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Odd gracefulness of the chain of trim kite graph and bipartite graph *

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Abstract In graph theory, graph labeling technique was first introduced in the mid 1990's. Most of the graph labeling methods had their origin from the graceful labeling which was first introduced by Rosa (A. Rosa (1967). On certain valuations of the vertices of a graph, Theory of Graphs (Internat. Symposium, Rome, July 1966), Gordon and Breach, New York and Dunod Paris, 349–355) in the year 1967. Odd graceful labeling is one such method introduced by Gnanjothi (R.B. Gnanajothi, Topics in Graph Theory, Ph. D. Thesis, Madurai Kamaraj University, Tamil Nadu, India (1991)) in the year 1991. In this paper we prove that the chain of a trim kite graph and a bipartite graph is odd graceful.

Key words Graceful labeling, path, complete bipartite graph, ladder graph, trim kite graph.

2010 Mathematics Subject Classification Primary: 05C78, 05C90, 05C99.

1 Introduction

As stated above in the abstract that the concept of graceful labeling of a graph is originally due to Rosa [8], based on whose work later on some techniques of graceful labeling were introduced in the 1990's, like the work of Gnanajothi [5]. Labeling has many diverse applications in coding theory such as missile guidance code, design of good radar type codes and convolution codes. It also has a vast range of application in communication network, optimal circuit layouts, cryptography and traffic control systems. For the development of the concepts in this paper, in which we aim to prove that the chain of trim kite graph and a bipartite graph is odd graceful, we record below some essential preliminary concepts and definitions.

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A graph G with q edges is said to be odd graceful if there is an injection f from $V(G)$ to $\{0, 1, 2, \ldots, (2q-1)\}$ such that, when each edge xy is assigned the label $|f(x) - f(y)|$, the resulting edge labels are $\{1, 3, 5, \ldots, (2q-1)\}$. Gnanjothi [5] proved that the graph of path P_n and cycle C_n is odd graceful if and only if n is even. Gnanjothi [5] stated a very famous conjecture that "all trees are odd graceful" and also proved this for all trees with order up to 10. In 2009 Barrientos [2] verified this conjecture for order up to 12. Kaneria et al. [7] proved that the join sum of complete bipartite graphs, the star of complete bipartite graphs and the path union of complete bipartite graphs are graceful. Jeba Jesintha and Ezhilarasi Hilda [6] showed thatthe disjoint union of two subdivided shell graphs is odd-graceful and the one vertex union of three subdivided shells are odd-graceful. Acharya et al. [1] proved that every bipartite graph G can be embedded in an odd-graceful graph H. The construction is done in such a way that if G is planar and odd-graceful, then so is H. Chawathe and Krishna [3] showed that all countably infinite bipartite graphs that are connected and are locally finite have odd-graceful labelings. Barrientos [2] conjectures that every bipartite graph is odd-graceful.

1.1 Basic definitions

Definition 1.1. Kite Graph: A kite graph is a 5 vertex graph obtained by joining a pendent edge to a diamond graph as shown below in Fig. 1.

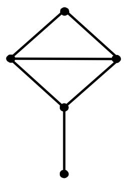

Fig. 1: The Kite Graph.

Definition 1.2. The Trim Kite Graph: The trim kite graph is a 6 vertex graph obtained by subdividing the card of the kite graph as shown below in Fig. 2.

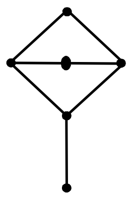

Fig. 2: The Trim Kite Graph.

Definition 1.3. Bipartite Graph: A bipartite graph is a graph whose vertices can be decomposed into two disjoint and independent sets U and V such that each edge connects a vertex in U to a vertex in V. Such a graph is represented schematically below in Fig. 3.

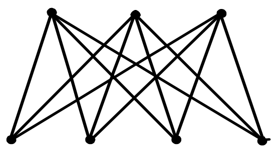

Fig. 3: The Bipartite Graph.

Definition 1.4. Chain of a Graph: A chain of a graph G is an alternate sequence of vertices and edges beginning and ending with vertices in which each edge is incident with the two vertices immediately preceding and following it.

2 Main Results

Theorem 2.1. *The chain of trim-kite graph and a bipartite graph is odd graceful.*

Proof. Let G be a graph obtained by joining a trim kite graph and a bipartite graph alternatively by a path. Let us denote the even vertices of the trim kite by $x_1^1, x_2^1, x_3^1, \ldots, x_1^n, x_2^n, x_3^n$ and the odd vertices of trim kite by $y_1^1, y_2^1, y_3^1, \ldots, y_1^n, y_2^n, y_3^n$.

Case(i): The even vertices of the isomorphic copies of the bipartite graph are denoted by $u_1^1, u_2^1, u_3^1, \ldots, u_r^1; \ldots; u_1^m, u_2^m, u_3^m, \ldots, u_r^m$ while its odd vertices are denoted by $v_1^1, v_2^1, v_3^1, \ldots, v_s^1; \ldots; v_1^m, v_2^m, v_3^m, \ldots, v_s^m$. (see Fig. 4.)

Case(ii): The even vertices of the non-isomorphic copies of the bipartite graph are denoted as $u_1^1, u_2^1, u_3^1, \ldots, u_{r1}^1; \ldots; u_1^m, u_2^m, u_3^m, \ldots, u_{rm}^m$ and the odd vertices of the non-isomorphic copies of bipartite graph by $v_1^1, v_2^1, v_3^1, \ldots, v_{s1}^1; \ldots; v_1^m, v_2^m, v_3^m, \ldots, v_{sm}^m$. (see Fig. 5.)

Here n is the number of trim kites in the chain, m is the number of bipartite graphs in the chain, r is the number of even vertices in the bipartite graph and s is the number of odd vertices in the bipartite graph as shown below in Fig. 4.

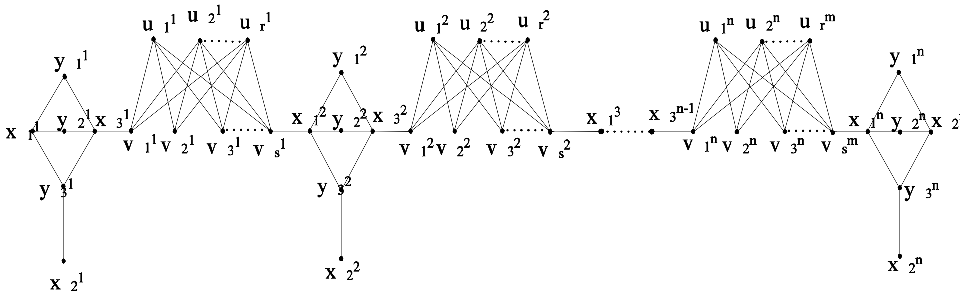

Fig. 4: The general graph G with chain of the trim kite graphs and isomorphic copies of the bipartite graphs.

The vertex labeling of the trim kite (Fig. 4) is given by:

$$f\left(x_1^k\right) = (k-1)\left[8 + 2(r+1)\right] \text{ for } 1 \le k \le n,$$

$$f\left(x_2^k\right) = 2 + (k-1)\left[8 + 2(r+1)\right] \text{ for } 1 \le k \le n,$$

$$f\left(x_3^k\right) = 8 + (k-1)\left[8 + 2(r+1)\right] \text{ for } 1 \le k \le n,$$

$$f\left(y_i^k\right) = (2q-1) - \left[2(k-1)(rs+1)\right] \text{ for } 1 \le k \le n, 1 \le i \le n.$$

The vertex labeling for the isomorphic copies of bipartite graph (Fig. 4) is given by:

$$f\left(u_i^k\right) = 10 + (k-1) + 2(i-1) \text{ for } 1 \le k \le m, 1 \le i \le r,$$

$$f\left(v_j^k\right) = (2q-7) - 2r(j-1) - \left[2(k-1)(rs+1)\right] \text{ for } 1 \le k \le m, 1 \le j \le s.$$

The vertex labeling of the trim kite (Fig. 5) is given as:

$$f\left(x_1^k\right) = 8(k-1) + 2\left[\sum_{a=1}^{k-1}(r_a+1)\right] \text{ for } 1 \le k \le n,$$

$$f\left(x_2^k\right) = 2 + 8(k-1) + 2\left[\sum_{a=1}^{k-1}(r_a+1)\right] \text{ for } 1 \le k \le n,$$

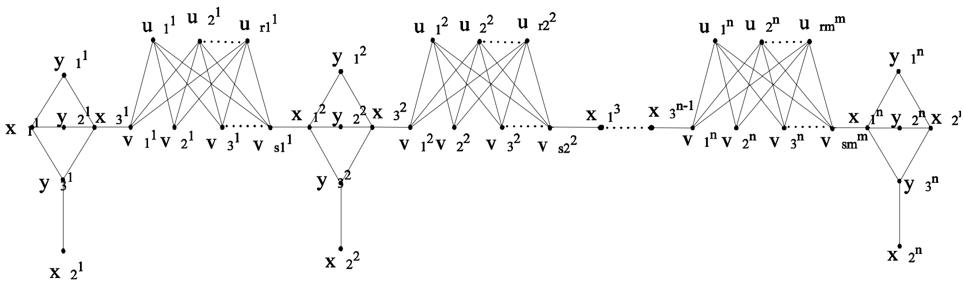

Fig. 5: The general graph G with chain of trim kite graph and non-isomorphic copies of bipartite graph.

$$f\left(x_3^k\right) = 8 + 8(k-1) + 2\left[\sum_{a=1}^{k-1}(r_a+1)\right] \text{ for } 1 \le k \le n,$$

$$f\left(y_i^k\right) = (2q-(2i-1)) - \left[2\sum(rs+1)\right] \text{ for } 2 \le k \le n, 1 \le i \le 3.$$

The vertex labeling for the non-isomorphic copies of bipartite graph (Fig. 5) is given as:

$$f\left(u_i^k\right) = [(2i-2)+10] + 2\,(k-1)\left[\sum_{a=1}^{k-1}(r_a+1)\right] + 8 \text{ for } 1 \le k \le m, 1 \le i \le r,$$

$$f\left(v_j^k\right) = (2q-(2j-2)) - 2r\,(j-1) - 2\left[\sum_{a,b=1}^{k-1}(r_a s_b+1)\right] \text{ for } 1 \le k \le m, 1 \le j \le s.$$

From the above equations it is clearly seen that the vertex labels are distinct and the edge sets get the odd labels from 1 to $2q-1$. Thus the graph G is odd graceful.

□

Illustrations:
Case (i): $n = 4, m = 3, r = 3, s = 4, q = 70 \text{and} (2q-1) = 139$ (see Fig. 6).

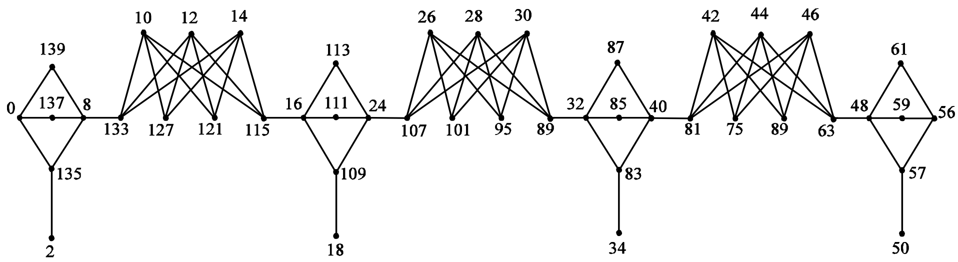

Fig. 6: Illustration of a graph G with chain of trim kite graph and isomorphic copies bipartite graph.

Case (ii): $n = 4, m = 3, r_1 = 3, s_1 = 4, r_2 = 4, s_2 = 5, r_3 = 2, s_2 = 3, q = 72$ and $(2q-1) = 143$ (see Fig. 7).

3 Conclusion

In this paper we have shown that the chain of trim kite graph and a bipartite graph is odd graceful.

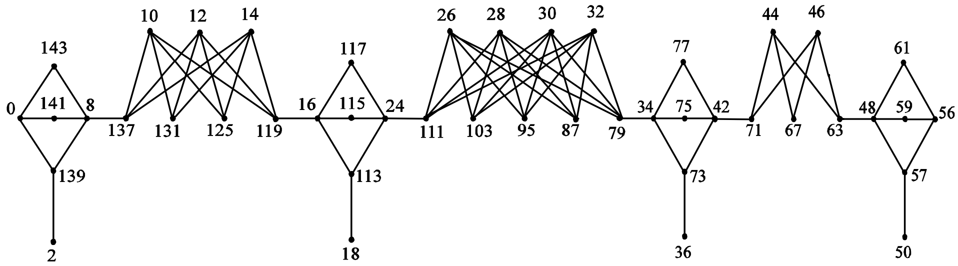

Fig. 7: Illustration of a graph G with chain of trim kite graph and non-isomorphic copies of bipartite graph.

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