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# On generalized star $\omega\alpha I$-closed sets in ideal topological spaces *

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**Abstract** In this paper, we introduce a new class of sets named as generalized star $\omega\alpha I$-closed sets in ideal topological spaces and study some of their properties. Further we also define and study the concept of $\omega\alpha I$-open sets in ideal topological spaces and discuss their properties.

**Key words** $\omega$-closed sets, $\omega\alpha I$-closed sets, $\omega\alpha I$-open sets.

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## 1 Introduction

In [7, 13] the theory of ideal topological spaces was introduced by Kuratowski and Vaidyanathswamy. Ideals in topological space have been considered since 1930. In the year 1990 Jankovic and Hamlett [6] obtained new topologies using older ones and introduced $I$-open sets in ideal topological spaces which initialized the application of topological ideals in the generalization of most fundamental properties in general topology.
An ideal $I$ is a nonempty collection of subsets of $X$ closed with respect to a finite union. $(X, \tau, I)$ is an ideal topological space (ITS) and, in short, it is called an ideal space. For a subset $A$ of $X$, the local function of $A$ is defined as: $A^* = \{x \in X : U \cap A \text{ does not belong to } I \text{ for every } U \in \tau(x)\}$, where $\tau(x)$ is the collection of all nonempty open sets containing $x$. From this point onwards now we simply write $A^*$ instead of $A^*(I)$ to avoid any chance of confusion. A Kuratowski closure operator $\mathrm{cl}^*(.)$ for a topology $\tau^*(I, \tau)$ is termed as a $*$-topology, finer than $\tau$, and is determined by $\mathrm{cl}^*(A) = A \cup A^*$. If $A \subseteq X$, $\mathrm{cl}(A)$ and $\mathrm{int}(A)$ will denote the closure and interior of $A$ in $(X, \tau)$ respectively and $\mathrm{cl}^*(A)$ and $\mathrm{int}^*(A)$ will denote the closure and interior of $A$ respectively in $(X, {}^*\tau)$.
In 2013 Carpintero et al. [1] introduced the concept of the generalization of $\omega$-closed sets via operators and ideals. In 2014 Maragathavalli and Vinodhini [8] introduced the concept of $\alpha$-generalized closed sets in ideal topological spaces.
In 2016, Ravi et al. [10] presented the concept of $g^{\#}$- closed sets in ideal topological spaces. In 2008 Navaneethakrishnan and Joseph [9] introduced the concept $G$-closed sets in ideal topological spaces. In 2002 Hatir and Noiri [4] presented the concept of decomposition of continuity via idealization. In 1996

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Dontchev [2] discussed about pre-$I$-open sets with reference to a decomposition of $I$-continuity. The aim of this paper is to extend the notion of sets in ideal topological spaces by introducing the concept of the $\omega\alpha I$-closed sets in ideal topological spaces and to study some of their basic properties.

## 2 Preliminaries

Throughout this paper $(X, \tau)$ represent topological spaces. For a subset $A$ of a space $(X, \tau)$, $\mathrm{cl}(A)$, $\mathrm{int}(A)$ denote the closure of $A$ and the interior of $A$ respectively. We recall the following definitions.

**Definition 2.1.** [6] Let $(X, \tau)$ be a topological space. Let $I$ be an ideal defined on $X$. Then the space $(X, \tau, I)$ is termed as an ideal topological space, which satisfies the following two conditions:

1. If $A \in I$ and $B \subseteq A \Rightarrow B \in I$.
2. If $A \in I$ and $B \in I$, then $A \cup B \in I$.

**Definition 2.2.** [5,6] A subset $A$ of an ideal topological space $(X, \tau, I)$ is termed as

1. a pre-$I$-closed set if $\mathrm{cl}^*(\mathrm{int}(A)) \subseteq A$. If $A \subseteq (\mathrm{int}(\mathrm{cl}^*(A))$ then $A$ is called a pre-$I$-open set.
2. a semi-$I$-closed set if $\mathrm{int}(\mathrm{cl}^*(A)) \subseteq A$. If $A \subseteq (\mathrm{cl}^*(\mathrm{int}(A))$ then $A$ is called a semi-$I$-open set.
3. an $\alpha$-$I$-closed set if $\mathrm{cl}^*(\mathrm{int}(\mathrm{cl}^*(A))) \subseteq A$. If $A \subseteq (\mathrm{int}(\mathrm{cl}^*(\mathrm{int}(A)))$ then $A$ is called an $\alpha$-$I$-open set.
4. a $\beta$-$I$-closed set if $(\mathrm{int}(\mathrm{cl}^*(\mathrm{int}(A))) \subseteq A$. If $A \subseteq (\mathrm{cl}^*(\mathrm{int}(\mathrm{cl}^*(A)))$ then $A$ is called a $\beta$-$I$-open set.
5. a regular-$I$-closed set if $A = \mathrm{cl}^*(\mathrm{int}(A))$. If $A = (\mathrm{int}(\mathrm{cl}^*(A))$ then $A$ is called a regular-$I$-open set.

**Lemma 2.3.** *[6] Let $(X, \tau, I)$ be an ideal topological space. Let $A, B$ be subsets of $X$. Then the following properties hold:*

1. $A \subseteq B \Rightarrow A^* \subseteq B^*$,
2. $A^* = \mathrm{cl}(A^*) = \mathrm{cl}(A) = \mathrm{cl}^*(A)$,
3. $(A \cup B)^* = A^* \cup B^*$,
4. $(A \cap B)^* \subseteq A^* \cap B^*$,
5. $(A^*)^* \subseteq A^*$.

**Definition 2.4.** [3,5,6,8–12,14,15] A subset $A$ of a topological space $X$ is said to be:

1. a generalized closed ($-$closed) set if $\mathrm{cl}(A) \subseteq U$ whenever $A$ is a subset of $U$ and $U$ is open in the space $X$; a $g$-open set is the complement of a $g$-closed set.
2. generalized pre regular-closed (gpr-closed) set if $\mathrm{pcl}(A) \subseteq U$ whenever $A$ is a subset of $U$ and $U$ is regular open in the space $X$.
3. an $\omega$-closed set if $\mathrm{cl}(A) \subseteq U$ whenever $A$ is a subset of $U$ and $U$ is semi-open in the space $X$.
4. a $g\hat{}$-closed set if $\mathrm{cl}(A) \subseteq U$ whenever $A$ is a subset of $U$ and $U$ is semi-open in the space $X$.
5. a $g^{\#}$-closed set if $\mathrm{cl}(A) \subseteq U$ whenever $A$ is a subset of $U$ and $U$ is $g$-open in the space $X$.
6. an $\alpha$ generalized closed ($\alpha g$-closed) set if $\alpha\mathrm{cl}(A) \subseteq U$ whenever $A$ is a subset of $U$ and $U$ is open in the space $X$.
7. a generalized pre-closed (gp-closed) set if $\mathrm{pcl}(A) \subseteq U$ whenever $A$ is a subset of $U$ and $U$ is open in the space $X$.
8. a regular generalized closed (rg-closed) set if $\mathrm{cl}(A) \subseteq U$ whenever $A$ is a subset of $U$ and $U$ is regular open in the space $X$.
9. a generalized semi-pre-closed (gsp-closed) set if $\mathrm{spcl}(A) \subseteq U$ whenever $A$ is a subset of $U$ and $U$ is open in the space $X$.
10. a strongly $g$-closed ( $g^*$-closed) set if $\mathrm{cl}(A) \subseteq U$ whenever $A$ is a subset of $U$ and $U$ is $g$-open in the space $X$.
11. a generalized star pre-closed ( $g^*p$-closed) set if $\mathrm{pcl}(A) \subseteq U$ whenever $A$ is a subset of $U$ and $U$ is $g$-open in the space $X$.
12. a pre generalized star closed ( $pg^*$-closed) set if $\mathrm{pcl}(A) \subseteq U$ whenever $A$ is a subset of $U$ and $U$ is $\omega\alpha$-open in the space $X$.

## 3 On generalized star $\omega\alpha I$-closed sets in ideal topological spaces

In this section we introduce the concept of $\omega\alpha I$-closed set and study some of their properties.

**Definition 3.1.** A subset $B$ of an ideal topological space $(X, \tau, I)$ is called a generalized star $\omega\alpha I$-closed (briefly $g^*\omega\alpha I$-closed)set if $B^* \subseteq D$ whenever $B$ is a subset of $D$ (i.e., $B \subseteq D$) and $D$ is $\omega\alpha$-open in the space.

**Example 3.2.** Let $X = \{u, v, w\}$ and $I = \{\emptyset, \{v\}\}$. Take $\tau = \{\emptyset, \{u\}, \{v, w\}, X\}$ and $\tau^c = \{\emptyset, \{v, w\}, \{u\}, X\}$. Therefore, $g^*\omega\alpha I$ closed sets of $X$ are $\{\emptyset, \{u\}, \{w\}, \{v, w\}, \{u, w\}, X\}$. Here $B = \{v\}$ is a $g^*\omega\alpha$-closed set but it is not a $g^*\omega\alpha I$-closed set.

**Example 3.3.** Let $X = \{u, v, w\}$ and $I = \{\emptyset, \{v\}\}$. Take $\tau = \{\emptyset, \{u, v\}, X\}$ and $\tau^c = \{\emptyset, \{w\}, X\}$. Therefore, $g^*\omega\alpha I$ closed sets of $X$ are $\{\emptyset, \{w\}, \{v, w\}, \{u, w\}, X\}$. Here $B = \{u, w\}$ is a $g^*\omega\alpha I$-closed set but it is not a $g^*\omega\alpha$-closed set.

**Theorem 3.4.** *Every regular closed set in the ideal topological space $(X, \tau, I)$ is $g^*\omega\alpha I$-closed.*

**Proof.** Let $B$ be a regular closed set in the ideal topological space $(X, \tau, I)$. Let $D$ be any $\omega\alpha$-open set in $X$ such that $B^* \subseteq D$. Since every open set is $\omega\alpha$-open, so $B^* \subseteq \mathrm{cl}(B) \subseteq D$. Now, $B^* \subseteq \mathrm{cl}^*(B) \subseteq \mathrm{cl}(B) \subseteq D$. This shows that $B$ is a $g^*\omega\alpha I$-closed set. Hence every regular closed set in the ideal topological space is a $g^*\omega\alpha I$-closed set. In general the converse of this theorem does not hold. $\square$

**Example 3.5.** Let $X = \{u, v, w\}$ and $I = \{\emptyset, \{v\}\}$. Take $\tau = \{\emptyset, \{u\}, \{v, w\}, X\}$ and $\tau^c = \{\emptyset, \{v, w\}, \{u\}, X\}$. Therefore, $g^*\omega\alpha I$-closed sets of $X$ are $\{\emptyset, \{u\}, \{w\}, \{v, w\}, \{u, w\}, X\}$. Here $B = \{u, w\}$ is a $g^*\omega\alpha I$-closed set but it is not a regular closed set.

**Theorem 3.6.** *Every $\alpha g$-closed, $gp$-closed, $gsp$-closed, $gpr$-closed, $rg$-closed, $g^*p$-closed, and pre $g^*$-closed set in the space is $g^*\omega\alpha I$-closed set in the ideal topological space $(X, \tau, I)$.*

**Proof.** It follows from that, every open set is $\omega\alpha$-open set in the space $(X, \tau, I)$. In general the converse of this theorem does not hold. $\square$

**Example 3.7.** Let $X = \{u, v, w\}$ and $I = \{\emptyset, \{v\}\}$. Take $\tau = \{\emptyset, \{u\}, \{v\}, \{u, v\}, X\}$ and $\tau^c = \{\emptyset, \{v, w\}, \{u, w\}, \{w\}, X\}$. Therefore, $g^*\omega\alpha I$ closed sets of $X$ are $\{\emptyset, \{u\}, \{w\}, \{v, w\}, \{u, w\}, X\}$. Here $B = \{u\}$ is a $g^*\omega\alpha I$-closed set but it is not an $\alpha g$-closed, $gp$-closed, $gsp$-closed, $gpr$-closed, $rg$-closed, $g^*p$-closed, and pre $g^*$-closed set.

**Theorem 3.8.** *The class of $g^*\omega\alpha I$-closed set in the ideal topological space $(X, \tau, I)$ is independent of the class of $\alpha$-closed, semi-closed, $\omega\alpha$-closed, $g$-closed and $g^*$-closed sets in the space $(X, \tau, I)$.*

**Proof.** The proof is obvious from the definitions mentioned in the Preliminaries (section 2). $\square$

**Example 3.9.** Let $X = \{u, v, w\}$ and $I = \{\emptyset, \{w\}\}$. Take $\tau = \{\emptyset, \{w\}, \{u, w\}, X\}$ and $\tau^c = \{\emptyset, \{u, v\}, \{v\}, X\}$. Therefore, $g^*\omega\alpha I$ closed sets of $X$ are $\{\emptyset, \{v\}, \{u, v\}, \{v, w\}, X\}$. Here $B = \{u\}$ is an $\alpha$-closed, semi-closed, $\omega\alpha$-closed set but it is not a $g^*\omega\alpha I$-closed set.

**Example 3.10.** Let $X = \{u, v, w\}$ and $I = \{\emptyset, \{v\}\}$. Take $\tau = \{\emptyset, \{u\}, \{v, w\}, X\}$ and $\tau^c = \{\emptyset, \{v, w\}, \{u\}, X\}$. Therefore, $g^*\omega\alpha I$ closed sets of $X$ are $\{\emptyset, \{u\}, \{w\}, \{v, w\}, \{u, w\}, X\}$. Here $B = \{u, w\}$ is a $g^*\omega\alpha I$-closed set but it is not an $\alpha$-closed, semi-closed, $\omega\alpha$-closed set.

**Example 3.11.** Let $X = \{u, v, w\}$ and $I = \{\emptyset, \{v\}\}$. Take $\tau = \{\emptyset, \{u\}, \{v\}, \{u, v\}, X\}$ and $\tau^c = \{\emptyset, \{v, w\}, \{u, w\}, \{w\}, X\}$. Therefore, $g^*\omega\alpha I$ closed sets of $X$ are $\{\emptyset, \{u\}, \{w\}, \{v, w\}, \{u, w\}, X\}$. Here $B = \{u\}$ is a $g^*\omega\alpha I$-closed set but it is not a $g$-closed and a $g^*$-closed set.

**Theorem 3.12.** *The union of two $g^*\omega\alpha I$-closed sets is a $g^*\omega\alpha I$-closed set in any ideal topological space $(X, \tau, I)$.*

**Proof.** Let $M$ and $N$ be two $g^*\omega\alpha I$-closed sets in an ideal topological space $(X, \tau, I)$. Let $D$ be an $\omega\alpha$-open set in the space, such that $M \cup N \subseteq D$. Then $M \subseteq D$ and $N \subseteq D$. Since $M$ and $N$ are $g^*\omega\alpha I$-closed sets, $M^* \subseteq D$ and $N^* \subseteq D$ whenever $M^* \cup N^* \subseteq (M \cup N)^* \subseteq D$, $D$ is $\omega\alpha$-open. Hence, $M \cup N$ is $g^*\omega\alpha I$-closed set in the ideal topological space. □

**Theorem 3.13.** *If a subset $B$ of a space $X$ is $g^*\omega\alpha I$-closed in the ideal topological space $(X, \tau, I)$, then $B^* - B$ does not contain any non empty closed set in the space.*

**Proof.** Let $E$ be a closed set contained in $B^* - B$, such that $B^* - B \subseteq X - E$ and $X - E$ is open, so that we have an $\omega\alpha$-open set with $B \subseteq X - E$. But $B$ is $g^*\omega\alpha I$-closed in the ideal topological space. Therefore, $B^* \subseteq X - E$, consequently $E \subseteq X - B^*$. Then $E \subseteq B^*$. Thus $E \subseteq B^* \cap (X - B^*) = \emptyset$. That is, $E = \emptyset$. Hence the proof. □

**Theorem 3.14.** *If a subset $B$ of a space $X$ is $g^*\omega\alpha I$-closed in the ideal topological space $(X, \tau, I)$, then $B^* - B$ does not contain any non empty $\omega\alpha$-closed set in the space.*

**Proof.** The proof follows from Theorem 3.13 and the fact that every closed set is a $g^*\omega\alpha I$-closed set in the space $(X, \tau, I)$. □

**Example 3.15.** Let $X = \{p, q, r, s\}, I = \{\emptyset, \{q\}\}$ and $\tau = \{\emptyset, \{r, s\}, X\}$. Then let $(X, \tau)$ be a topological space and $I$ be an ideal. Consider the set $B = \{p, r\}$ then $B^* - B = X - \{p, r\} = \{q, s\}$. It does not contain any non empty $\omega\alpha$-closed set. But $B$ is not a $g^*\omega\alpha I$-closed set in the ideal topological space $(X, \tau, I)$.

**Theorem 3.16.** *If $B$ is $\omega\alpha$-open and $g^*\omega\alpha I$-closed set of the space $(X, \tau, I)$, then $B$ is closed set of the space $(X, \tau, I)$.*

**Proof.** Let $B$ be an $\omega\alpha$-open and a $g^*\omega\alpha I$-closed set in the ideal topological space $(X, \tau, I)$. Then $B^* \subseteq B$. Hence, $B$ is closed. □

**Theorem 3.17.** *If $B$ is a $g^*\omega\alpha I$-closed set in the ideal topological space $(X, \tau, I)$, then $B$ is closed iff $B^* - B$ is $\omega\alpha$-closed in the space.*

**Proof.** Let $B$ be a closed subset of the ideal topological space $(X, \tau, I)$. Then $B^* = B$ and so $B^* - B = \emptyset$, which is $\omega\alpha$-closed. Conversely, suppose $B^* - B$ is $\omega\alpha$-closed. Since, $B$ is $g^*\omega\alpha I$-closed in the ideal topological space $(X, \tau, I)$, by Theorem 3.14 $B^* - B$ does not contain any non empty $\omega\alpha$-closed set which implies that $B^* - B = \emptyset$. That is $B^* - B = \emptyset \Rightarrow B^* = B$. Hence, $B$ is closed. □

**Theorem 3.18.** *If $B$ is $g^*\omega\alpha I$-closed set in the ideal topological space $(X, \tau, I)$ and $B \subseteq C \subseteq B^*$ then $C$ is also $g^*\omega\alpha I$-closed set in the space.*

**Proof.** Let $D$ be an $\omega\alpha$-open set in the space $(X, \tau, I)$ such that $C \subseteq D$ then $B \subseteq D$. Since, $B \subseteq D$ and $D$ is $\omega\alpha$-open set then $B^* \subseteq D$. Then $B^* \subseteq \mathrm{cl}^*(B) = B^*$. Since $C \subseteq B^*$, thus $C^* \subseteq B^* \subseteq D$. Hence $C$ is $g^*\omega\alpha I$-closed set in the ideal topological space $(X, \tau, I)$. In general the converse of this theorem does not hold. □

**Example 3.19.** Let $X = \{u, v, w\}, I = \{\emptyset, \{w\}\}$ and $\tau = \{\emptyset, \{w\}, \{u, w\}, X\}$. Let $(X, \tau)$ be a topological space and $I$ be an ideal. The $g^*\omega\alpha I$-closed sets of $X$ are $\{\emptyset, \{v\}, \{u, v\}, \{v, w\}, X\}$. Consider the set $B = \{v\}$ and $C = \{v, w\}$ such that $B$ and $C$ are $g^*\omega\alpha I$-closed sets in the ideal topological space $(X, \tau, I)$ but $B \subseteq C$ is not contained in $B^*$.

**Theorem 3.20.** *Let $B \subseteq Y \subseteq X$ and if $B$ is $g^*\omega\alpha I$-closed set in the ideal topological space $(X, \tau, I)$, then $B$ is $g^*\omega\alpha I$-closed relative to $Y$.*

**Proof.** Let $B \subseteq Y \cap D$ where $D$ is an $\omega\alpha$-open set in the space $(X, \tau, I)$. Then $B \subseteq D$ and hence $B^* \subseteq D$. This implies that $Y \cap B^* \subseteq Y \cap D$. Thus $B$ is a $g^*\omega\alpha I$-closed set in the space $(X, \tau, I)$ relative to $Y$. □

**Theorem 3.21.** *If a subset $B$ of a topological space $X$ is both semi-open and an $\omega$-closed set then it is a $g^*\omega\alpha I$-closed set in the ideal topological space $(X, \tau, I)$.*

**Proof.** Let $B$ be a semi-open and $\omega$-closed set in the space $(X, \tau, I)$. Let $B \subseteq D$ and $D$ be an $\omega\alpha$-open set in the space $(X, \tau, I)$. Now $B \subseteq B$, by hypothesis, $B^* \subseteq B$ then $B^* \subseteq B \subseteq D$. Thus $B$ is a $g^*\omega\alpha I$-closed set in the ideal topological space $(X, \tau, I)$. If $B$ is both semi-open and $g^*\omega\alpha I$-closed set in the space $(X, \tau, I)$ then, $B$ need not be an $\omega$-closed set as can be seen from the Example 3.22 below. □

**Example 3.22.** Let $X = \{u, v, w\}$ and $I = \{\emptyset, \{v\}\}$. Take $\tau = \{\emptyset, \{u\}, \{v\}, \{u, v\}, X\}$ and $\tau^c = \{\emptyset, \{v, w\}, \{u, w\}, \{w\}, X\}$. Therefore, $g^*\omega\alpha I$ closed sets of $X$ are $\{\emptyset, \{u\}, \{w\}, \{v, w\}, \{u, w\}, X\}$. Here $B = \{u\}$ is a semi-open and a $g^*\omega\alpha I$- closed set but it is not an $\omega$-closed set.

## 4 Generalized star $\omega\alpha I$-open sets in ideal topological spaces

**Definition 4.1.** A subset $B$ of a topological space $(X, \tau, I)$ is said to be a generalized star $\omega\alpha I$-open (or, briefly, a $g^*\omega\alpha I$-open) set if its complement is a $g^*\omega\alpha I$-closed set in the space $(X, \tau, I)$.

**Theorem 4.2.** *A subset $B$ of a topological space $X$ is $g^*\omega\alpha I$-open iff $D \subseteq \mathrm{int}^*(B)$ whenever $D$ is $\omega\alpha$-closed and $D \subseteq B$.*

**Proof.** Assume that $B$ is a $g^*\omega\alpha I$-open set in the ideal topological space $(X, \tau, I)$ and $D$ is an $\omega\alpha$-closed set such that $D \subseteq B$. Then $X - B$ is a $g^*\omega\alpha I$-closed set in the ideal topological space $(X, \tau, I)$. Moreover, $X - B \subseteq X - D$ and $X - D$ is an $\omega\alpha$-open set in the space $X$. This implies that $\mathrm{cl}^*(X - B) \subseteq X - D$. But $\mathrm{cl}^*(X - B) = X - \mathrm{int}^*(B)$. Thus $X - \mathrm{int}^*(B) \subseteq X - D$. So $D \subseteq \mathrm{int}^*(B)$. Conversely, suppose $D \subseteq \mathrm{int}^*(B)$ whenever $D$ is an $\omega\alpha$-closed set and $D \subseteq B$. To prove that $B$ is $g^*\omega\alpha I$-open in the ideal topological space $(X, \tau, I)$. Let $J$ be an $\omega\alpha$-open set of the space $X$ such that $X - B \subseteq J$, then $X - J \subseteq B$. Now $X - J$ is a $g^*\omega\alpha I$-closed set containing $B$, so that $X - J \subseteq \mathrm{int}^*(B)$, $X - \mathrm{int}^*(B) \subseteq J$ but $\mathrm{cl}^*(X - B) = X - \mathrm{int}^*(B)$. Thus $\mathrm{cl}^*(X - B) \subseteq J$, that is $X - B$ is a $g^*\omega\alpha I$-closed set in the ideal topological space $(X, \tau, I)$. Hence $B$ is a $g^*\omega\alpha I$-open set in the ideal topological space $(X, \tau, I)$. □

**Theorem 4.3.** *If $\mathrm{int}^*(B) \subseteq C \subseteq B$ and $B$ is $g^*\omega\alpha I$-open set in the space, then $C$ is $g^*\omega\alpha I$-open set in the ideal topological space $(X, \tau, I)$.*

**Proof.** If $\mathrm{int}^*(B) \subseteq C \subseteq B$, then $X - B \subseteq X - C \subseteq X - \mathrm{int}^*(B) = \mathrm{cl}^*(X - B)$. Since, $X - B$ is a $g^*\omega\alpha I$-closed set, then by Theorem 3.18, $X - C$ is also a $g^*\omega\alpha I$-closed set in the ideal topological space $(X, \tau, I)$. Therefore, $C$ is a $g^*\omega\alpha I$-open set in the ideal topological space $(X, \tau, I)$. □

**Theorem 4.4.** *If $B$ is a $g^*\omega\alpha I$-closed set in the space, then $B^* - B$ is a $g^*\omega\alpha I$-open set in the ideal topological space $(X, \tau, I)$.*

**Proof.** Let $B$ be a $g^*\omega\alpha I$-closed set in the ideal topological space $(X, \tau, I)$. Let $E$ be an $\omega\alpha$-open set such that $E \subseteq B$. Since $B$ is $g^*\omega\alpha I$-closed, then by Theorem 3.13, $B^* - B$ does not contain any non empty closed set in the space. Thus $E = \emptyset$. Then $E \subseteq \mathrm{int}^*(B^* - B)$. Therefore by Theorem 3.4, $B^* - B$ is a $g^*\omega\alpha I$-open set in the ideal topological space $(X, \tau, I)$. □

**Theorem 4.5.** *A subset $B$ is $g^*\omega\alpha I$-open set in the ideal topological space $(X, \tau, I)$ iff $J = X$ every time $J$ is $\omega\alpha$-open and $\mathrm{int}^*(B) \cup (X - J) \subseteq J$.*

**Proof.** Let $B$ be a $g^*\omega\alpha I$-open set in the ideal topological space $(X, \tau, I)$. Let $D$ be an $\omega\alpha$-open set and $\mathrm{int}^*(B) \cup (X - D) \subseteq D$. This gives $X - D \subseteq (X - \mathrm{int}^*(B)) \cap (X - (X - B)) = X - \mathrm{int}^*(B) - (X - B) = \mathrm{cl}^*(X - B) - (X - B)$. Since $X - B$ is $g^*\omega\alpha I$-closed and $X - D$ is $\omega\alpha$-closed in the space $(X, \tau, I)$, then by Theorem 3.13 it follows that $X - D = \emptyset$. Therefore, $X = D$. Conversely, suppose $E$ is $\omega\alpha$-closed and $E \subseteq B$. Then $\mathrm{int}^*(B) \cup (X - B) \subseteq \mathrm{int}^*(B) \cup (X - E)$. It follows that $\mathrm{int}^*(B) \cup (X - E) = X$ and hence, $E \subseteq \mathrm{int}^*(B)$. Therefore, $B$ is $g^*\omega\alpha I$-open in the ideal topological space $(X, \tau, I)$. □

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