

Applications of mathematics: a perspective *

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Abstract Mathematics has played a significant role in the development of all fields of life. It is a science which started with the beginning of life and has provided tools for the development of all sciences. We use the concepts of mathematics from day to day life to higher levels of research in almost every field such as transportation, architecture, medical science, or space science, etc. Thus mathematics has truly acted as the mother of all sciences and nourished their offspring with its concepts.

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1 Introduction

From very beginning of the civilization, we have ushered in the era of science and technology. A close look of this journey from the dawn of civilization to this stage reveals that mathematics has played a significant role at every stage. Whether it is a field of transportation, architecture, medical science or space science, it has provided tools for all fields. It has played an instrumental role in saving human life by giving X-rays, MRI and CT-scans. It has helped in saving millions of lives by giving contributions to the field of study of climate change. It is seen in the cases of 'Amphan' and 'Nisarg' cyclones which recently hit the coastal areas of West Bengal and Mumbai. Early prediction saved millions of lives and prevented a greater loss.

Mathematicians have contributed to explain the applications of mathematics in nature. They proved that nature is also following rules to prevent chaos. For example, the concept of countably finite and countably infinite sets is very much evident in nature. Similarly, to overcome the word exact, the concept of limit was introduced. So, we can say that mathematics is closely related with nature.

Counting was one of the most ancient problem that our civilization was encountered with. The earliest civilized men started counting on fingers which resulted in the origin of natural numbers. With the inclusion of zero and negative numbers, the set of integers was formed. But these numbers proved insufficient to support all the purposes, so the ratio of these numbers were taken in account. Thus the rational numbers were invented. But still the set of rational numbers was not found adequate. To explain continuous problems, real numbers came into existence with the introduction of irrational numbers. With the concept of real numbers the concept of limit was naturally elaborated and explained. In the 17th century, the calculus helped in giving insights into many natural phenomena around us like

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Fig. 1: A 12-hour clock.

house planning, construction designing, locomotion monitoring, velocity, acceleration etc. This is how mathematics proved to be a catalytic tool in explaining our day-to-day life phenomenon from the very beginning till today. This increasing application of mathematics in our mundane activities and its analysis is the main objective of this article.

2 Applications of mathematics

Though there are countless examples of the applications of mathematics only some of them are mentioned below:

2.1 Applications of modular mathematics

Modular mathematics [4] is widely used in number theory and other branches of mathematics such as Abstract Algebra and Linear Algebra. It is a mathematics of wrap. A common use of this is observed in the 12-hour clock (Fig. 1). As the number exceeds 12, it is divided by 12 and the remainder gives the time. Similar approach is applied to know the day of a week or a year.

A wide use of modular mathematics is found in error detection. For error AIR Tickets ID numbers, ISSN, ISBN, Bank ID numbers follow modular arithmetic. AIR Tickets ID numbers consists of 15 digits. The last digit is chosen in such a way that it is the modulo 7 of the first 14 digits number. But it cannot detect all single digit error such as the Transposition of $0 \leftrightarrow 7, 1 \leftrightarrow 8, 2 \leftrightarrow 9$ and vice-versa.

2.1.1 United Parcel Code (UPC)-

The United parcel Code is used to give identification to retail items. A 12 digit number digit is used for this purpose. The first six digits of this number give the information about the manufacturer and next five digits give the information about the product. The last digit is a check digit. In the following picture 2 is check digit. An item with UPC check digit satisfies the condition

$$(a_1, \dots, a_{12}) * (3, 1, 3, 1, \dots, 3, 1) = 0 \pmod{10}.$$

For Fig. 2 we see that

$$(0 \cdot 3 + 2 \cdot 1 + 0 \cdot 3 + 3 \cdot 1 + 3 \cdot 5 + 7 \cdot 1 + 1 \cdot 3 + 2 \cdot 1 + 2 \cdot 3 + 6 \cdot 1 + 8 \cdot 3 + 2 \cdot 1) \pmod{10} = 0.$$

The UPC scheme can detect all type of errors including the single digit error and the error due to the transposition of two digits. But an error can remain undetected when $a - b = 5$. For sophisticated calculation weights are given to various positions of the sequence of numbers a_1, a_2, \dots, a_{n-1} and one extra digit a_n is added such that $(a_1, \dots, a_n) * (w_1, \dots, w_n) = 0 \pmod{k}$.

The ISBN 10 (International Standard Book Number of ten digits) also follows this technique in which $(a_1, a_2, a_3, \dots, a_{10}) * (10, 9, 8, \dots, 1) = 0 \pmod{11}$, where a_1, a_2, \dots represent different information.



Fig. 2: A United Parcel Code.



Fig. 3: The old Chrysler logo.

3 Symmetric groups and their applications

Though groups are widely used in advanced mathematics, the applications of dihedral groups are widely observed in art and nature. The concept of symmetry and rotation are reflected in decorative designs for floor covering, pottery and buildings. Corporation logos are also rich sources of symmetry, e.g., the old logo of Chrysler has D_5 symmetry (Fig. 3) and the Mercedes-Benz logo has D_3 symmetry (Fig. 4) [2, 3].

The concept of symmetry is also used by chemists to classify molecules by symmetry, e.g. the ammonia molecule (NH_3) has a pyramidal structure with D_3 symmetry (Fig. 5). Mineralogists study the structure of crystals with the help of these groupoids. These groups are also quite helpful in the classification of organisms in different phyla. For example, the phylum Echinodermata contains many sea animals such as starfish, sea cucumber, feather stars and sea collars. These animals have five-pointed star-like structure showing D_5 symmetry (Fig. 6).

4 Applications of linear algebra

4.1 Application in construction of skyscrapers and architecture

As soon as construction comes to mind, the concepts of stress and strain suddenly strike us. One should know about the flexibility of an iron beam, which obeys Hooke's law that states that the produced strains are proportional to the applied stresses. If $\mathbf{x} = (x_1, x_2, x_3)'$ are the strains produced at different places and $\mathbf{W} = (w_1, w_2, w_3)'$ are the corresponding stresses applied at the respective places and if $\mathbf{F} = (f_{ij})_{3 \times 3}$ be the matrix of proportionality constants, then Hooke's law can be mathematically



Fig. 4: The logo of Mercedes-Benz.

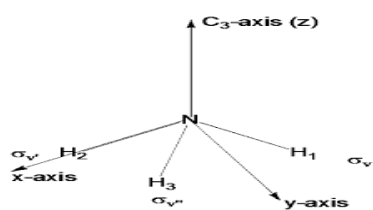


Fig. 5: The structure of ammonia molecule.



Fig. 6: The structure of star fish.

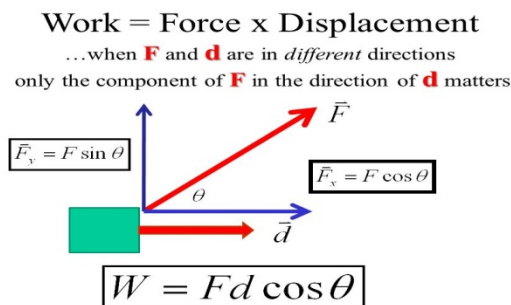


Fig. 7: The work done by a force acting on a body.

expressed as

$$\mathbf{x} = \mathbf{F}\mathbf{W},$$

or, more elaborately as

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix},$$

from which the strains can be calculated provided f_{ij} 's are known. So we can think of increasing the applications of linear algebra in present day-to-day tower and bridge construction projects, especially, in urban area during the proposed mega smart cities projects being currently implemented in India.

4.2 Application of linear algebra in network problem and traffic flow problem

Flow axiom-the flow into a node equals the flow away from the node. Let us consider a grid of one-way street in a given city. The intersection of roads (nodes) are denoted by numbers by 1, 2, 3, ... and in coming flow of traffic per hour on the roads is denoted by numbers such as 200, 300, etc. satisfying this axiom. Simultaneous equations are formed like the following

$$x_1 + x_3 = 200, x_1 + x_2 = 200, x_3 + x_4 = x_7, x_2 + x_4 = 500 + x_3, x_5 + 400 = x_6 + 200, x_6 + 300 = x_7,$$

which can be solved by row reducing the augmented matrix of the system to the echelon form to obtain the value of the concerned variables. Thus with the help of linear algebra, traffic planners and engineers can know which road can have heavy traffic.

The linear algebra techniques also apply in solving the electrical network problems by the application of Kirchhoff's laws.

4.3 World wide web searching

Beginning in 1998 the web search engine Google started to use the hyperlink structure to improve their search analysis. This is known as link analysis and is based on fundamental concepts of linear algebra. This approach resulted in remarkably accurate web searches and it revolutionized the design of search engines. HITS algorithms with many variations were developed. Google is based on the Page Rank Algorithm, which is a link-analysis algorithm similar to HITS.

4.4 Mathematical expressions for formulas

Many formulas can be written in mathematical terms. An example is the relation between work and force. Let \vec{F} be the force acting on a body which causes a displacement \vec{d} of the body, if θ be the angle between \vec{F} and \vec{d} then work W done by the force is given by (see Fig. 7)

$$W = \vec{F} \cdot \vec{d} = |\vec{F}| |\vec{d}| \cos \theta$$

Similarly the law of cosine can be applied to determine the final position of a ball after undergoing a glancing collision with another ball.

5 Application of mathematics in the field of in economics

As discussed above mathematics has contributed in every field, the field of economics is not an exception to this rule. Several branches of mathematics such as linear algebra and differential equations have proved very helpful in this field. Many mathematical relations in the field of economics such as the relation between the price and the demand, the rise in temperature and the sale of commodities can be approximated as linear equations. These equations can be solved with the help of linear algebra. How the production of one industry becomes the input of other industries can be displayed with the help of an input-output matrix in which the columns give the input to each sector and each row shows the outputs of the different sectors. When an economy have a number of economic sectors then each sector produces goods some of which serve as inputs for the the other sectors of the economy and finally some sectors produce consumer goods to be sold to the customers. The input-output relations between the various sectors of an economy can be represented by linear equations which can be solved by suitable tools of linear algebra. Similarly the tools developed in differential equations [1] and operations research are also widely used in economics.

6 Conclusion

In this informative article we made an elementary attempt to explain the applications of mathematics in various day-to-day life activities so that more and more people may come forward to learn mathematics owing to its varied applications.

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