Original Article

Bulletin of Pure and Applied Sciences.
Vol. 36E (Math & Stat.), No.2, 2017.P.311-315
Print version ISSN 0970 6577
Online version ISSN 2320 3226
DOI: 10.5958/2320-3226.2017.00033.9

A LINEAR POPULATION MODEL FOR DIABETES MELLITUS

Vanitha R*, Sophia Porchelvi R**

Author Affiliation:
*Associate Professor, Dept .of Mathematics, A.D.M. College for Women (Autonomous), Nagapattinam, Tamil Nadu 611001, India
E-mail: vanithaadmc@gmail.com
**Associate Professor, Dept .of Mathematics, A.D.M. College for Women (Autonomous), Nagapattinam, Tamil Nadu 611001, India
E-mail: sophiaporchelvi@gmail.com

Corresponding Author:
Vanitha R, Associate Professor, Dept .of Mathematics, A.D.M. College for Women (Autonomous), Nagapattinam, Tamil Nadu 611001, India

E-mail: vanithaadmc@gmail.com

Received on 09.11.2017, **Accepted on** 09.12.2017

Abstract

Diabetes and its associated complications have been increasing at an alarming rate worldwide. Prevalence of diabetes is higher in developed countries than in the developing countries, however during the past two decades diabetes mellitus was reported higher in developing countries. A mathematical study of the size of population of diabetes mellitus is carried out in this paper. By appropriate definition of a parameter , the mathematical model may be classified as linear or non-linear. Here, linear differential equations are developed and solved by numerical methods.

Keywords: Population model, diabetes mellitus , Numerical methods.

AMS Mathematics Subject Classification:34C35, 65C20,

1. INTRODUCTION

Diabetes mellitus has emerged as a major health care problem worldwide. According to International Diabetes Federation (IDF)[9], India has earned the dubious distinction of being the diabetic capital of the world and there is an estimated forty million people with diabetes in India today. Unfortunately, over half of these people remain undiagnosed as diabetes is a "silent " disease. In 1994 ,Boutayeb and Kerfati made mathematical modeling on diabetology[1].

Diabetes is caused by increased levels of glucose in blood. When carbohyrdrate is taken by the body it gets converted into glucose which is transmitted to blood cells through blood with the help of insulin.

Insulin has an important role for the control of intermediatery metabolism. In case of normal human being , insulin is released from β cells in the pancreas which usually absorb the excess glucose present in the blood and convert it into energy or fuel.Insulin also stimulates liver to absorb and store any glucose left over [6]. Diabetes Mellitus (DM) is a polygenic metabolic and heterogeneous disease condition in which the body does not succeed in producing enough insulin, characterized by abnormal glucose homeostasis. The number of adults with a clinical diagnosis of diabetes has been increasing dramatically worldwide. Prevalence of diabetes is higher in developed and in developing countries, but the major increase in people with diabetes was reported to occur in developing countries , especially Asian countries, in the age group between 45-65 years. There are more women than men with diabetes, especially in developed countries[7]. Several sub types of the (DM) exist with multiple underlying pathogenic pathways and have been classified based on the etiology[4]. As a clinical condition, there are two major types, type I diabetes (**IDDM-** *Insulin* Dependent Diabetes Millitus) and type II diabetes (**NIDDM**-Non Insulin Dependent Diabetes Millitus). Type II diabetes (T2D) is found to be the most prevalent form of diabetes accounting for accounts for 90 % of total diabetes cases [5].

The three main types of diabetes and its risk factors are given in [8] as follows:

Type 1 diabetes	**Type 2 diabetes**	**Gestational diabetes**
• Risk factors: family history of diabetes, genetics, infections and other environmental influences • Appears very suddenly and is currently incurable • Without insulin, a person with type 1 diabetes will die	Risk factors: excess body weight, physical inactivity, poor nutrition, genetics, family history of diabetes, past history of gestational diabetes and older age • Can go unnoticed and undiagnosed for years • Can often be managed with dietary changes and increasing physical activity. In some cases ,medication is required	• Appears during pregnancy • Can lead to serious health risks for both the mother and child • Associated with an increased risk of both mother and child developing type 2 diabetes later in life

The majority of people with diabetes are NIDDM and estimates of the exact proportion vary from 75% to 90% given by the British Diabetic Association. People with NIDDM who are treated with insulin may be classified wrongly as having IDDM and the overall proportion of NIDDM to IDDM will be distorted. In addition NIDDM symptoms are sometimes not evident and diabetes might not be diagnosed until complications develop[3].

2. MATHEMATICAL MODEL

A mathematical model of diabetes mellitus will be analyzed which contains diabetes mellitus with complication and without complication. The model will monitor the size of a population of diabetics and will give the number of people with complications as a function of time. The case when probability of a diabetic developing a complication is taken to be constant classifies the model as linear (Boutayeb et al.[2]) and the case when this probability is allowed to vary at a rate proportional to the fraction of diabetes with complications classifies the model as non-linear. The paper will concentrate only linear model and numerical method is used to solve the equations.

Suppose that

C: C(t) number of diabetics with complication.
D: D(t) number of diabetics without complication
N: N(t) =C(t) +D(t) –the size of the population of diabetic at time t
I: Incidence of *diabetes mellitus* (assumed constant); number of cases diagnosed in time t
μ : Natural Mortality Rate.
λ :Probability of Diabetic person developing complications..
Υ :The rate at which complications are cured.
ν :The rate at which diabetes patients with complications become severely disabled
δ: The mortality rate due to complications.

Patients who already have complications upon diagnosis with diabetes mellitus are placed immediately in class C .

A diagramatic representation of the model is shown in Figure 1

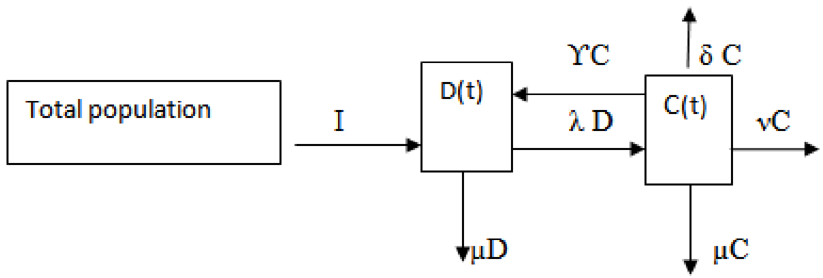

Figure 1: The mathematical model

The diagram shows that I cases are diagnosed in a time interval of length t and are assumed to have no complications upon diagnosis. In the same time interval , the number of sufferers without complications , D = D(t) ,is seen **to decrease** by the amounts μD (Natural Mortality) and λ D (sufferers who develop complications and who have complications at diagnosis) ,**and to increase** by the amount Υ D (sufferers whose complications are cured). During this time interval, the number of diabetic with complications is **increased** by the afore- mentioned amount λ D; it is **decreased** by the aforementioned amount ΥC and by the amounts μC (Natural Mortality), νC (patients who become severely disabled and whose disabilities cannot be cured. and δ C(those who die from their complications). These rates of changes are formalized by the ordinary differential equations (ODEs)

$$D'(t) = \frac{dD}{dt} = I - (\lambda + \mu)D(t) + \Upsilon C(t), \quad (1)$$

$$C'(t) = \frac{dC}{dt} = \lambda D(t) - (\Upsilon + \mu + \nu + \delta) C(t), \quad (2)$$

Let $\alpha = \lambda + \mu$ and $\beta = \Upsilon + \mu + \nu + \delta$, Equations (1),(2) becomes the initial value problem (IVP) as

$$D'(t) = I - \alpha D(t) + \Upsilon C(t), t>0 ; D(0)=D_0 \quad (3)$$

$$C'(t) = \lambda D(t) - \beta C(t), t>0 ;C(0)= C_0 \quad (4)$$

Now by Euler –Cauchy method the solution of (3) and (4) is given by

$$D_{n+1} = D_n + \frac{1}{2}[k_{11}+ k_{12}] \quad (5)$$

$$C_{n+1} = C_n + \frac{1}{2}[k_{21}+ k_{22}] \quad (6)$$

Where ,

$k_{11} = h\ F(t_0,D_0,C_0)$; $k_{21}= h\ G(t_0,D_0,C_0)$

$k_{12}= h\ F(t_0+h, D_0+k_{11}, C_0+k_{21})$; $k_{22}= h\ G(t_0+h, D_0+k_{11}, C_0+k_{21})$

In the case when the probability of a diabetic person developing a complication , λ will be estimated to have the constant value $\lambda= C_0/N_0$, the differential equation in (3),(4) are linear in C(t) and D(t) . It can be solved by any method with parameter value per year :Υ =0.08 , μ =0.02 , ν =0.05, δ = 0.05. [2]

3. THE CRITICAL POINT AND ITS PROPERTY

Since N=C+D, (3) & (4) to be

$$C'(t) = -(\lambda+\beta) C(t) + \lambda N(t) ;C(0) = C_0 \quad (7)$$

$$N'(t) = I(t) - (\nu +\delta) C(t) -\mu N(T), t>0; N(0) =N_0 \quad (8)$$

The initial value problem (7) and (8) may write in the matrix-vector form as

$$x'(t) = Ax(t)+b(t) , t>0 ; x(0) = X_0 \quad (9)$$

in which $x(t) = \begin{bmatrix} C(t) \\ N(t) \end{bmatrix}$, $A = \begin{pmatrix} -\lambda - \beta & \lambda \\ \nu - \delta & -\mu \end{pmatrix}$,

$$b(t) = \begin{bmatrix} 0 \\ I(t) \end{bmatrix}, \quad X_0 = \begin{bmatrix} C0 \\ N0 \end{bmatrix} \tag{10}$$

Suppose that I is the steady-state value of the incidence then the model reaches its critical point when dC/dt and dN/ dt given in (8) and (9) vanish simultaneously, that is when

$$\lambda N - (\lambda+\beta) C = 0 \tag{11}$$
$$I - \mu N - (\nu +\delta) C = 0 \tag{12}$$

Solving these $C = \frac{\lambda I}{\mu\lambda+\mu\beta+\lambda\delta+\lambda\nu}$

and $$N = \frac{(\lambda+\beta)I}{\mu\lambda+\mu\beta+\lambda\delta+\lambda\nu} \tag{13}$$

The eigen values of the matrix A ,χ_1, χ_2 , are the roots of the characteristic equation

$$\chi^2 + (\lambda +\beta +\mu) \chi + \mu(\lambda +\beta)+ \lambda(\nu +\delta) = 0 . \tag{14}$$

The discriminant Δ ,of this equation is given by
$\Delta= (\lambda +\beta +\mu)^2 -4[\mu(\lambda +\beta)+ \lambda(\nu +\delta)]$ and ,and solving equation (14) gives
$$\chi_1 = [- (\lambda +\beta +\mu) + \Delta^{1/2}] / 2 \quad \text{and} \quad \chi_2 = [- (\lambda +\beta +\mu) - \Delta^{1/2}] / 2 \tag{15}$$

and it is easy to verify that , when the parameters of the model are such that
(a) $\Delta > 0$, χ_1, χ_2 are both real and negative;
(b) $\Delta = 0$, $\chi_1 = \chi_2$ are real and negative;
(c) $\Delta < 0$, χ_1, χ_2 are complex with negative real parts.

It may be concluded , therefore ,that the critical point (C,N) is stable.[3].

4. NUMERICAL SOLUTIONS

Figure 2 shows the percentage of controlled diabetic patients for 100 years. Matlab R2013a is used to solve the Eq. (3). The behavior of graph reached the steady state after 50 years.

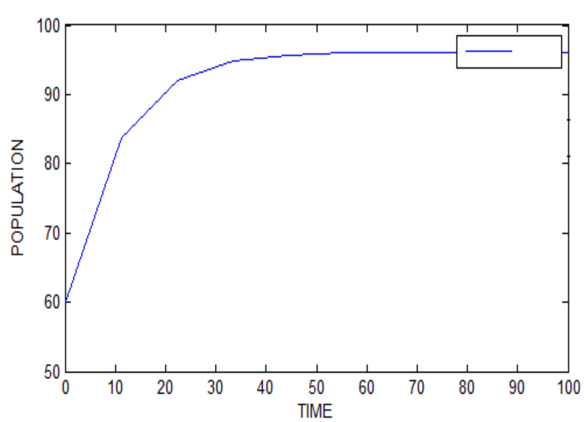

Figure 2: Graph of the Percentage of Controlled Diabetics Patients for 100 Years

Figure 3 shows the percentage of uncontrolled diabetic patients for 100 years. Matlab R2013a is used to solve the Eq. (4). The behavior of graph reached the steady state after 50 years.

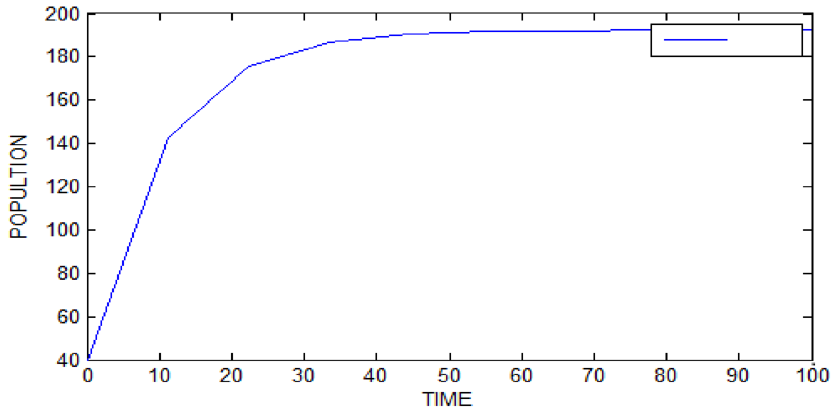

Figure 3: Graph of the Percentage of Uncontrolled Diabetics Patients for 100 Years

5. CONCLUSION

In this paper a linear population model for diabetic patients is analysed. To monitor the percentage of controlled and uncontrolled diabetic patients ,numerical method was used. From the graph it is concluded that stability is obtained in the 50 years and analysis is done for 100 years.

Appendix

Solution of the Mathematical Model:
By taking $I = 25$, $D_0 = 60$, $C_0=40$, $\lambda= C_0 /N_0 = 0.4$, $\Upsilon =0.08$, $\mu =0.02$, $\nu =0.05$, $\delta = 0.05$, $\beta = \Upsilon+ \mu+ \nu+ \delta= 0.2$, $\alpha = \lambda + \mu =0.5$ and using the following MATLAB command , solution and curves of controlled and uncontrolled diabetic population are obtained .

```
>> [x,y]=dsolve('Dx=25-0.42*x+0.08*y','Dy=0.4*x-0.2*y')
>> inits='x(0)=60,y(0)=40';
>> [x,y]=dsolve('Dx=25-0.42*x+0.08*y','Dy=0.4*x-0.2*y',inits)
>> t=linspace(0,100,10);
>> xx=eval(vectorize(x));
>> plot(t,xx)
 >>t=linspace(0,100,10);
>>yy =eval(vectorize(y));
>> plot(t,yy)
```

REFERENCES

1. A.Boutayeb and A.Kerfati ,Mathematical models in diabetology ,, Modelling, Measurement and control, C 44(2)(1994) 53-63.
2. A.Boutayeb ,E.H.Twizell,K.Achouyab and A.Chetouani, A mathematical model for the burden of diabetes and its complications, BioMedical Engineering OnLine,3:20 (2004).
3. A.Boutayeb ,E.H.Twizell,K.Achouyab and A.Chetouani,(2006),A non-linear Population Model of Diabetes Mellitus , J.Appl.Math . & Computing,**21**(1),127-139
4. Americann Diabetes Association . Diabe Care ,2009;32:13-61.
5. Rolo AP , Palmeira CM.Toxicol Appl Pharma,2006;212:167-178.
6. Sandhya ,Deepak Kumar & Prerna Pandit, An Ordinary Differential Equation Model Of Diabetic Population in NewDelhi, India n Journal of Mathematical Sciences Vol.7,No.1,(June 2011):45-50.
7. Waild S,Roglic G, Green A, Sicree R, King H. Diabe Care,2004;27:1047- 53.
8. WHO., Definition ,Diagnosis and Classification of Diabetes Mellitus and its Complications. Geneva:Department of Non-Communicable Disease Surveillance ,1999.
9. International Diabetes Fedreration [IDF], IDF Report 2003,www.idf.org/home/index.cfm.