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ANALYSIS OF MHD BOUNDARY LAYER SLIP FLOW AND HEAT TRANSFER ALONG A STRETCHING CYLINDER

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Abstract

An analysis has been carried out to obtain the effect of axi-symmetric MHD slip flow and heat transfer of a viscous incompressible fluid along a stretching cylinder having free stream velocity. We have incorporated similarity transformations in our study to mould the partial differential equations corresponding to the momentum and heat into a set of non-linear ordinary differential equations. Using Runge-Kutta Fehlberg method, these equations have been solved numerically with shooting technique. In the present study, the effects of curvature parameter, slip parameter, Prandtl number, magnetic parameter, free stream parameter and temperature exponent parameter on flow and heat transfer characteristics have been discussed. The effect with variations of these parameters on velocity and temperature profiles has been investigated. We found that slip parameter and free stream parameter has a substantial effect on MHD fluid flow.

Keywords: Partial Slip, MHD, Stretching Cylinder, Boundary Layer, Free Stream.

1. INTRODUCTION

The study of heat transfer and boundary layer flow over stretching surfaces is of much interest for new researchers because of its industrial purpose such as formation of paper, wire and plastic sheets etc. Initially, the flow characteristics have been examined by [1] over stretching surfaces. Many authors extended the work of [1] by dealing with heat transfer characteristics along with the flow behaviour in various physical situations in [2-5].

Firstly, the flow characteristics over stretching cylinder have been investigated by [6]. In our study, we consider the thin cylinder, so boundary layer thickness and radius may be regarded as having the same order. Due to this, the flow may be considered as axisymmetric instead of two-dimensional and hence curvature term is included in the transformed equations. It affects the temperature and velocity fields and hence affects the heat transfer rate and skin friction coefficient at the surface. Similarity solutions of such type of flows reported in [7-10] in different physical circumstances.

Upto now, magnetic impact has not been considered although it has many applications in industries like petroleum refining process, power generation process and cooling of objects etc. This effect on stretching materials has been studied in [11-14]. In [15] and [16] the magnetic impact with porous and free stream velocity reported on linear and non-linear stretching surfaces respectively.

All of these studies do not consider the slip stretched surface although in some of the engineering and industrial manufacturing processes, like spinning motion and filtration process, the slip surfaces have been used. The impact of slip surface with distinct conditions has been recognized by [17] and [18] and they discussed the effect of viscous flow on stretching surfaces. The effect of slip flow on stretching cylinder has been done in [19-23], and these studies reported that velocity of fluid reduces in presence of slip surface. Also, the slip effect on unsteady flow has been studied by [24].

It may be noted that the effort has yet been done to examine the together impact of MHD and slip flow case in the existence of free stream over stretching surfaces. The motivation of the current analysis is to introduce the combined effect of MHD slip flow having free stream velocity on fluid characteristics. In the current analysis, we try to investigate the impact of slip MHD flow on stretching cylinder in the presence of free stream.

2. FORMULATION OF THE PROBLEM

A steady, axi-symmetric two-dimensional MHD slip flow of a viscous incompressible fluid over a continuously stretching horizontal cylinder of radius *a* in the presence of free stream velocity has been considered (shown in Fig.1). It is assumed that the magnetic field B_0 is applied in the radial direction *r*. The induced magnetic field is negligible as it is so small as compared to the applied magnetic field. We consider the stretched surface has the velocity $u_w = c\left(\frac{x}{l}\right)$, $T_w = T_\infty + T_0\left(\frac{x}{l}\right)^n$ be the temperature on the boundary and $u_\infty = b\left(\frac{x}{l}\right)$ is the free stream velocity of the fluid. Here, c, T_0 be the reference velocity and temperature respectively on the surface, *b* is the reference velocity far away from the surface, T_∞ be the ambient temperature, *l* is the characteristic length and *n* is the temperature exponent.

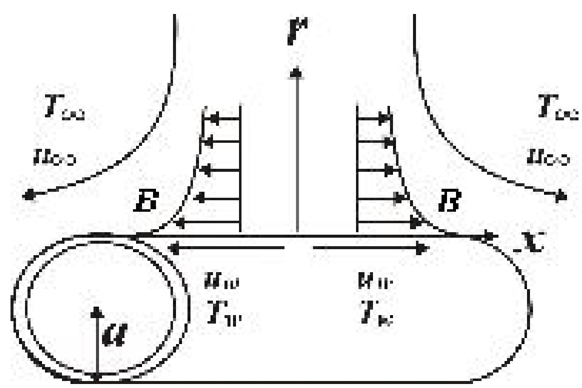

Figure 1: Schematic diagram

The governing equations of continuity, momentum and energy for such type of flow are given as follows

$$\frac{\partial}{\partial x}(ru) + \frac{\partial}{\partial r}(rv) = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial r} = u_\infty\frac{\partial u_\infty}{\partial x} + \frac{\nu}{r}\frac{\partial}{\partial r}(r\frac{\partial u}{\partial r}) - \frac{\sigma B_0^2}{\rho}(u - u_\infty) \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial r} = \frac{\alpha}{r}\frac{\partial}{\partial r}(r\frac{\partial T}{\partial r}) \tag{3}$$

where *u*,*v* are the velocity components along *x* and *r* directions respectively, ρ is kinematic viscosity, σ is the fluid density, σ is electrical conductivity, B_0 is the intensity of magnetic field, α is the thermal diffusivity and *T* be the fluid temperature.

The boundary conditions for this problem are:

$$u = u_w(x) + B_1 \nu \frac{\partial u}{\partial r}, v = 0, T = T_w(x) \text{ at } r = a \tag{4}$$

$$u \to u_\infty = b\left(\frac{x}{l}\right), T \to T_\infty \text{ as } r \to \infty \tag{5}$$

where B_1 is the slip velocity.

The equation of continuity is satisfied if we consider the stream function ψ (x,r) such that $u = \frac{1}{r}\frac{\partial \psi}{\partial r}$ and $v = -\frac{1}{r}\frac{\partial \psi}{\partial x}$. Introducing the similarity variables as $\eta = \frac{r^2 - a^2}{2a}\left(\frac{u_w}{\nu x}\right)^{1/2}$, $\psi = (u_w \nu x)^{1/2} a f(\eta)$ and $\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}$.

Using these similarity variables, the momentum and the energy equations (2) and (3) can be transformed to the corresponding ordinary differential equations as

$$(1 + 2\gamma\eta)f'''(\eta) + 2\gamma f''(\eta) + f(\eta)f''(\eta) - f'(\eta)^2 - M(f'(\eta) - \lambda) + \lambda^2 = 0 \tag{6}$$

$$(1 + 2\gamma\eta)\theta''(\eta) + 2\gamma\theta'(\eta) + Pr(f(\eta)\theta'(\eta) - nf'(\eta)\theta(\eta)) = 0 \tag{7}$$

The corresponding boundary conditions are

$$f(0) = 0,\ f'(0) = 1 + Bf'',\ \theta(0) = 1 \tag{8}$$

and $$f'(\infty) \to 0,\ \theta(\infty) \to 0 \tag{9}$$

where primes denote differentiation with respect to η, $B = B_1\sqrt{\frac{c\nu}{l}}$ is the slip parameter, $\gamma = \frac{1}{a}\left(\frac{\nu l}{c}\right)^{1/2}$ is the curvature parameter, $M = \frac{\sigma B_0^2 l}{\rho c}$ is the magnetic parameter, $\lambda = \frac{b}{c}$ is the ratio of free stream velocity to the stretching velocity and $\Pr = \frac{\nu}{\alpha}$ is the Prandtl number.

The physical quantities of interest here are the skin friction coefficient C_f and the local Nusselt number Nu which are defined as:

$C_f = 2(R_e)^{-1/2}\,\bar{x} f''(0)$ and $Nu = -(R_e)^{-1/2}\,\bar{x}\theta'(0)$ where $\bar{x} = x/l$ and $R_e = lU_0/\nu$.

3. RESULTS AND DISCUSSION

The set of non-linear coupled ordinary differential equations (6) and (7) subject to boundary condition (8-9) constitute a two-point boundary value problem. In order to solve these equations numerically, we follow one of the most efficient numerical schemes, that is, Runge-Kutta Fehlberg integration scheme with shooting techniques. In our computations, we have carried out a parametric study showing influence of several non-dimensional parameters, namely slip parameter *B*, free stream parameter λ, Prandtl number *Pr*, curvature parameter γ, temperature exponent *n* and magnetic parameter *M* on temperature and velocity profile of the boundary layer flow. For the validation of numerical method used, we have compared our results on $-\theta'(0)$ with those of Ishak and Nazar (2009) and Elbashbeshy et al. (2012) for γ=0 (stretching flat plate), *B*=0 (no-slip condition), γ=0 and in the absence of magnetic field with different values of *n*. They are found to be in a good agreement as shown in Table 1.

Table 1: Comparison of $-\theta'(0)$ for a few values of temperature exponent *n* when *Pr*=1

n	Ishak and Nazar (2009)	Elbashbeshy et al.(2012)	Present study
0	0.5820	0.5820	0.5819
1	1.0000	1.0000	1.0000
2	1.3333	1.3333	1.3332

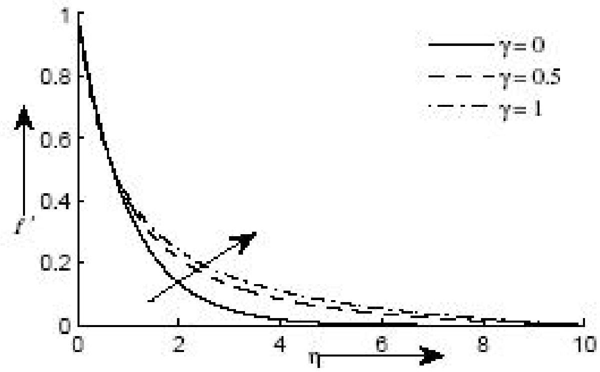

Figure 2: The velocity profile $f'(\eta)$ for various values of γ having no-slip condition in absence of λ

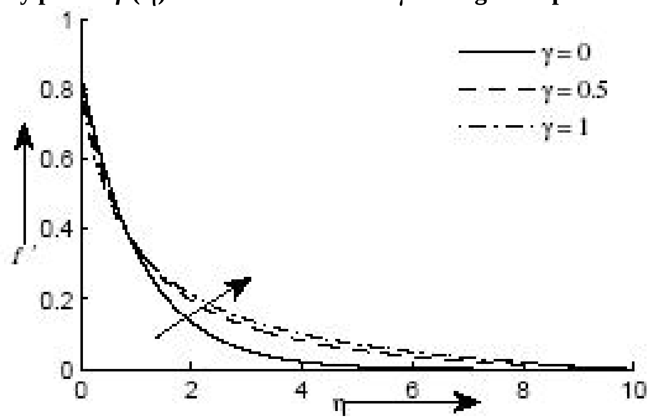

Figure 3: The velocity profile $f'(\eta)$ for various values of γ having slip condition in absence of λ

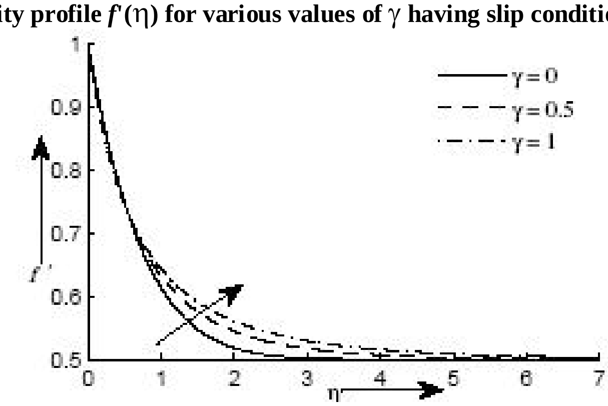

Figure 4: The velocity profile $f'(\eta)$ for various values of γ having no-slip condition, when λ<1

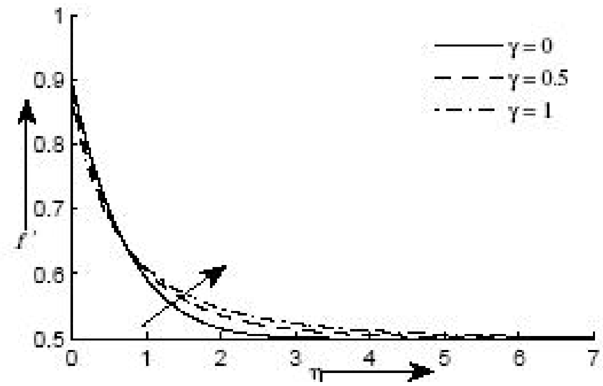

Figure 5: The velocity profile $f'(\eta)$ for various values of γ having slip condition, when λ<1

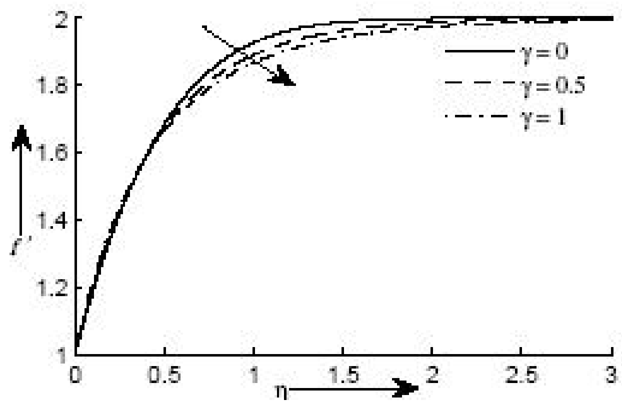

Figure 6: The velocity profile $f'(\eta)$ for various values of γ having no-slip condition, when λ>1

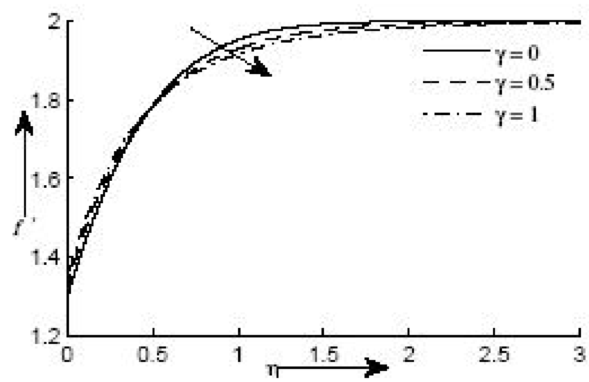

Figure 7: The velocity profile $f'(\eta)$ for various values of γ having slip condition, when λ>1

Figures 2, 4 and 6 show the effects of curvature parameter λ on velocity profiles having no-slip boundary condition for λ=0, λ<1 and λ>1 respectively. Here, λ<1 and λ>1 implies that the free stream velocity is less than and greater than the stretching velocity respectively. The velocity increases as we increase the curvature parameter λ when λ<1(fig 2 and 4), while it shows an reverse trend in velocity when λ>1 (shown by the arrow in fig. 6). The reverse trend is obtained due to the greater free stream velocity than stretching velocity. Figures 3, 5 and 7 show the effects of curvature parameter λ on velocity profiles in slip boundary condition for λ=0, λ<1 and λ>1 respectively. It is clear from the figures 3 and 5 that for $\lambda<1$ (in presence of slip parameter), the fluid velocity decreases near the surface up-to a certain distance as curvature parameter λ increases but after crossing some distance, the velocity profiles behaves the same as behave in the no-slip condition. The velocity decreases due to the slip effect at the boundary of the stretching surface. For $\lambda>1$, we obtain an reverse trend in the velocity (fig. 7). Also, we see that from all of these figures that the velocity gradient increase and vanishes after a large distance from the surface as we increase the free stream parameter λ. The variation between the the velocity profiles has been reduced as we increase the free stream parameter λ. In the presence of slip parameter, there is a transition phase occur in velocity profiles near the surface which explains that the velocity is affected by slip effect near the surface.

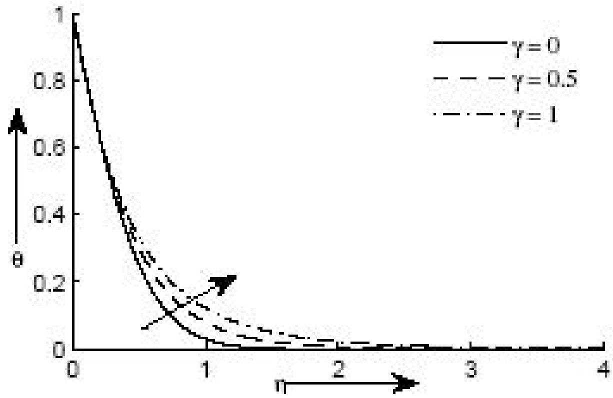

Figure 8: The temperature profile $\theta(\eta)$ for various values of γ having no-slip condition in absence of λ

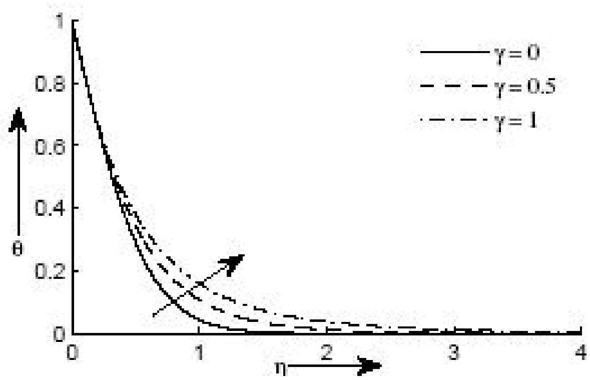

Figure 9: The temperature profile $\theta(\eta)$ for various values of □ having slip condition in absence of λ

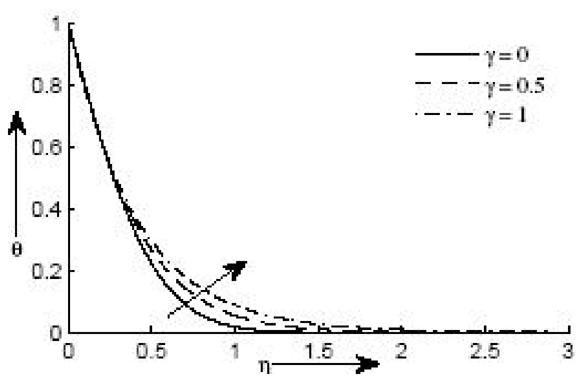

Figure 10: The temperature profile $\theta(\eta)$ for various values of γ having no-slip condition, when $\lambda<1$

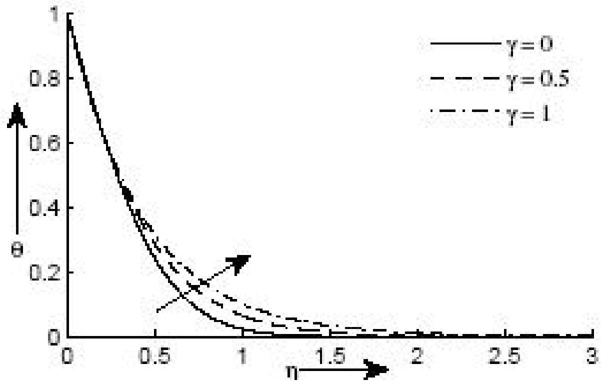

Figure 11: The temperature profile $\theta(\eta)$) for various values of γ having slip condition, when λ<1

The effects on temperature profile of the curvature parameter γ having no-slip condition at boundary of the surface for λ=0, λ<1 and λ>1 are shown by the figures 8, 10 and 12 respectively, while figures 9, 11 and 13 show the same in slip condition at the surface. In all these cases, the temperature gradient decreases and hence temperature increases as we increase the curvature parameter γ.

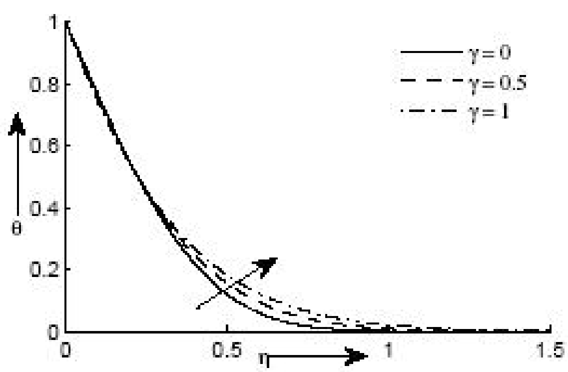

Figure 12: The temperature profile $\theta(\eta)$ for various values of γ having no-slip condition, when λ>1

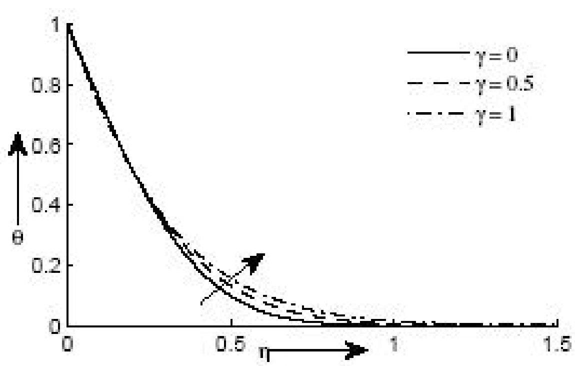

Figure 13: The temperature profile $\theta(\eta)$ for various values of γ having slip condition, when λ>1

Figures 14, 15 and 16 show the velocity profiles for various values of the magnetic parameter M for λ=0, λ<1 and λ>1 respectively having slip condition at surface. The velocity decreases with an increase in the magnetic parameter M when λ<1, while it increases when λ>1. The magnetic parameter M has an significant effect on fluid flow. The presence of magnetic field causes high restriction to the fluid which reduce the fluid velocity when $\lambda<1$ (shown by the arrow in fig. 14 and 15), while the free stream effect has overcome the magnetic effect when $\lambda>1$. So, we get an reverse trend of velocity (fig. 16) for $\lambda>1$. However, the magnetic field opposes the transport phenomenon (reverse the temperature process). But the magnetic field enhances the temperature at all points leading to increase in the thermal boundary layer thickness. Figure 17 show the temperature increases as we increase the magnetic parameter *M.*

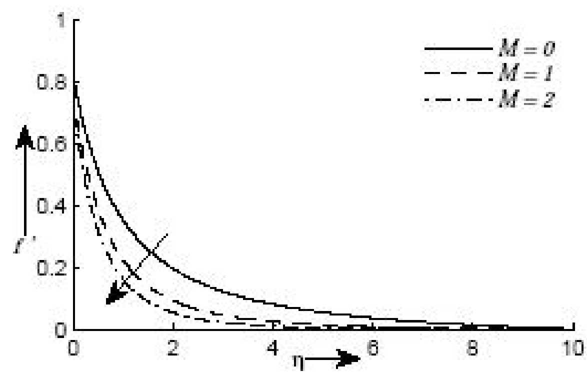

Figure 14: The velocity profile $f'(\eta)$ for various values of M having slip condition in absence of λ

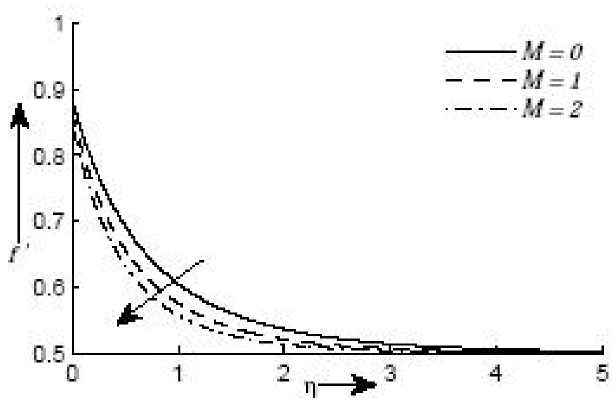

Figure 15: The velocity profile $f'(\eta)$ for various values of M having slip condition, when $\lambda<1$

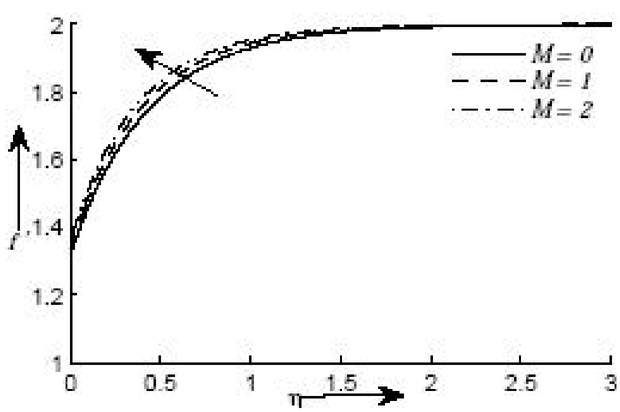

Figure 16: The velocity profile $f'(\eta)$ for various values of M having slip condition, when $\lambda>1$

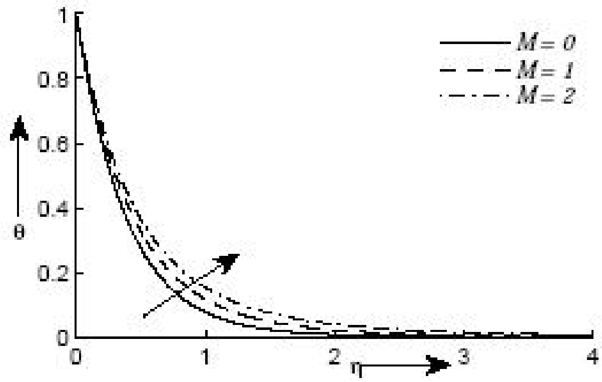

Figure 17: The temperature profile $\theta(\eta)$ for various values of M having slip condition, in absence of λ

Figures 18, 19 and 20 show the effect on temperature profiles for various values of Prandtl number *Pr* for λ=0, λ<1 and λ>1 respectively having slip condition at the surface. In all the cases, the temperature is found to decrease as we increase the Prandtl number *Pr*. This shows that an increase in Prandtl number reduces the thermal boundary layer thickness. Prandtl number signifies the ratio of momentum diffusivity to thermal diffusivity. Fluids with lower Prandtl number will possess higher thermal conductivity and thicker thermal boundary layer structure. Hence, large Prandtl number can be used to decrease the thermal boundary layer thickness in conducting flows. The temperature curves for the various values of *Pr* show greater separation for λ=0 (fig. 18) as compared to λ<1 (fig. 19) and λ>1 (fig. 20).

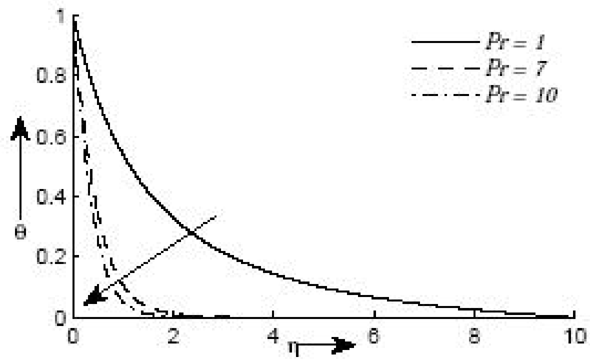

Figure 18: The temperature profile $\theta(\eta)$ for various values of *Pr* having slip condition in absence of λ

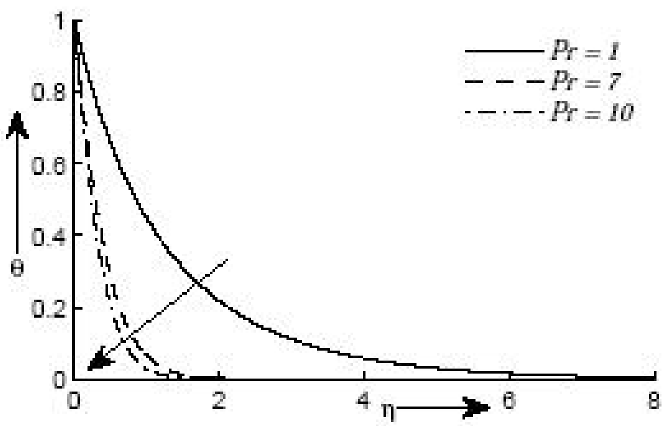

Figure 19: The temperature profile $\theta(\eta)$ for various values of *Pr* having slip condition, when λ<1

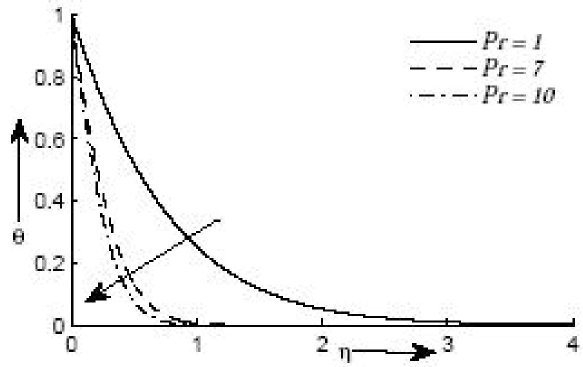

Figure 20: The temperature profile θ(η) for various values of *Pr* having slip condition, when λ>1

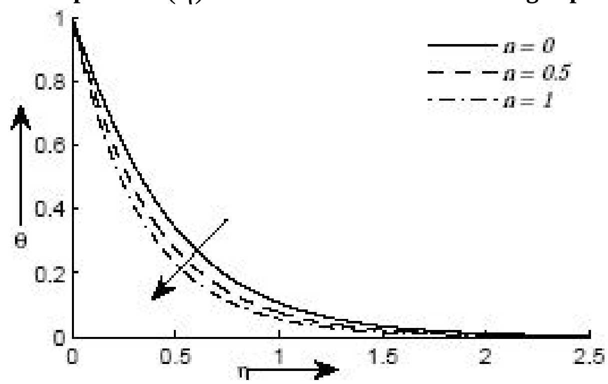

Figure 21: The temperature profile θ(η) for various values of *n* having slip condition in absence of λ

Figures. 21, 22 and 23 show the temperature profiles for various values of the temperature exponent *n* for λ=0, λ<1 and λ>1 respectively having slip condition at the surface. It is clear from the figures that the temperature decreases with the increase in *n* (shown by the arrow). Also, we observe that the temperature decreases with the increase in λ for a fixed value of *n*.

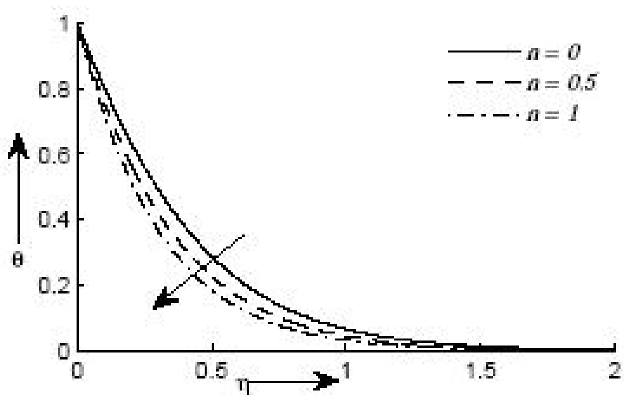

Figure 22: The temperature profile θ(η) for various values of *n* having slip condition, when λ<1

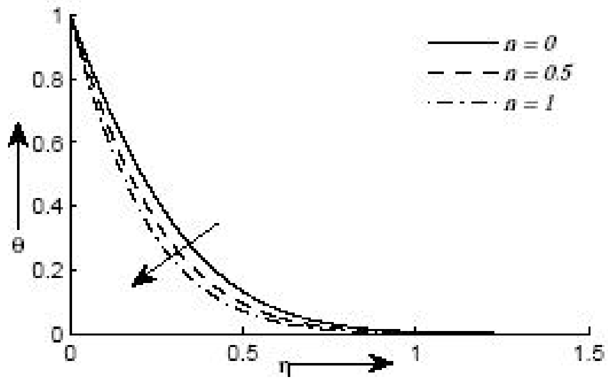

Figure 23: The temperature profile $\theta(\eta)$ for various values of *n* having slip condition, when $\lambda>1$

4. CONCLUSIONS

In the present study, numerical results have been reported for steady, axi-symmetric, MHD slip flow of a viscous and incompressible fluid along a continuously stretching horizontal cylinder in the presence of free stream. The effects of Prandtl number *Pr*, magnetic parameter *M*, curvature parameter γ, temperature exponent *n* in presence of slip velocity have been studied. The key findings of the present study in the presence of free stream are as follows:

1. The velocity increases with an increase in γ and decreases with an increase in *M* and *B* for $\lambda<1$ while reverse trend of velocity has been found for $\lambda>1$.
2. The temperature increases with the increase in γ, *M*, and decreases with the increase in *Pr* and *n*.
3. The boundary layer thickness decreases as we increase the λ.

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