}{2},\frac{z}{2})$$

for some $L{<}1$ and for all $x,y,z \in X$. Let $f : X \to Y$ be a mapping with $f(0) = 0$ and satisfying (3.2). Then $C(x) = F - \lim_{n\to\infty} \frac{1}{8^n} f(2^n x)$ exists for all $x \in X$ and defines a cubic mapping $C : X \to Y$ such that

$$F(f(x) - C(x),t) \geq \frac{(2-2L)8kt}{(2-2L)8kt + \eta\Psi(x,0,0)} \tag{3.9}$$

for all $x \in X$, $t > 0$, where $\eta = \frac{1}{|\rho_1|} + \frac{1}{|\rho_2|}$.

Proof. It follows from (3.4) that, $F(\frac{1}{8}f(x) - f(\frac{x}{2}), \frac{\eta t}{16k}) \geq \frac{t}{t+\Psi(x,0,0)}$ for all $x \in X$ and all $t > 0$. Now consider the linear mapping $A : S \to S$ such that

$$Ag(x) = \frac{1}{8}f(2x)$$

for all $x \in X$, where (S, d) is the generalized metric space as defined in the Theorem 3.2. Then $d(f, Af) \leq \frac{\eta}{16k}$. Hence

$$d(f, C) \leq \frac{\eta}{16k - 16kL}$$

which ratifies the inequality (3.9) and the rest of the proof can be easily generated from the previous Theorem 3.2. □

Corollary 3.5. *Let $\varsigma \geq 0$ and p be a real number with $0 < p < 3$. Let X be a normed vector space with norm $||.||$ and (Y, N) be a fuzzy normed vector space. Let $f : X \to Y$ be a mapping with $f(0) = 0$ and satisfying (3.7). Then $C(x) = F - \lim_{n\to\infty} \frac{1}{8^n} f(2^n x)$ exists for all $x \in X$ and a cubic mapping $C : X \to Y$ such that*

$$F(f(x) - C(x), t) \geq \frac{(2k^3 - 2k^p)8kt}{(2k^3 - 2k^p)8kt + \eta\varsigma k^3||x||^p} \tag{3.10}$$

for all $x \in X$, $t > 0$, where, $\eta = \frac{1}{|\rho_1|} + \frac{1}{|\rho_2|}$.

Proof. The proof follows from the Theorem 3.4 by taking $\Psi(x, y, z) = \varsigma(||x||^p + ||y||^p + ||z||^p)$ for all $x, y, z \in X$ and $L = |k|^{p-3}$ and thus we get the desired result. □

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