Bulletin of Pure and Applied Sciences
Section - E - Mathematics & Statistics
Website : https : //www.bpasjournals.com/

Bull. Pure Appl. Sci. Sect. E Math. Stat.
38E(Special Issue)(2S), 92–98 (2019)
e-ISSN:2320-3226, Print ISSN:0970-6577
DOI: 10.5958/2320-3226.2019.00085.7

Cordial labelings for comb related graph *

S. Bala[1], S. Sundarraj[2] and K. Thirusangu[3]

1,2,3. S.I.V.E.T. College, Gowrivakkam, Chennai-600073, Tamil Nadu, India.
1. E-mail: yesbala75@gmail.com , 2. E-mail: sjsundar89@gmail.com
3. E-mail: kthirusangu@gmail.com

Abstract In this paper we investigate the existence of homo-cordial, hetro-cordial and cup(V)-cordial labeling for the extended triplicate graph of a comb.

Key words Homo-cordial labeling, Hetro-cordial labeling, Cup(V)-cordial labeling, Extended Triplicate Graph of a Comb.

2010 Mathematics Subject Classification 05C78, 05C99.

1 Introduction

The beginnings of graph theory can be traced back to the first paper written by Leonhard Euler in 1736 about the solution of unsolved problem of his day known as the Konigsberg bridge problem. Graph theory has various applications in the fields of computer programming, maps, network models in operation research, social network, security of a system and so on. The concept of graph labeling was introduced by Rosa [7] in 1967. A graph labeling is an assignment of integers to the vertices (or) edges (or) both subject to certain conditions. If the domain of the mapping is the set of vertices (or edges), then the labeling is called a vertex labeling (or an edge labeling). The concept of cordial labeling was introduced by Cahit [2] in 1967. A function $\phi : V \to \{0,1\}$ is said to be a cordial labeling if each edge uv has the label $|\phi(u) - \phi(v)|$ such that the number of vertices labeled 0 and the number of vertices labeled 1 differ by at most one and the number of edges labeled 0 and the number of edges labeled 1 differ by at most one. A graph which admits cordial labeling is called cordial graph. In 2015 Nellai Murugan and Mathubala [5] discussed the concept of homo-cordial labeling. A homo-cordial labeling of a graph G with vertex set V is a bijection $\phi : V \to \{0,1\}$ such that the induced function $\phi* : E \to \{0,1\}$ given by

$$\phi*(uv) = \begin{cases} 1, & \text{if } \phi(u) = \phi(v), \\ 0, & \text{otherwise.} \end{cases}$$

with the condition that $|v_\phi(0) - v_\phi(1)| \le 1$ and $|e_\phi(0) - e_\phi(1)| \le 1$. The graph that the admits a homo-cordial labeling is called a homo-cordial graph. In the very same year Nellai Murugan and Selva Vidhya [6] introduced the concept of hetro-cordial labeling of graphs. A hetro-cordial labeling

* Communicated, edited and typeset in Latex by *Lalit Mohan Upadhyaya* (Editor-in-Chief).
Received March 16, 2019 / Revised October 29, 2019 / Accepted December 04, 2019. Online First Published on June 03, 2020 at https://www.bpasjournals.com/.
Corresponding author S. Bala, E-mail: yesbala75@gmail.com
Refereed Proceedings of the National Conference on Mathematics and Computer Applications held at the Department of Mathematics, Women's Christian College, Chennai, India from January 29, 2019 to January 30, 2019.

of a graph G with vertex set V is a bijective function $\phi : V \to \{0,1\}$ such that the induced function $\phi* : E \to \{0,1\}$ given by

$$\phi * (uv) = \begin{cases} 0, & \text{if } \phi(u) = \phi(v), \\ 1, & \text{if } \phi(u) \neq \phi(v). \end{cases}$$

with the condition that $|v_\phi(0) - v_\phi(1)| \leq 1$ and $|e_\phi(0) - e_\phi(1)| \leq 1$. The graph that the admits a hetro-cordial labeling is called a hetro-cordial graph. The concept of cup(V)-cordial labeling of a graph was also introduced earlier in the year 2011 by Nellai Murugan and Iyadurai Selvaraj [4]. A cup(V)-cordial labeling of a graph G with vertex set V is a bijective function $\phi : V \to \{0,1\}$ such that the induced function $\phi* : E \to \{0,1\}$ is defined by

$$\phi * (uv) = \begin{cases} 0, & \text{if } \phi(u) = \phi(v) = 0, \\ 1, & \text{otherwise.} \end{cases}$$

with the condition that $|v_\phi(0) - v_\phi(1)| \leq 1$ and $|e_\phi(0) - e_\phi(1)| \leq 1$. The graph that the admits a cup(V)-cordial labeling is called a cup(V)-cordial graph. In this paper we aim to discuss the homo-cordial, the hetro-cordial and cup(V)-cordial labeling of the Extended Triplicate Graph of Comb.

2 The structure of the extended triplicate graph of comb

Let $P_m, m \geq 3$ be a path graph with m vertices and $m-1$ edges. Comb graph is defined as $P_m \odot mK_1$ with the vertex set and edge set as $V_1 = \{(v_i \bigcup u_i) | 1 \leq i \leq m\}$ and $E_1 = \{(v_i v_j \bigcup v_j v_i) | 1 \leq i \leq m\}$ respectively. It is denoted as $(\text{Comb})_m$. Clearly, a comb graph has $2m$ vertices and $2m-1$ edges.

In 2018 Bala et al. [1] and others introduced the concept of Extended Triplicate Graph of Comb which is described by the algorithm below:

Algorithm 2.1. Input : Comb graph
procedure (structure of TG $(\text{Comb})_m$) for $i = 1$ to m

$$X \leftarrow \left\{v_i, v_i^{'}, v_i^{''}, u_i, u_i^{'}, u_i^{''}\right\}$$

end for
for $i = 1$ to $m-1$ do
$E_1 \leftarrow \left\{u_i u_{i+1}^{'} \bigcup u_i^{'} u_{i+1}^{''}\right\}$
end for
for $i = 2$ to m do

$$E_2 \leftarrow \left\{u_i^{'} u_{i-1}^{''} \bigcup u_i u_{i-1}^{'}\right\}$$

end for
for $i = 1$ to m do

$$E_3 \leftarrow \left\{u_i v_i^{'} \bigcup v_i u_i^{'} \bigcup v_i^{'} u_i^{''} \bigcup u_i^{'} v_i^{''}\right\}$$

end
for$Y \leftarrow E_1 \bigcup E_2 \bigcup E_3$
end procedure
Output: TG$(\text{Comb})_m$

From the structure of TG$(\text{Comb})_m$, it is clear that the triplicate graph of a comb is disconnected with $6m$ vertices and $8m-4$ edges.To make it as a connected graph, include a new edge $v_1 v_m$ to the edge set of TG$(\text{Comb})_m$ if $m \equiv 0\,(mod2)$ and if $m \equiv 1\,(mod2)$, include a new edge $v_1 u_m$ in the edge set of TG$(\text{Comb})_m$. This new graph is called the Extended Triplicate Graph of Comb and it is denoted by ETG$(\text{Comb})_m$. By the construction, the extended triplicate graph of comb has $6m$ vertices and $8m-3$ edges.

Illustration 2.2. The structure of ETG$(\text{Comb})_5$ is shown below in Fig.1.

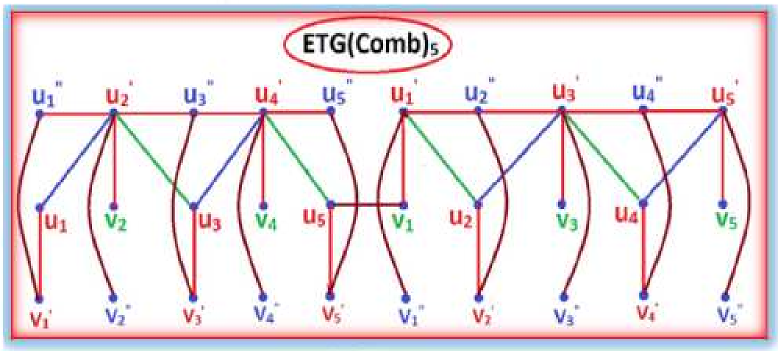

Fig. 1: The $\text{ETG}(\text{Comb})_5$.

3 Homo-cordial labeling

In this section we prove the existence of Homo-cordial labeling for $\text{ETG}(\text{Comb})_m$ by presenting algorithms.

Algorithm 3.1. Input: Extended triplicate graph of Comb
Procedure: Homo-cordial labeling for E $\text{TG}(\text{Comb})_m$
for $i = 1$ to n do
if $i \equiv 1\,(mod 2)$

$$v_i^{'} \leftarrow v_i \leftarrow u_i^{''} \leftarrow 1$$
$$u_i^{'} \leftarrow u_i \leftarrow v_i^{''} \leftarrow 0$$

else

$$v_i^{'} \leftarrow v_i \leftarrow u_i^{''} \leftarrow 0$$
$$u_i^{'} \leftarrow u_i \leftarrow v_i^{''} \leftarrow 1$$

end if
end for
end procedure
Output: labeled vertices of $\text{ETG}(\text{Comb})_m$

Theorem 3.2. *The ETG*$(\text{Comb})_m$ *admits the homo-cordial labeling.*

Proof. From the construction of the extended triplicate graph of a comb, we know that $\text{ETG}(\text{Comb})_m$ has $6m$ vertices and $8m-3$ edges. The vertices are labeled by defining a function $\phi : V \to \{0, 1\}$ as given in Algorithm3.1. In order to obtain the labels for the edges, define the induced map $\phi* : E \to \{0, 1\}$ such that for any $v_iv_j \in E, \phi * (v_iv_j) = \{\phi\,(v_i) + \phi\,(v_j)\}\,(mod 2)$. Thus,

(i) For $1 \le i \le n-1$

$$\phi * \left(u_i u_{i+1}^{'}\right) \leftarrow 0$$

(ii) For $1 \le i \le n-1$

$$\phi * \left(u_i^{'} u_{i+1}^{''}\right) \leftarrow 1$$

(iii) For $2 \le i \le n$

$$\phi * \left(u_i^{'} u_{i-1}^{''}\right) \leftarrow 1$$

(iv) For $2 \le i \le n$

$$\phi * \left(u_i u_{i-1}^{'}\right) \leftarrow 0$$

(v) For $1 \leq i \leq n$

$$\phi * \left(u_i v_i^{'}\right) \leftarrow \phi * \left(v_i u_i^{'}\right) \leftarrow 0$$

(vi) For $1 \leq i \leq n$

$$\phi * \left(u_i^{'} v_i^{''}\right) \leftarrow \phi * \left(v_i^{'} u_i^{''}\right) \leftarrow 1$$

$u_n v_1 = 0$ if $n \equiv 1\,(mod 2)$
$v_n v_1 = 0$ if $n \equiv 0\,(mod 2)$

□

Illustration 3.3. The ETG$(\text{Comb})_5$ and its homo-cordial labeling.

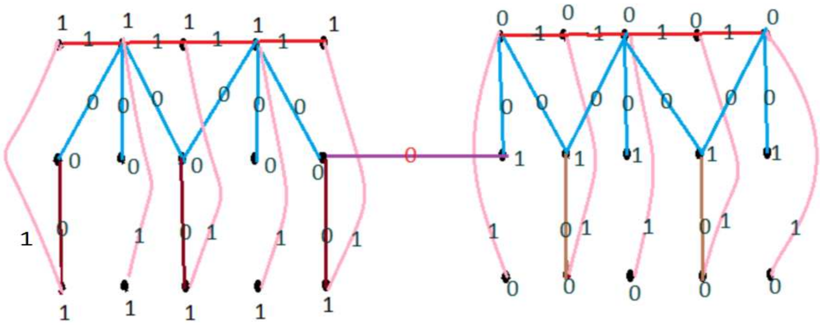

Fig. 2: The homo-cordial labeling of the ETG$(\text{Comb})_5$.

4 Hetro-cordial labeling

In this section we prove the existence of hetro-cordial labeling for ETG$(\text{Comb})_m$ by presenting algorithms.

Algorithm 4.1. Input: Extended triplicate graph of Comb
Procedure Hetro cordial labeling for ETG$(\text{Comb})_m$
for $i = 1$ to n do
if $i \equiv 1\,(mod 2)$

$$v_i^{'} \leftarrow v_i \leftarrow u_i^{''} \leftarrow 1$$
$$u_i^{'} \leftarrow u_i \leftarrow v_i^{''} \leftarrow 0$$

else

$$v_i^{'} \leftarrow v_i \leftarrow u_i^{''} \leftarrow 0$$
$$u_i^{'} \leftarrow u_i \leftarrow v_i^{''} \leftarrow 1$$

end if
end for
end procedure
Output: labeled vertices of ETG$(\text{Comb})_m$

Theorem 4.2. *The ETG$(\text{Comb})_m$ admits hetro-cordial labeling.*

Proof. From the construction of the extended triplicate graph of a comb, we know that ETG$(\text{Comb})_m$ has $6m$ vertices and $8m - 3$ edges.
The vertices are labeled by defining a function $\phi : V \rightarrow \{0, 1\}$ as given in the Algorithm4.1. In order to obtain the labels for the edges, define the induced map $\phi* : E \rightarrow \{0, 1\}$ such that for any $v_i v_j \in E, \phi * (v_i v_j) = \{\phi(v_i) + \phi(v_j)\}\,(mod 2)$. Thus,

(i) For $1 \leq i \leq n-1$

$$\phi * \left(u_i u_{i+1}^{'}\right) \leftarrow 1$$

(ii) For $1 \leq i \leq n-1$

$$\phi * \left(u_i^{'} u_{i+1}^{''}\right) \leftarrow 0$$

(iii) For $2 \leq i \leq n$

$$\phi * \left(u_i^{'} u_{i-1}^{''}\right) \leftarrow 0$$

(iv) For $2 \leq i \leq n$

$$\phi * \left(u_i u_{i-1}^{'}\right) \leftarrow 1$$

(v) For $1 \leq i \leq n$

$$\phi * \left(u_i v_i^{'}\right) \leftarrow \phi * \left(v_i u_i^{'}\right) \leftarrow 1$$

(vi) For $1 \leq i \leq n$

$$\phi * \left(u_i^{'} v_i^{''}\right) \leftarrow \phi * \left(v_i^{'} u_i^{''}\right) \leftarrow 0$$

$u_n v_1 = 1$ if $n \equiv 1\,(mod2)$
$v_n v_1 = 1$ if $n \equiv 0\,(mod2)$.

□

Illustration 4.3. The ETG(Comb)$_5$ and its hetro-cordial labeling.

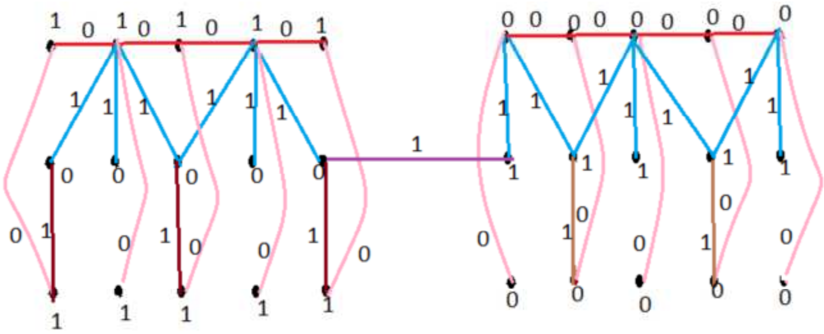

Fig. 3: The hetro-cordial labeling of the ETG(Comb)$_5$.

5 Cup(V)-cordial labeling

In this section we prove the existence of Cup-cordial labeling for ETG$(Comb)_m$ by presenting algorithms.

Algorithm 5.1. Input: Extended triplicate graph of Comb
Procedure Cup (V)-cordial labeling for ETG$(Comb)_m$
for $i = 1$ to n do
if $i \equiv 0\,(mod2)$

$$u_i^{''} \leftarrow u_i \leftarrow v_i^{'} \leftarrow 1$$
$$u_i^{'} \leftarrow v_i \leftarrow v_i^{''} \leftarrow 0$$

else

$$u_i^{''} \leftarrow u_i \leftarrow v_i^{'} \leftarrow 0$$
$$u_i^{'} \leftarrow v_i \leftarrow v_i^{''} \leftarrow 1$$

end if

end for
end procedure
Output: labeled vertices of $\text{ETG}(\text{Comb})_m$.

Theorem 5.2. *The* $ETG(\text{Comb})_m$ *admits Cup*(V)*-cordial labeling.*

Proof. From the construction of the extended triplicate graph of a comb, we know that $\text{ETG}(\text{Comb})_m$ has $6m$ vertices and $8m-3$ edges. The vertices are labeled by defining a function $\phi : V \to \{0, 1\}$ as given in the Algorithm 5.1. In order to obtain the labels for the edges, define the induced map $\phi* : E \to \{0, 1\}$ such that for any $v_i v_j \in E, \phi * (v_i v_j) = \{\phi(v_i) + \phi(v_j)\} \, (mod 2)$. Thus,

(i) For $1 \le i \le n-1$

$$\phi * \left(u_i u_{i+1}^{'}\right) = \begin{cases} 0, & \text{odd} \\ 1, & \text{otherwise} \end{cases}$$

(ii) For $1 \le i \le n-1$

$$\phi * \left(u_i^{'} u_{i+1}^{''}\right) = \begin{cases} 1, & \text{odd} \\ 0, & \text{otherwise} \end{cases}$$

(iii) For $2 \le i \le n$

$$\phi * \left(u_i^{'} u_{i-1}^{''}\right) = \begin{cases} 1, & \text{odd} \\ 0, & \text{otherwise} \end{cases}$$

(iv) For $2 \le i \le n$

$$\phi * \left(u_i u_{i-1}^{'}\right) = \begin{cases} 0, & \text{odd} \\ 1, & \text{otherwise} \end{cases}$$

(v) For $1 \le i \le n$

$$\phi * \left(u_i v_i^{'}\right) = \phi * \left(v_i^{'} u_i^{''}\right) = \begin{cases} 0, & \text{odd} \\ 1, & \text{otherwise} \end{cases}$$

(vi) For $1 \le i \le n$

$$\phi * \left(u_i^{'} v_i^{''}\right) = \phi * \left(v_i u_i^{'}\right) = \begin{cases} 1, & \text{odd} \\ 0, & \text{otherwise} \end{cases}$$

$u_n v_1 = 0$ if $n \equiv 1 \, (mod 2)$
$v_n v_1 = 0$ if $n \equiv 0 \, (mod 2)$.

□

Illustration 5.3. The $\text{ETG}(\text{Comb})_5$ and its Cup(V)-cordial labeling.

6 Conclusion

In this paper, we proved existence of homo-cordial labeling, hetro-cordial labeling and cup(V) -cordial labeling for the extended triplicate graph of comb.

References

[1] Bala, S., Thirusangu, K. and Suresh, D. (2018). Some graph labelings in the framework of triplication, *International Journal of Mathematical Archive*, 9(2), 23–28.

[2] Cahit, I. (1987). Cordial graphs: a weaker version of graceful and harmonious graphs, *Ars Combin.*, 23, 201–207.

[3] Gallian, J.A. (2018). A dynamic survey of graph labeling, *The Electronic Journal of Combinatorics*, 18, #DS6.

[4] Nellai Murugan, A. and Iyadurai Selvaraj, P. (2011). Path Related Cup Cordial Graphs, *Indian Journal of Applied Research*, 4(8), 433–436.

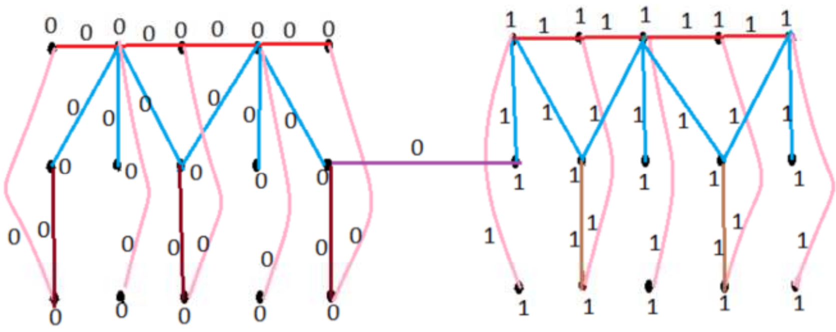

Fig. 4: The Cup(V)-labeling of the ETG(Comb)$_5$.

[5] Nellai Murugan, A. and Mathubala, A. (2015). Cycle-Related Homo-Cordial Graphs, *International Journal of Multidisciplinary Research and Development*, 2(10), 84–88.

[6] Nellai Murugan, A. and Selva Vidhya,V. (2015). Path related hetro-cordial graphs, *International Journal of Emerging Technologies in Engineering Research*, 2(3), 9–14.

[7] Rosa, A. (1967). On certain valuations of the vertices of a graph, Theory of graphs (International Symposium, Rome, July 1996), Gordon and Breach, NewYork and Dunod Paris, 349–355.