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Implementation of Finite Volume Method (FVM) in 1-D Diffusion Equation

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ABSTRACT

In this work finite volume method has been used in 1 D steady state diffusion equation in a plate. Then the step by step procedures of the central differencing scheme for numerical solution are described. we increase the number of nodes and solve the system of our discretized algebraic linear equations. Finally we compare the numerical solution obtained by finite volume techniques and analytical solution. The finite volume technique adopted in this paper shows minimum error.

KEYWORDS: Finite Volume Method, Node, Discretized Method, Temprature

1.INTRODUCTION

Computational fluid dynamics (CFD) derives numerical solutions for the analysis of mass, momentum, and heat transport phenomena, also associated phenomena like reaction chemistry and thermodynamics turbo machinery, chemical manufacturing, poer generation, weather simulation, biological engineering, meteorology, aerospace, reaction chemistry, predicting fluid flow and heat transfer [7],[8]. Recently, due to the revolution of computer technology, abundant computational grid techniques have been developed which is very efficient to solve numerous engineering problems [3],[4]. Among these numerical grid techniques, finite difference method (FDM) is used as a common numerical technique to solve numerous engineering problems [9]. Again, finite element method (FEM) is another kind of commonly used numerical technique which has been applied to solve many heat transfer problems [12]. A generalized transfer equation for a dependent variable which can be mass, concentration, heat and momentum is given by [11], where discretization technique is applied for CFD analysis [8].

The Finite Volume Method (FVM) is most well known in CFD. it provides as the discretization method. Even though the methodology developed many years ago by the finite difference and finite element methods. The FVM played an important role in the fluid flow simulation problems. The FVM is more popular as it provides solution which does not need to transfer the physical and computational coordinate system. We accept this finite volume method because it is suitable to solving flows in irregular geometrical domains. These effective results of the FVM allowed to keep its popularity as it is applied in different context for its simple mathematical formation. The basic idea of this finite volume method about to collect the different equation for finite finite-sized control volume covering evwery nodal point on the specific mesh. Then the volume integral is converting into surface intgrals that could be estimate as fluxes at the surfaces of this each finite volume.

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2. DIFFUSION EQUATION

The finite volume method of computational fluid dynamics about the technique of discretization for partial differential equations that appears from physical conservation laws. These equations can be variety types in nature like parabolic , hyperbolic or elliptic. The general transport equation can be defined as

$$\frac{\partial \rho T}{\partial t} + div(\rho T v) = div(kgradT) + S_T \qquad (1.01)$$

Where ρ is the density, T is denoted as the conserved quantity and diffusion coefficient is denoted by k. $div(\rho T v)$ is known as the net rate of flow of T out of fluid element (conservation) $div(kgrad(T))$ is rate of increase of Tdue to diffusion. S_T is rate of increase of φ due to source .$\frac{\partial \rho T}{\partial t}$ is rate of increase of T of fluid element transient.

In this article describes the identification of the transport equations governing the flow of fluid and the transfer of heat and the formal control volume integration. The finite volume (or control volume) method by taking into account the easiest transport process of all pure diffusion in the time independent state. We get the steady state diffusion equation through deleting transient and convective terms. General transport equation reduces to $div(kgradT) + S_T = 0$

$$\text{Or}\frac{d}{dx}\left(k\frac{dT}{dx}\right) + S_T = 0 \qquad (1.02)$$

The finite volume method's main step is the control volume integration which is different from other CFD techniques. Yields the following form

$$\begin{aligned} &\int_{CV} div(kgradT)dV + \int_{CV} S_T\, dV = 0 \\ &= \int_A n.(kgradT)dA + \int_{CV} S_T\, dV = 0 \end{aligned} \qquad (1.03)$$

The approximation method which are needed as to obtained discretised equations in steady state one dimensional diffusion equation.

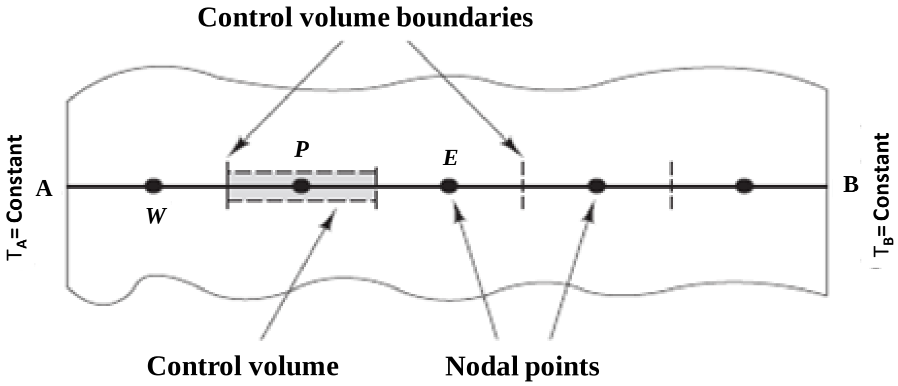

Figure 1:

2.1 Generation of Grid

At the very first the domain is divided into small domains where all small domains are equal in terms of distance. These equal small domains are called as elements as well. Then the nodal point is identified between the middle posotion of the every elements. The volume is created according to thje nodal points. We named P as nodal point for every elements of general control volume. If the p point is identified , the left side of the point west side (W)and the right side of the point will be the east side (E). The volume of east side identified as e and the west side volume is identified as w. The accurate disntance of WP, PE, is $\delta x_{WP}\ and\ \delta x_{PE}$ respectively. In the same way distances between face w and point P and between P and face e are indicated by δx_{wp} and δx_{pe} respectively .The width of the control volume is $\Delta x = \delta x_{we}$ displays fig (1.2).

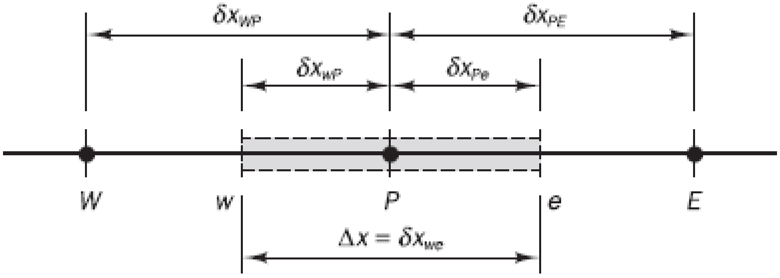

Figure 2:

2.2 Discretisation

Integrate the general governing equation is the crux of the finite volume method. The every nodal points of each elements used for over a control volume to yield a discretsed equation. The control volume integral at the nodal point p is explained as

$$\int_{\Delta V} \frac{d}{dx}(k\frac{dT}{dx})dV + \int_{\Delta V} SdV = (kA\frac{dT}{dx})_e - \left(kA\frac{dT}{dx}\right)_w + \bar{S}\Delta V = 0 \quad (1.04)$$

Where A is the cross-sectional area of the cross-section of control volume face, ΔV is the volume and $\bar{S}$ is the average value of source S over the control volume. Equation (2.04) states that the diffusive flux Fick law of diffusion of T leaving the east face minus the diffusive flux of T entering the west face is equal to the generation of T. That means it constitutes a balance equation for T over the control volume. The gradient $\frac{dT}{dx}$ at west (w) & east (e) and diffusion coefficient k are required in order to derive useful the discretized equation. The central differencing scheme is used to the diffusive coefficient of T In a uniform grid linearly interpolated values for k_w and k_e are given by

$$k_w = \frac{k_W + k_P}{2} \quad (1.05a)$$

$$k_e = \frac{k_P + k_E}{2} \quad (1.05b)$$

and the diffusive flux terms are evaluated as

$$(kA\frac{dT}{dx})_e = k_e A_e (\frac{T_E - T_P}{\delta x_{PE}}) \quad (1.06)$$

$$(kA\frac{dT}{dx})_w = k_w A_w (\frac{T_P - T_W}{\delta x_{WP}}) \quad (1.07)$$

The source term *S* diffusion equation is the function of the dependent variable. In the finite volume method source term is written as linear form.

$$\bar{S}\Delta V = S_u + S_p T_p \quad (1.08)$$

Substitution of mention equations (1.06) (1.07) and into equation (1.04)

$$k_e A(\frac{T_E - T_P}{\delta x_{PE}}) - k_w A\left(\frac{T_P - T_A}{\delta x_{WP}}\right) + (S_u + S_p T_p) = 0 \quad (1.09)$$

This can be rearranged as

$$(\frac{k_e}{\delta x_{PE}} A_e + \frac{k_w}{\delta x_{WP}} A_w - S_p) T_P = (\frac{k_w}{\delta x_{WP}} A_w) T_W + (\frac{k_e}{\delta x_{PE}} A_e) T_E + S_u \quad (1.10)$$

In the equation (1.10) the coefficients of T_W and T_E is identified as a_W and a_E and the coefficient of T_P as a_P the above equation can be written as

$$a_P T_P = a_W T_W + a_E T_E + S_u \quad (1.11)$$

Where

a_W	a_E	a_P
$\frac{k_w}{\delta x_{WP}} A_w$	$\frac{k_e}{\delta x_{PE}} A_e$	$a_W + a_E - S_P$

The values of $S_u$andS_P can be obtained from the source model (1.08):$\bar{S}\Delta V = S_u + S_p.T_P$ Equations (1.11) and (1.08) represent the discretised form of equation (1.02). This type of discretised equation is central to all further developments

2.3 Solution of Equation

The system of linear equation fixed at the nodal points to solve the problem. The results temperture at the nodal point p obtained while soved algebraic equations using any suitable matrix solution technique in MATLAB.

3. NUMERICAL EXAMPLE

A large plate of thickness L=2 cm with constant thermal conductivity k = 0.5 w/m.k and uniform heat general q = 1000kw/m3 includes boundary condition with sourcesfigure below. The faces A and B are at temperatures of 100 C and 200C respectively . Assuming that the dimension in the Y-and Z- directions are so large that temperature gradients are significant in the x-direction only calculate the steady state temperature distribution. To compare the numerical result with the analytical solution. The governing equation is

$$\frac{d}{dx}\left(k\frac{dT}{dx}\right) + q = 0$$

(1.12)

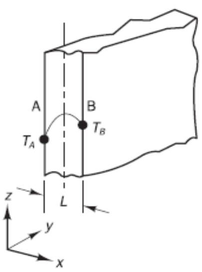

Figure 3:

Solution

The method of solution is calculated using a simple grid. The domain is split into five control volume with δx = 0.004m. Aunit area is consider the area of yz- plane is unity.

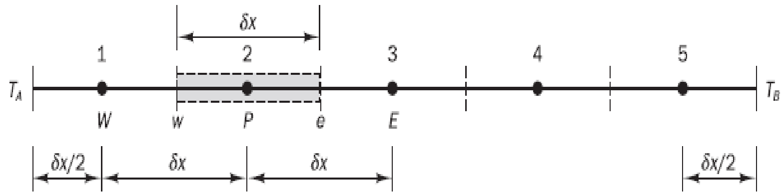

Figure 4 : The grid used

There are five nodes in the grids. We describe nodes 2, 3 and 4 for each one of temperature values to the east and west are available as nodal values. The System of linear equations of the form (1.11) can be readily written usual integration of the governing equation for control volumes surrounding these nodes.

$$\int_{\Delta V}^{\cdot} \frac{d}{dx}(k\frac{dT}{dx})dV + \int_{\Delta V}^{\cdot} qdV = 0 \qquad (1.13)$$

The source term of the above equation is second integral. That is valued by solving the average generation (i.e. $\bar{S}\Delta V = q\Delta V$) within each control volume. Equation (1.13) could be written as

$$\left[\left(kA\frac{dT}{dx}\right)_e - \left(kA\frac{dT}{dx}\right)_w\right] + q\Delta V = 0 \qquad (1.14)$$

$$k_e A(\frac{T_E - T_P}{\delta x}) - k_w A\left(\frac{T_P - T_W}{\frac{\delta x}{2}}\right) + qA\delta x = 0 \qquad (1.15)$$

Rearranged the equation (1.15) using to $k_e = k_w = k$ yield the system of equation

$$\frac{kA}{\delta x}T_E - \frac{kA}{\delta x}T_P - \frac{2kA}{\delta x}T_P + \frac{2kA}{\delta x}T_W + qA\delta x = 0$$

$\left(\frac{kA}{\delta x}+\frac{kA}{\delta x}\right)T_P = \frac{k}{\delta x}T_E+\frac{k}{\delta x}T_W+qA\delta x$ (1.16)

$a_P T_P = a_W T_W + a_E T_E + S_u$ (1.17)

Where

a_W	a_E	S_u	S_P	a_P
$\frac{kA}{\delta x}$	$\frac{kA}{\delta x}$	qAδx	0	a_W+$a_E - S_p$

E

quation (1.17) is conclusive for control volume at the nodal points 2,3,4.

As the objective to sorround the boundary conditions at nodes 1 and 5. we applied the linear approximation to get temperatures between a boundary point and the adjacent nodal point. At node 1 the temperature at west boundary is given. Integration of eq(1.12) at the control volume surrounding node 1 gives

$\left[\left(kA\frac{dT}{dx}\right)_e - \left(kA\frac{dT}{dx}\right)_w\right] + q\Delta V = 0$ (1.18)

Introducing of the linear approximation for temperatures between A and P yield

$[k_e A(\frac{T_E-T_P}{\delta x}) \; - k_A A\left(\frac{T_P-T_A}{\frac{\delta x}{2}}\right)] + qA\delta x = 0$ (1.19)

The equation(1.19) can be rearranged using $k_e = k_A = k$ to give the linear equation for boundary node 1

$\frac{kA}{\delta x}T_E - \frac{kA}{\delta x}T_P - \frac{2kA}{\delta x}T_P + \frac{2kA}{\delta x}T_A + qA\delta x = 0$

$\left(\frac{kA}{\delta x}+\frac{2kA}{\delta x}\right)T_P = \frac{kA}{\delta x}T_E + \frac{2kA}{\delta x}T_A + qA\delta x$ (1.20)

$a_P T_P = 0.T_W + a_E T_E + S_u$ (1.21)

Where

a_w	a_E	S_u	S_P	a_p
0	$\frac{kA}{\delta x}$	$qA\delta x + \frac{2kA}{\delta x}T_A$	$-\frac{2kA}{\delta x}$	$a_w + a_E - S_P$

At node 5 we have similar way to node 1.The temperature is given on the east face of the control volume.

$\left[\left(kA\frac{dT}{dx}\right)_e - \left(kA\frac{dT}{dx}\right)_w\right] + q\Delta V = 0$ (1.22)

$[k_e A(\frac{T_B-T_P}{\delta x/2}) \; - k_w A\left(\frac{T_P-T_W}{\delta x}\right)] + qA\delta x = 0$ (1.23)

The equation (1.23) can be rearranged using $k_e = k_w = k$ to give the linear equation for boundary node 5:

$\frac{2kA}{\delta x}T_B - \frac{2kA}{\delta x}T_P - \frac{kA}{\delta x}T_P + \frac{kA}{\delta x}T_W + qA\delta x = 0$

$\left(\frac{2kA}{\delta x}+\frac{kA}{\delta x}\right)T_P = \frac{kA}{\delta x}T_W + \frac{2kA}{\delta x}T_B + qA\delta x$ (1.24)

$a_P T_P = a_W T_W + 0.T_E + S_u$ (1.25)

Where

a_w	a_E	S_u	S_P	a_p
$\frac{kA}{\delta x}$	**0**	$qA\delta x + \frac{2kA}{\delta x}T_B$	$-\frac{2kA}{\delta x}$	$a_w + a_E - S_P$

We put the A=1 , k = 0.5 W/m.K, q = 1000kW/m^3 and $\delta x = 0.004m$ in equation (1.17) (1.21) (1.25) and gives the coefficients of the system of linear equations calculated in below

$375T_1 = 125T_2 + 25004$

$250T_2 = 125T_1 + 125T_3 + 4$

$250T_3 = 125T_2 + 125T_4 + 4$

$250T_4 = 125T_3 + 125T_5 + 4$

$375T_5 = 125T_4 + 125T_5 + 50004$

This set of equation can be rearranged as

$$\begin{bmatrix} 375 & -125 & 0 & 0 & 0 \\ -125 & 250 & -125 & 0 & 0 \\ 0 & -125 & 250 & -125 & 0 \\ 0 & 0 & -125 & 250 & -125 \\ 0 & 0 & 0 & -125 & 375 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \end{bmatrix} = \begin{bmatrix} 25004 \\ 4 \\ 4 \\ 4 \\ 50004 \end{bmatrix}$$

The result to above arranged system of linear equation is

$$\begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \end{bmatrix} = \begin{bmatrix} 110.040 \\ 130.088 \\ 150.104 \\ 170.088 \\ 190.040 \end{bmatrix}$$

No. Nodes = 10

The domain is divided into ten small element control volume giving = 0.002 m. consider area of yz- plane is unity. Put in numerical values for A=1 , k = 0.5 W/m.K, q = 1000kW/m^3 and $\delta x = 0.004m$ everywhere gives the coefficients of the system of linear equations summarised in below

Node	a_W	a_E	S_u	S_P	$a_P = a_W + a_E - S_P$
1	0	250	50002	-500	750
2,3,4,5,6,7,8,9	250	250	2	0	500
10	250	0	100002	-500	750

The resulting sets of algebraic equations are

$750T_1 = 250T_2$+50002 ,	$500T_2 = 250T_1$+$250T_3 + 2$,	$500T_3 = 250T_2$+$250T_4 + 2$
$500T_4 = 250T_3$+$250T_5 + 2$,	$500T_5 = 250T_4$+$250T_6 + 2$	$500T_6 = 250T_5$+$250T_7 + 2$
$500T_7 = 250T_6$+$250T_8 + 2$	$500T_8 = 250T_7$+$250T_9 + 2$	$500T_9 = 250T_8$+$250T_{10} + 2$
$500T_{10} = 250T_9$+100002		

The resulting sets of algebraic equations are

T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	T_{10}
110.020	115.052	125.076	135.092	145.100	155.100	165.092	175.076	185.052	195.020

The analytical solution for mentioned problem may be acquired by integrating equation (1.02) two times w.r.t. to x and by specific application of the boundary conditions. The results comes as

$$T = \left[\frac{T_B - T_A}{L} + \frac{q}{2k}(L - x)\right]x + T_A$$

Table 1:

No. Nodes = 5

Nodes	**1**	**2**	**3**	**4**	**5**
X(m)	0.002	0.006	0.010	0.014	0.018
FVM solution	110.040	130.088	150.104	170.088	190.040
Analytical solution	110.036	130.084	150.100	170.084	190.036
Absolute Error	0.004	0.004	0.004	0.004	0.004

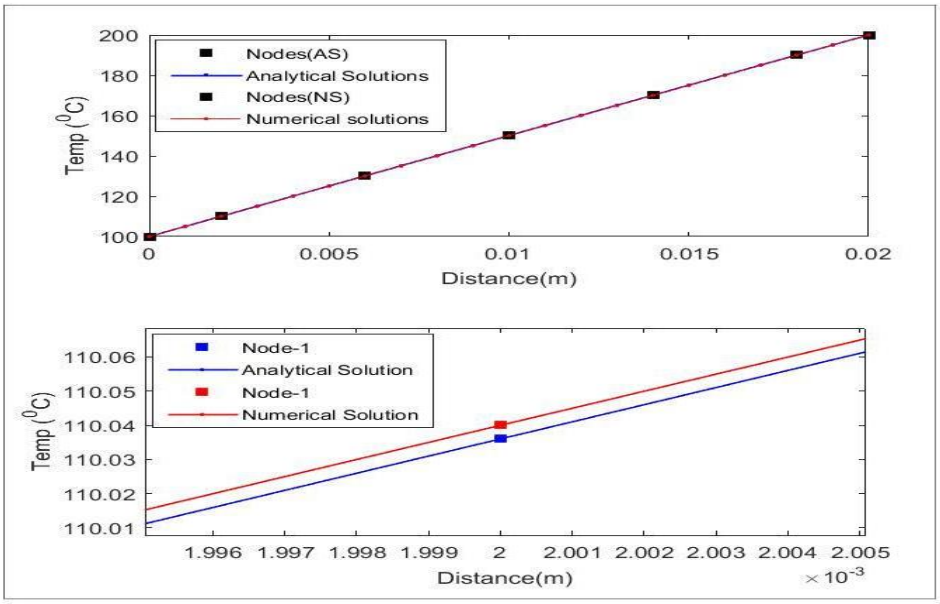

Figure 5 : Comparison of analytical results with the numerical solution

Above diagram seen that FVM and analytical solution coincides. But magnifying we get the below diagram which have absolute error is 0.004.

Table 2:
No. Nodes = 10

Nodes	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
X(m)	0.001	0.003	0.005	0.007	0.009	0.011	0.013	0.015	0.017	0.019
FVM solution	105.020	115.052	125.076	135.092	145.100	155.100	165.092	175.076	185.052	195.020
Analytical solution	105.019	115.051	125.075	135.091	145.099	155.099	165.091	175.075	185.051	195.019
Absolute Error	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001

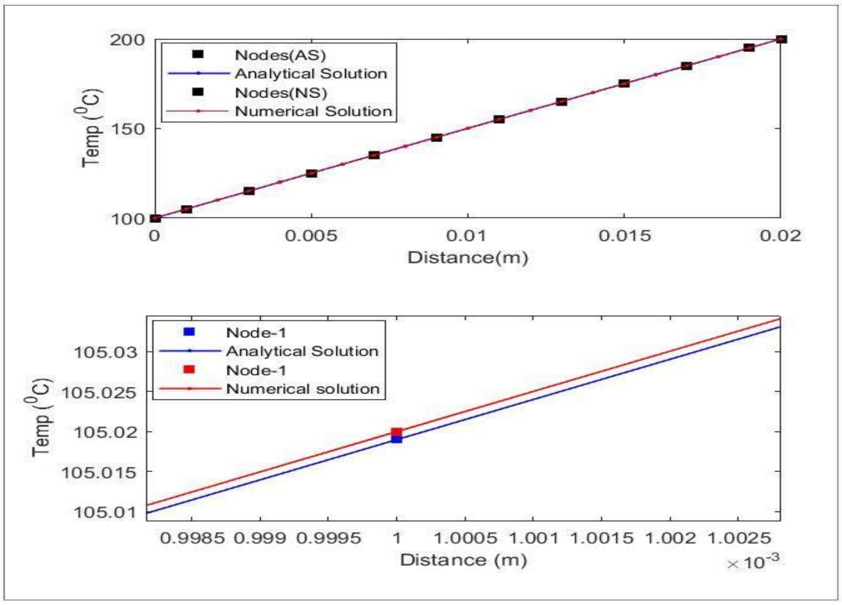

Figure 6: Comparison of analytical results with the numerical solution

Above diagram seen that FVM and analytical solution coincides. But magnifying we get the below diagram which have absolute error is 0.001.

Table 3:

Nodes	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
X(m)	0.001	0.003	0.005	0.007	0.009	0.011	0.013	0.015	0.017	0.019
FVM solution	105.020	115.052	125.076	135.092	145.100	155.100	165.092	175.076	185.052	195.020
ΔT		10.032	10.024	10.016	10.008	10.000	9.992	9.984	9.976	9.968
Analytical solution	105.019	115.051	125.075	135.091	145.099	155.099	165.091	175.075	185.051	195.019
ΔT		10.032	10.024	10.016	10.008	10.000	9.992	9.984	9.976	9.968

4. CONCLUSION

- From the table 1 and table 2 we see that when the number of nodes increases, the absolute error decreases.
- From table 3 we see that the rate of change of temperature increases in the direction of flow from B to A.

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