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Odd gracefulness of the chain of trim kites and star graph *

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Abstract Gnanajothi (Gnanajothi, R.B. (1991). Topics in Graph Theory, Ph.D. Thesis, Madurai Kamaraj University, Tamil Nadu, India) defined a graph G with q edges to be odd graceful if there is an one-one function f from $V(G)$ to $\{0, 1, 2, \ldots, (2q-1)\}$ such that, when each edge xy is assigned the label $|f(x) - f(y)|$, the resulting edge labels are $\{1, 3, \ldots, (2q-1)\}$. A graph which reveals an odd graceful labeling is called an odd graceful graph. In this paper, we prove that the graphs of chain of Trim kite and Star graph are odd graceful.

Key words Odd graceful labeling, Kite graph, Trim Kite, Star graph.

2010 Mathematics Subject Classification 05C78.

1 Introduction

The first graph labeling method is the graceful labeling introduced by Rosa [9] in 1967. A graph G with q edges is said to be odd graceful if there is an one-one function $f$from its vertex set $V(G)$ to the set $\{0, 1, 2, \ldots, (2q-1)\}$ such that resulting edge labels are $\{1, 3, \ldots, (2q-1)\}$according to the definition of Gnanajothi [4]. In fact, an odd-graceful labeling is an injection f from $V(G)$ to $\{0, 1, 2, \ldots, (2q-1)\}$such that, when each edge xy is assigned the label $|f(x) - f(y)|$, the resulting edge labels are $\{1, 3, \ldots, (2q-1)\}$. A variety of graphs are proved to be odd graceful by Gnanajothi [4]. Other authors who have contributed to the odd gracefulness of different classes of graphs are Eldergill [2], Jeba Jesintha et al. [6], Barrientos [1]. A dragon is formed by joining the end point of a path to a cycle (Koh et al. [10] call these tadpoles); Kim and Park [8] call them kites. For an exhaustive survey on odd graceful labeling refer to the dynamic survey by Gallian [2]. Other relevant references are [3,7]. In this paper we define a new graph called the trim kite which is obtained from the Kite graph and we prove the odd graceful labeling of chain of trim kites and star graph.

Definition 1.1. A **Kite graph** is a graph which has 5-vertices as shown in Fig. 1.

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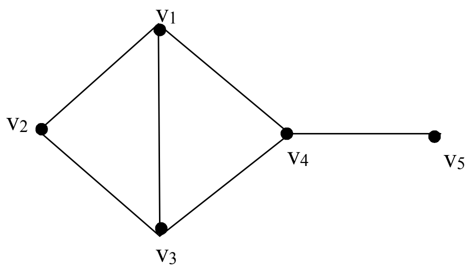

Fig. 1: The Kite Graph.

Definition 1.2. The Trim kite is obtained by subdividing the chord in the kite graph and it is a 6-vertex graph as shown in Fig. 2.

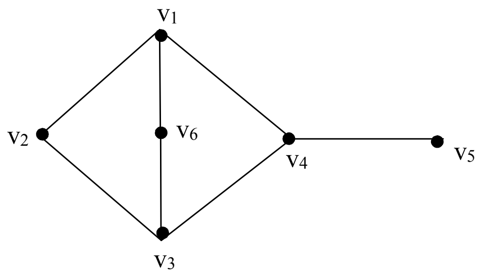

Fig. 2: The Trim Kite Graph.

Definition 1.3. The star graph S_n of order n, is a special type of graph in which $n-1$ vertices have degree 1 and a single vertex is of degree $n-1$ as shown in Fig. 3.

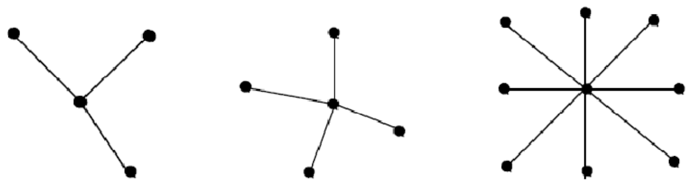

Fig. 3: The Star Graph.

2 Main result

In this section, we prove that the graph which is obtained by attaching the chain of Trim kite and Star graphis odd graceful.

Theorem 2.1. *The graph obtained by attaching the chain of Trim kite and isomorphic copies of Star graph admits odd graceful labeling.*

Proof. Let G be a chain of trim kite and isomorphic copies of star graph. We describe G as follows: let $|V(G)| = p, |E(G)| = q$. The vertices of the first copy of star graph are denoted by $u_1^1, u_2^1, \ldots, u_j^1$ where, $j = 1, 2, \ldots, r$. The vertices of the second copy of the star graph are denoted by $u_1^2, u_2^2, \ldots, u_j^2$ where, $j = 1, 2, \ldots, r$. In general, the vertices of isomorphic copies of the star graph are denoted by

$u_1^i, u_2^i, \ldots, u_j^i$ where, $i = 1, 2, \ldots, n$ and $j = 1, 2, \ldots, r$. The central vertices of star graph are denoted by v_1^i where $i = 1, 2, \ldots, n$. The vertices of the trim kite attached to the central vertex of the star graph are denoted by x_1^i and x_2^i where, $i = 1, 2, \ldots, n$. The tail of the trim kite is denoted by z_1^i where, $i = 1, 2, \ldots, n$. The other vertices of the trim kite are denoted by y_1^i, y_2^i, y_3^i where, $i = 1, 2, \ldots, n$. The description of G is shown in Fig. 4.

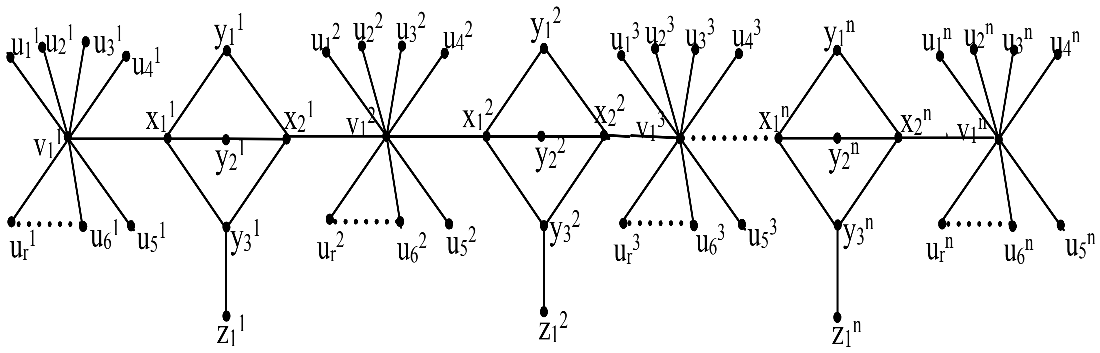

Fig. 4: Chain of star graph and trim kite.

The vertex labeling of the star graph is defined as follows:

$$f\left(v_1^k\right) = (2q-1) - 8(k-1) \qquad \text{for } 1 \le k \le n$$

$$f\left(u_i^k\right) = (2i-2) + (k-1)(2r+8) \qquad \text{for } 1 \le i \le r, 1 \le k \le n$$

The vertex labeling of the trim kite graph is defined as follows.

$$f\left(x_1^k\right) = 2r + (k-1)(2r+8) \qquad \text{for } 1 \le k \le n$$

$$f\left(x_2^k\right) = (2r+6) + (k-1)(2r+8) \qquad \text{for } 1 \le k \le n$$

$$f\left(y_i^k\right) = 2q - (2i+1) - 8(k-1) \qquad \text{for } 1 \le i \le \ r, 1 \le k \le n$$

$$f\left(z_1^k\right) = (2q-5) - 6(k-1) \qquad \text{for } 1 \le k \le n$$

From these equations we see that the vertex labels are distinct. Now we compute the edge labels for the graph G.

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$$f\left(y_i^k\right) = 2q - (2i+1) - 8(k-1) \qquad \text{for } 1 \le i \le \ r, 1 \le k \le n$$

$$f\left(z_1^k\right) = (2q-5) - 6(k-1) \qquad \text{for } 1 \le k \le n$$

From these equations we see that the vertex labels are distinct. Now we compute the edge labels for the graph G.

□

We illustrate the proof of the above theorem as follows:

Illustration 2.2. Let $r = 8, q = 42, p = 39$.

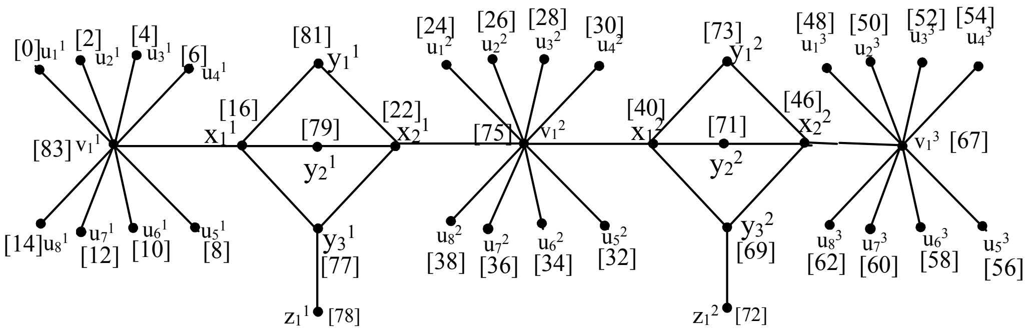

Fig. 5: Chain of star graph and trim kite.

3 Conclusion

In this paper we proved the odd gracefulness of a chain of isomorphic copies of star graph and trim kite. In a future work we intend to prove that the non isomorphic copies of star graph and trim kite is also odd graceful.

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