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Edge product cordial labeling on duplication of prism graph *

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Abstract For a graph $G = (V(G), E(G))$, an edge labeling function f: $E(G) \rightarrow \{0,1\}$ induces a vertex labeling function $f^*: V(G) \rightarrow \{0,1\}$ such that $f^*(v)$ is the product of the labels of the edges incident to v. This function f is called the edge product cordial labeling of G if the edges with label 1 and label 0 differ by at most 1 and the vertices with label 1 and label 0 also differ at most by 1. In this paper, we obtain an edge product cordial labeling of the duplication of prism graph.

Key words Cordial graph, Product cordial graph, Edge product cordial graph, Prism graph, Web graph, Duplication of vertex by edge.

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1 Introduction

Graph labeling is an assignment of integers to the vertices or edges of a graph subject to certain constraints, this was first introduced by Alex Rosa [3] in 1967. Later on other labelings came into existence, one such labeling is the Cordial Labeling introduced by Cahit [1]. The concept of Product Cordial Labeling was first introduced by Sundaram et al. [5]. The edge product cordial labeling was introduced by Vaidya and Barasara [6]. Vaidya and Barasara [7] proved that the following graphs are edge product cordial: C_n for n odd, trees with order greater than 2, unicyclic graphs of odd order; $C_n(t)$, the one point union of t copies of C_n for t even or t and n both odd, Helms, closed helms, webs, flowers, gears, shells S_n for odd n, double triangular snakes DT_n, quadrilateral snakes Q_n and double quadrilateral snakes DQ_n. There are also graphs which are not edge product cordial: C_n for n even, K_n for $n \leq 4$, $K_{m;n}$ for m, $n \leq 2$, wheels, the one point union of t copies of C_n for t odd and n even, shells S_n for even n, double fans. Vaidya and Barasara [8] proved that the shell graph S_n is product cordial for odd n and it is not product cordial for even n. Vaidya and Prajapathi [9] proved that the graph obtained by the duplication of a vertex in P_n and by the duplication of a vertex by an edge in P_n is a prime graph. For an exhaustive survey on edge product cordial labeling we refer to Gallian's survey [2]. In this paper, we obtain the edge product cordial labeling of the duplication of prism graph

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for $n \leq 4$, $K_{m,n}$ for $m, n \leq 2$, wheels, the one point union of t copies of C_n for t odd and n even, shells S_n for even n and double fans.

2 Preliminary definitions

We give below some preliminary definitions.

Definition 2.1. A function $f : V\ (G) \rightarrow \{0,\ 1\}$ is called binary vertex labeling of a graph G and $f(v)$ are called the labels of the vertex v of G under f. For an edge $e = (u,\ v)$, the induced function f:$E(G)\rightarrow$ $\{0,\ 1\}$ is defined as $f(e) = |f(u) - f(v)|$. Let $v_f\ (0)$, $v_f\ (1)$ be the number of vertices of G having labels 0 and 1 respectively under f and $e_f\ (0), e_f\ (1)$ be the number of edges of G having labels 0 and 1 respectively under f. A binary vertex labeling f of a graph G is called a cordial labeling [1] if $|v_f\ (0) - v_f\ (1)| \leq 1$ and $|e_f\ (0) - e_f\ (1)| \leq 1$. A graph which admits a cordial labeling is called a cordial graph [1].

Definition 2.2. Let f be a function from $V(G) \rightarrow$\{0,1\}. For each edge uv, assign the label $f(u)f(v)$. f is called a product cordial labeling if $|v_f\ (0) - v_f\ (1)| \leq 1$, $|e_f\ (0) - e_f (1)| \leq 1$ where $v_f\ (i)$ and $e_f\ (i)$ denote the number of vertices and edges respectively labeled with i(i=0,1). A graph with a product cordial labeling [5] is called a product cordial graph.

Definition 2.3. For a graph $G = (V\ (G),\ E\ (G))$, an edge labeling function $f :\ E(G) \rightarrow \{0,\ 1\}$ induces a vertex labeling function $f^* : V\ (G) \rightarrow \{0,\ 1\}$ defined as $\prod f\ (e_i)$ for $\{e_i \in E(G)/e_i$ is incident to $v\}$. Now denoting the number of vertices of G having label i under f^* as $v_f\ (i)$ and the number of edges of G having label i under f as $e_f\ (i)$, then f is called an edge product cordial labeling of graph G if $|v_f\ (0) - v_f\ (1)| \leq 1$ and $|e_f\ (0) - e_f\ (1)| \leq 1$. A graph G is called an edge product cordial graph if it admits an edge product cordial labeling [6].

Definition 2.4. A prism graph [2] Y_n, $n\geq$ 3 is a Cartesian product graph $C_n\times$ P_2, where C_n is a cycle graph of order n and P_2 is path graph of order 2.

Definition 2.5. The duplication of a vertex [4] V_j by a new edge $e = {V_j}', {V_j}''$ in a graph G produces a new graph $G^{''}$ such that $N\left({V_j}'\right) = V_j, {V_j}''$ and $N\left({V_j}''\right) = V_j, {V_j}'$.

3 The main result

In this section we obtain the edge product cordial labeling of the duplication of a prism graph, which is the principal result of this paper.

Theorem 3.1. *The duplication of prism graph Y_n by its outer vertices and joining the new vertices obtained admits the edge product cordial labeling.*

Proof. Consider a prism graph Y_n having vertices $v_1, \ldots, v_{2n}$ and edges $e_1, \ldots, e_{4n}$ (Fig. 1). Now duplicating the outer vertices of a graph G is obtained by the duplication of a vertex V_j by an edge $V_j^{'}$, and $V_j^{''}$. And G contains cycle C_3 whose vertices are V_j,$V_j^{'}$,$V_j^{''}$. Let $G^{'}$ be a graph obtained from G by joining all the C_3's by equivalent edges. The graph H has $v_1, \ldots, v_{4n}$ vertices and $e_1, \ldots, v_{7n}$ edges (Fig. 2).
Now to prove that $G^{'}$ is an edge product cordial graph, we consider the following two cases:
Case 1: When n is odd:

$$f\ (e_i) = 1, \quad 1 \leq i\ \leq \lfloor 5n/2 \rfloor + n,$$

$$f\ (e_i)\ = 0, \quad \text{otherwise.}$$

In view of the above defined labeling pattern we have,

$$v_f\ (0) = v_f\ (1) + 1 = 2n,$$

$$e_f\ (0) = e_f\ (1) = \ \lfloor 3n/2 \rfloor + 2n + 1.$$

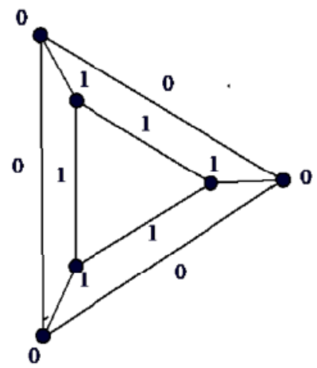

Fig. 1: Prism graph Y_3.

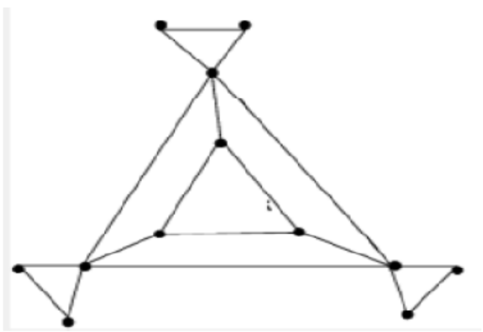

Fig. 2: The duplication of prism graph.

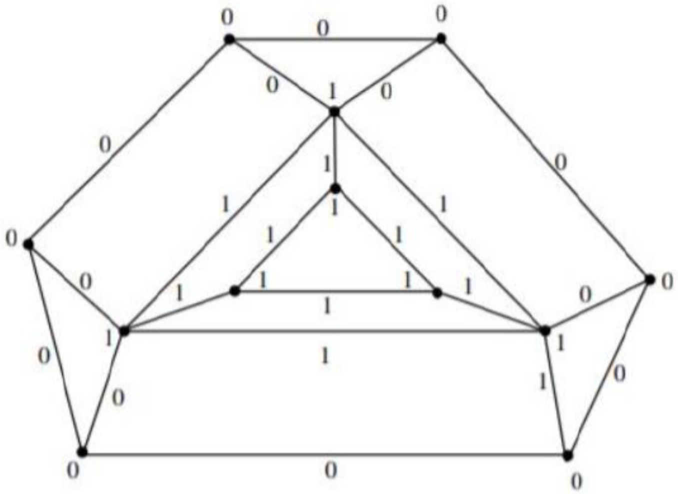

Fig. 3: The duplication of a prism graph by its outer vertices and by joining the new vertices.

Case 2: When n is even:

$$f\left(e_i\right)=1, \quad 1 \leq i \ \leq 3n+\lfloor n/2\rfloor,$$
$$f\left(e_i\right)=0, \quad \text{otherwise.}$$

From the above defined labeling pattern we have,

$$v_f\left(0\right)=v_f\left(1\right)=2n,$$

$$e_f\left(0\right)=e_f\left(1\right)= \ 3n+ \ \lfloor n/2\rfloor.$$

Thus in all the cases we have $\left|v_f\left(0\right)-v_f\left(1\right)\right| \leq 1$ and $\left|e_f\left(0\right)-e_f\left(1\right)\right| \leq 1$. Therefore the edge product cordial labeling is satisfied for $G^{'}$. □

We illustrate the above Theorem 3.1 in Fig. 3.

Theorem 3.2. *The duplicating prism graph Y_n with triangles admits an edge product cordial labeling.*

Proof. Consider a prism graph Y_n having vertices $v_1, \ldots, v_{2n}$ and edges $e_1, \ldots, e_{4n}$. Now by duplicating the outer vertices a graph G is obtained by the duplication of a vertex V_j by an edge $V_j^{'}$ and $V_j^{''}$. And G contains the cycle C_3 whose vertices are V_j,$V_j^{'}$,$V_j^{''}$. Let H be a graph obtained from G by joining all the C_3's by equivalent edges. The graph H has $v_1, \ldots, v_{5n}$ vertices and $e_1, \ldots, e_{9n}$ edges. We prove that H is an edge product cordial graph for which we consider the following two cases:
Case 1: When n is odd:

$$f\left(e_i\right)=1, \quad 1 \leq i \ \leq \lfloor 9n/2\rfloor+n,$$

$$f\left(e_i\right) \ =0, \quad \text{otherwise.}$$

In view of the above defined labeling pattern we have,

$$v_f\left(0\right)=v_f\left(1\right)+1=3n- \ \lfloor n/2\rfloor,$$

$$e_f\left(0\right)=e_f\left(1\right)+1= \ \lfloor 9n/2\rfloor.$$

Case 2: When n is even:

$$f\left(e_i\right)=0, \quad 1 \leq i \ \leq \lfloor 3n/2\rfloor+3n,$$

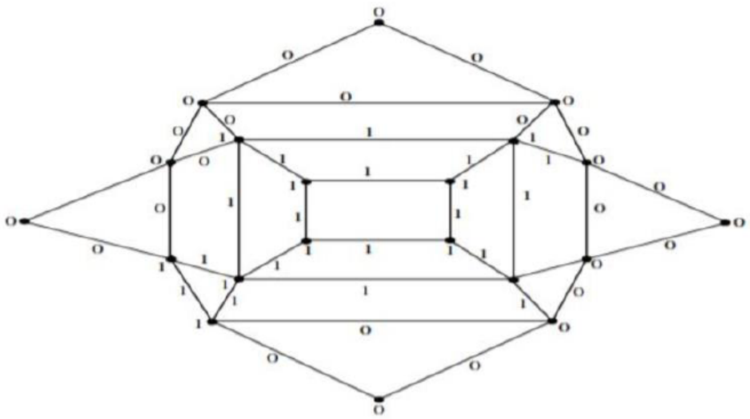

Fig. 4: The duplication of a prism graph with triangles.

$$f\left(e_i\right)=1, \qquad \text{otherwise.}$$

From the above defined labeling pattern we observe that

$$v_f\left(0\right)=v_f\left(1\right)=3n-n/2,$$

$$e_f\left(0\right)=e_f\left(1\right)=\ 3n/2+3n.$$

Thus in all the cases we have $|v_f\left(0\right)-v_f\left(1\right)|\leq 1$ and $|e_f\left(0\right)-e_f\left(1\right)|\leq 1$. Therefore, the edge product cordial labeling is satisfied for the duplicated prism graph with triangles. □

We illustrate the above Theorem 3.2 in Fig. 4 where we show the duplication of prism graph with triangles and its edge product cordial labeling.

4 Conclusion

In this paper we have proved the edge product cordial labeling of the duplication of a prism graph.

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References

[1] Cahit, I. (1987). Cordial graphs: a weaker version of graceful and harmonious graphs, *Ars Combin.*, 23, 201–207.

[2] Gallian, J. (2017). A dynamic survey of graph labeling, *The Electronic Journal of Combinatorics*, 1–415.

[3] Rosa, A. (1967). On certain valuations of the vertices of a graph, Theory of Graphs (International Symposium, Rome, July 1966), Gordon and Breach, N.Y. and Dunod Paris, 349 -355.

[4] Sampathkumar, E. (1973). On duplicate graphs, *Journal of the Indian Math. Soc.*, 37, 285–293.

[5] Sundaram, M., Ponraj, R. and Somasundaram, S. (2004). Product cordial labeling of graphs, *Bull. Pure Appl. Sci. Sect. E Math. Stat.*, 23, 155–163.

[6] Vaidya, S.K and Barasara, C.M. (2012). Edge product cordial labeling of graphs, *J. Math. Comput. Sci.*, 2(5), 1436–1450.

[7] Vaidya, S.K and Barasara, C.M. (2013). Some new families of edge product cordial graphs, *Advanced Modeling Optimization*, 15(1), 103–111.

[8] Vaidya, S.K. and Barasara, C.M. (2012). Further results on product cordial graphs, *International Journal of Mathematics and Soft Computing*, 2(2), 67–74.

[9] Vaidya, S.K. and Prajapathi, U.M. (2013). Prime labeling in the context of duplication of graph elements, *International Journal of Mathematics and Soft Computing*, 3(1), 13–20.