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PROPAGATION OF SH WAVES IN TWO MICROMORPHIC HALF SPACES

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Abstract

In this paper an attempt is made to study the propagation of SH waves in two micro-morphic half spaces. The period equation is obtained. It is observed that three additional waves are found which are not encountered in classical elasticity.
Keywords: SH Waves – Micro-morphic half spaces

1. INTRODUCTION

The classical theory of elasticity neglects the effect of the distribution of couples over the surface across which different parts of a continuum interact mechanically with each other. The micro polar theory developed by Eringen and Eringen [1] is based on the condition that microemotion of the medium is restricted to micro-rotation. To analyze the mechanical behavior of micromorphic solid it requires 12 second order partial differential equations in 12 unknowns involving 18 elastic constants. Koh[5] developed simpler theory by extending the concept of coincidence of principal directions of stress and strain in classical elasticity to the micro-elastic solid. Imposing a particular form of micro – isotropy Koh obtained special constraints on the elastic moduli, thereby reducing number 18 to 10 in the special case. In a subsequent formulation of the problem with respect to principal direction of the micro-strain one needs to consider only nine equations in nine unknowns.

The theory developed by Koh is known as micro – isotropic, micro-elastic theory. Assuming the micro-motion is restricted to micro-rotation and stress moment tensor has a particular form of anti – symmetry one obtain the equations of micropolar elasticity.

The propagation of plane waves in two semi-infinite media separated by a plane interface was discussed by Sommerfield [6], Jeffery [7], Muskant [8] and others have discussed wave propagation in case where distance of the point source from the plane is finite.

Wave propagation in semi-infinite micro polar isotropic elastic solid lying over another micro polar elastic solid is studied by M. Parameshwar Rao and B. Kesava Rao [9]. Reflection and Refraction of

SH waves at a corrugated interface between two dimensional transversely isotropic half spaces is studied by S.K. Tomar and S.L. Saini. [10]

In this paper we discuss the SH waves in two micromorphic elastic half spaces. This problem is of geophysical interest, particularly in investigations concerned with earthquakes and other phenomena in seismology. Since the propagation characteristics of earth vary with depth, the first approximation to the actual problem can be achieved by regarding the earth as formed of several layers in each of which physical additional waves are found. These three waves are dispersive and depend only on micromorphic constrants that is other than λ and μ.

2. BASIC EQUATIONS

The basic equations for micro-isotropic, micro elastic solids are obtained by Koh, Parameshwaran and Koh [11] are given as follows.

The constitutive equations are:

$$t_{(km)} = A_1 \tau_{pp} \delta_{Km} + 2A_2 e_{Km} \tag{1}$$

$$t_{[km]} = \sigma_{[km]} = 2A_3 \in_{pkm} \left(r_p + \phi_p\right) \tag{2}$$

$$\sigma_{[km]} = -A_4 \phi_{pp} \delta_{km} - 2A_5 \phi_{(km)} \tag{3}$$

$$t_{k(mn)} = B_1 \phi_{pp,k} \delta_{mn} + 2B_2 \phi_{(mn),k} \tag{4}$$

$$m_{(kl)} = -2\left(B_5 \phi_{l,k} + B_4 \phi_{k,l} + B_5 \phi_{p,p} \delta_{kl}\right) \tag{5}$$

Where

$$\begin{aligned} &A_1 = \lambda + \sigma_1, && B_1 = \tau_3 \\ &A_2 = \mu + \sigma_2, && 2B_2 = \tau_1 + \tau_{10} \\ &A_3 = \sigma_5, && B_3 = 2\tau_4 + 2\tau_9 + \tau_7 - \tau_{10} \\ &A_4 = -\sigma_1, && B_4 = -2\tau_4 \\ &A_5 = -\sigma_2 && B_5 = -2\tau_9 \end{aligned} \tag{6}$$

Subject to the conditions

$$\begin{aligned} &3A_1 + 2A_2 > 0, \quad A_2 > 0, \quad A_3 > 0 \\ &3A_4 + 2A_5 > 0, \quad A_5 > 0, \\ &3B_1 + 2B_2 > 0, \quad B_2 > 0, \quad B_3 > 0 \\ &-B_3 < B_4 < B_3, \quad B_3 + B_4 + B_5 > 0 \end{aligned} \tag{7}$$

The micro – displacement in the micro-elastic continuum is denoted by μ_k and the micro-deformation by ϕ_{mn}. For the linear theoty, we have the macro – strain $e_{km} = \mu_{(k,m)}$, the macro – rotation vector $r_k = \frac{1}{2} \epsilon_{pkm} \mu_{m,k}$, the micro-strain $\phi_{(mn)}$ and micro –rotation vector $\phi_p = = \frac{1}{2} \epsilon_{pkm} \phi_{km}$. The stress measures are the asymmetric stress (macro-stress) t_{mn}, the relative stress (micro-stress) σ_{km} and the stress moment t_{kmn}. Also the couple stress tensor $m_{kp} = \epsilon_{pkm} t_{kmn}$.

We indicate the symmetric part with () while the anti-symmetric part with []. λ, μ are classical material constants and σ_1, σ_2, σ_5, τ_3, τ_4, τ_7, τ_9 and τ_{10} are micro – isotropic and micro – elastic material constants. Further ϵ_{pkm} is the permutation symbol.

The equations of motion with the body forces and body couple are given by

$$(A_1 + A_2 - A_3)\,\mu_{p,pm} + (A_2 + A_3)\,\mu_{m,pp} + 2A_3\epsilon_{pkm}\phi_{p,k} + \rho f_m = \rho\frac{\partial^2\mu_m}{\partial t^2} \qquad (8)$$

$$B_1\phi_{pp,kk}\delta_{ij} + 2B_2\,\phi_{(ij),kk} - A_4\,\phi_{pp}\delta_{ij} - 2A_5\phi_{(ij)} + \rho f_{(ij)} = \frac{1}{2}\rho j\,\frac{\partial^2\phi_{(ij)}}{\partial t^2} \qquad (9)$$

$$2B_3\phi_{p,mm} + 2(B_4 + B_5)\,\phi_{m,mp} - 4A_3(r_p + \phi_p) - \rho l_p = \rho j\,\frac{\partial^2\phi_p}{\partial t^2} \qquad (10)$$

Where comma denotes partial derivative with respect to space variable (x_k) and repeated indices indicate summation.

Solution of the problem

Consider two micromorphic halfspaces with different mechanical properties perfectly welded along the x-axis (Fig. 1)

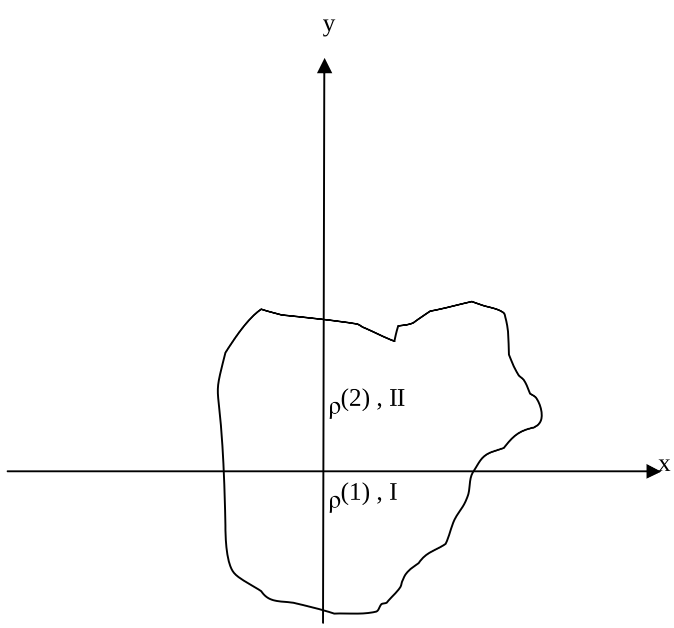

Fig.1.

Since we are considering time harmonic waves the macro displacement, micro rotation and macro-displacement are functions of x, y, t for both the media and are given by

$$u^{(1)} = v^{(1)} = 0, \qquad w^{(1)} = w^{(1)}(x, y, t)$$

$$\phi_1^{(1)} = \phi_1^{(1)}(x, y, t),\ \ \phi_2^{(1)} = \phi_2^{(1)}(x, y, t),\ \ \phi_3^{(1)} = 0$$

$$\phi_{(31)}^{(1)} = \phi_{(31)}^{(1)}(x,y,t), \quad \phi_{(32)}^{(1)} = \phi_{(32)}^{(1)}(x,y,t)$$

$$\phi_{33}^{(1)} = \phi_{33}^{(1)}(x,y,t), \varnothing_{11}^{1} = \varnothing_{(12)}^{(1)} = \phi_{(12)}^{(1)} = 0$$

for y < 0 and

$$u^{(2)} = v^{(2)} = 0, \; w^{(2)} = w^{(2)}(x,y,t) \tag{12}$$

$$\phi_1^{(2)} = \phi_1^{(2)}(x,y,t); \; \phi_2^{(2)} = \phi_2^{(2)}(x,y,t), \; \phi_3^{(2)} = 0$$

$$\phi_{(31)}^{(2)} = \phi_{(31)}^{(2)}(x,y,t), \; \phi_{(32)}^{(2)} = \phi_{(32)}^{(2)}(x,y,t)$$

$$\phi_{33}^{(2)} = \phi_{33}^{(2)}(x,y,t), \; \phi_{11}^{(2)} = \phi_{22}^{(2)} = \phi_{(12)}^{(2)} = 0$$

for y>0. Quantities with super suffix with (1) are corresponding to the medium I (y<0) and that of super suffix (2) corresponding to the medium II (y>0).

Under the absence of body forces and body couples the field equations (8) to (10) are reduce to

$$\left(A_2^{(i)} + A_3^{(i)}\right)\left(\frac{\partial^2 w^{(i)}}{\partial x^2} + \frac{\partial^2 w^{(i)}}{\partial y^2}\right) + 2A_3^{(i)}\left(\frac{\partial \phi_1^{(i)}}{\partial y} - \frac{\partial \phi_2^{(i)}}{\partial x}\right) = \rho^{(i)}\frac{\partial^2 w^{(i)}}{\partial t^2} \tag{13}$$

$$2B_3^{(i)}\left[\frac{\partial^2 \phi_1^{(i)}}{\partial x^2} + \frac{\partial^2 \phi_1^{(i)}}{\partial y^2}\right] + 2\left(B_4^{(i)} + B_5^{(i)}\right)\left[\frac{\partial^2 \phi_1^{(i)}}{\partial x^2} + \frac{\partial^2 \phi_2^{(i)}}{\partial x \partial y}\right]$$

$$-4A_3^{(i)}\left[\frac{1}{2}\frac{\partial w^{(i)}}{\partial y} + \phi_1^{(i)}\right] = \rho^{(i)} j^{(i)} \frac{\partial^2 \phi_1^{(i)}}{\partial t^2} \tag{14}$$

$$2B_3^{(i)}\left[\frac{\partial^2 \phi_2^{(i)}}{\partial x^2} + \frac{\partial^2 \phi_2^{(i)}}{\partial y^2}\right] + 2\left(B_4^{(i)} + B_5^{(i)}\right)\left[\frac{\partial^2 \phi_1^{(i)}}{\partial x \partial y} + \frac{\partial^2 \phi_2^{(i)}}{\partial y^2}\right]$$

$$-4A_3^{(i)}\left[\frac{-1}{2}\frac{\partial w^{(i)}}{\partial x} + \phi_2^{(i)}\right] = \rho^{(i)} j^{(0)} \frac{\partial^2 \phi_2^{(i)}}{\partial t^2} \tag{15}$$

$$B_1^{(i)}\left[\frac{\partial^2}{\partial x^2}\phi_{33}^{(i)} + \frac{\partial^2 \phi_{33}^{(i)}}{\partial y^2}\right] - A_4^{(i)}\phi_{33}^{(i)} = 0 \tag{16}$$

$$B_1^{(i)}\left[\frac{\partial^2}{\partial x^2}\phi_{33}^{(i)} + \frac{\partial^2}{\partial y^2}\phi_{33}^{(i)}\right] + 2B_2^{(i)}\left[\frac{\partial^2}{\partial x^2}\phi_{33}^{(i)} + \frac{\partial^2}{\partial y^2}\phi_{33}^{(i)}\right] - A_4\phi_{33}^{(i)} - 2A_5\phi_{33}^{(i)} = \frac{1}{2}\rho^{(i)} j^{(i)} \frac{\partial^2 \phi_{33}^{(i)}}{\partial t^2} \tag{17}$$

$$2B_2^{(i)}\left[\frac{\partial^2}{\partial x^2}\phi_{(32)}^{(i)}+\frac{\partial^2}{\partial y^2}\phi_{(32)}^{(i)}\right]-2A_5\phi_{32}^{(i)}=\frac{1}{2}\rho^{(i)}j^{(i)}\frac{\partial^2\phi_{32}^{(i)}}{\partial t^2} \tag{18}$$

$$2B_2^{(i)}\left[\frac{\partial^2}{\partial x^2}\phi_{(31)}^{(i)}+\frac{\partial^2}{\partial y^2}\phi_{(31)}^{(i)}\right]-2A_5^{(i)}\phi_{(31)}^{(i)}=\frac{1}{2}\rho^{(i)}j^{(i)}\frac{\partial^2\phi_{31}^{(i)}}{\partial t^2} \tag{19}$$

In view of (16) the equation (17) reduces to

$$2B_2\left[\frac{\partial^2}{\partial x^2}\phi_{33}^{(i)}+\frac{\partial^2}{\partial y^2}\phi_{33}^{(i)}\right]-2A_5\phi_{33}^{(i)}=\frac{1}{2}\rho^{(i)}j^{(i)}\frac{\partial^2}{\partial t^2}\phi_{33}^{(i)} \tag{20}$$

where i = 1 ,2. The field equations for the medium I corresponds to i =1 and for medium II corresponds to i=2.

The equations (13) to (15) are coupled equations and coupled in terms of $w^{(i)}, \phi_1^{(i)}$ and $\phi_2^{(i)}$.

As a trial solution, let us assume

$$w^{(i)}=A^{(i)}\exp\left(m^{(i)}y\right)\exp\left[iq\left(y-ct\right)\right]$$

$$\phi_1^{(i)}=B^{(i)}\exp\left(m^{(i)}y\right)\exp\left[iq\left(y-ct\right)\right] \tag{21}$$

$$\phi_2^{(i)}=C^{(i)}\exp\left(m^{(i)}y\right)\exp\left[iq\left(y-ct\right)\right]$$

where $A^{(i)}$, $B^{(i)}$, $C^{(i)}$ are constants, q is the wave number and c is phase velocity.
Substituting equations (21) in the equations (13) to (15) we get

$$\left[\left(A_2^{(i)}+A_3^{(i)}\right)\left(m^{(i)^2}-q^2\right)+\rho^{(i)}c^2q^2\right]A^{(i)}+2A_3^{(i)}m^{(i)}B^{(i)}-2A_3^{(i)}iqc=0 \tag{22}$$

$$-2A_3^{(i)}m^{(i)}A^{(i)}+\left[2B_3^{(i)}\left(m^{(i)^2}-q^2\right)-2\left(B_4^{(i)}+B_5^{(i)}\right)q^2-4A_3^{(i)}+\rho^{(i)}j^{(i)}q^2c^2\right]B^{(i)}$$

$$+\left[2\left(B_4^{(i)}+B_5^{(i)}\right)m^{(i)}iq\right]C^{(i)}=0 \tag{23}$$

$$\left(2A_3^{(i)}iq\right)A^{(i)}+\left[2\left(B_4^{(i)}+B_5^{(i)}\right)iqm^{(i)}\right]B^{(i)}+\left[2B_3^{(i)}\left(m^{(i)^2}-q^2\right)+2\left(B_4^{(i)}+B_5^{(i)}\right)m^{(i)^2}\right.$$

$$-4A_3^{(i)}+\rho^{(i)}j^{(i)}q^2\Big]C^{(i)}=0 \tag{24}$$

For i=1 we have a system of homogenous equations in $A^{(1)}$, $B^{(1)}$ and $C^{(1)}$. Similarly for i = 2 we have another system of homogeneous equations in $A^{(2)}$, $B^{(2)}$ and $C^{(2)}$. For the existence of non – trivial solution of the system of equations in $A^{(1)}$, $B^{(1)}$ and $C^{(1)}$, the determinant of the coefficient matrix should be zero. (i.e.) (25).

$$|a_{ij}|=0 \tag{25}$$

where

$$a_{11}=\left(A_2^{(1)}+A_3^{(1)}\right)\left(m^{(1)^2}-q^2\right)+\rho^{(1)}c^2q^2$$

$$a_{12}=2A_3^{(1)}m^{(1)}$$

$$a_{13}=-2A_3^{(1)}iq$$

$$a_{21}=-2A_3^{(1)}m^{(1)}$$

$$a_{22}=2B_3^{(1)}\left(m^{(1)^2}-q^2\right)-2\left(B_4^{(1)}+B_5^{(1)}\right)q^2-4A_3^{(1)}+\rho^{(1)}j^{(i)}C^2q^2 \tag{26}$$

$$a_{23}=2\left(B_4^{(1)}+B_5^{(1)}\right)m^{(1)}iq \quad a_{31}=2A_3^{(1)}iq$$

$$a_{32}=2\left(B_4^{(1)}+B_5^{(1)}\right)iqm^{(1)}$$

$$a_{33}=2B_3^{(1)}\left(m^{(1)^2}-q^2\right)+2\left(B_4^{(1)}+B_5^{(1)}\right)m^{(1)^2}-4A_3^{(1)}+\rho^{(1)}j^{(1)}q^2c^2$$

Expanding the determinant we get

$$\left[j^{(1)}\left(\theta^{(1)}+\delta^{(1)}\right)\left(m^{(1)^2}-q^2\right)+j^{(1)}q^{(2)}\xi^{(1)}-2\in^{(1)}\right].$$

$$\left[\theta^{(1)}j^{(1)}\left(m^{(1)^2}-q^2\right)-2\in^{(1)}+\rho^{(1)}j^{(1)}\xi^{(1)}q^2\right].$$

$$\left[\frac{C_1^{(1)^2}-C_1^{(1)^2}}{C_6^{(1)^2}}m^{(1)^2}-\frac{C_1^{(1)^2}-C_2^{(1)^2}-c^2}{C_6^2}q^2\right]+\in^{(1)^2}\left(m^{(1)^2}-q^2\right)=0 \tag{27}$$

where

$$C_1^{(1)^2}=\frac{A_1^{(1)}+2A_2^{(1)}}{\rho^{(1)}}; \qquad C_2^{(1)^2}=\frac{A_1^{(1)}+A_2^{(1)}-A_3^{(1)}}{\rho^{(1)}}$$

$$C_3^{(1)^2} = \frac{2A_3^{(1)}}{\rho^{(1)}} \qquad C_4^{(1)^2} = \frac{2B_3^{(1)}}{\rho^{(1)} j^{(1)}}; \qquad C_5^{(1)^2} = \frac{2\left(B_4^{(1)} + B_5^{(1)}\right)}{\rho^{(1)} j^{(1)}}$$

$$C_6^{(1)^2} = \frac{A_2^{(1)}}{\rho^{(1)}}; \qquad \xi^{(1)} = \frac{C^2}{C_6^{(1)^2}}$$

$$\in^{(1)} = \frac{C_3^{(1)^2}}{C_6^{(1)^2}}; \qquad \theta^{(1)} = \frac{C_4^{(1)^2}}{C_6^{(1)^2}}, \qquad \delta^{(1)} = \frac{C_5^{(1)^2}}{C_6^{(1)^2}} \tag{28}$$

Neglecting $\in^{(1)^2}$ term in (6.1.17) we obtain a set of approximate roots for $m^{(1)^2}$ and we suppose these roots be b_1^2, b_2^2 and b_3^2. Thus,

$$b_1^2 = \frac{2\in^{(1)}}{j^{(1)} + \left(\theta^{(1)} + \delta^{(1)}\right)} + \left(1 - \frac{\xi^{(1)}}{\theta^{(1)} + \delta^{(1)}}\right)\theta^2 \tag{29}$$

$$b_2^2 = \left[1 - \xi^{(1)}\left(1 - \frac{\in^{(1)}}{2}\right)\right]q^2 \tag{30}$$

$$b_3^2 = \left[\frac{2\in^{(1)}}{j^{(i)}\theta^{(i)}} + \left(1 - \frac{\xi^{(1)}}{\delta^{(i)}}\right)\right]q^2 \tag{31}$$

We assume that b_1, b_2 and b_3 are positive. The general solutions for displacements and rotation functions in the lower half space (y<0) are of form

$$w^{(1)} = m^{(1)} L_2 \exp\left(-b_2 y\right)\exp\left[iq(x - ct)\right] \tag{32}$$

$$\phi_1^{(1)} = \sum_{k=1}^{3} \lambda_k L_k \exp\left(-b_k y\right)\exp\left[iq\left(x - ct\right)\right] \tag{33}$$

$$\phi_2^{(1)} = \sum_{k=1}^{3} L_k \exp\left(-b_k y\right)\exp\left[iq\left(x - ct\right)\right] \tag{34}$$

where L_1, L_2 and L_3 are constants,

$$M^{(1)} = \frac{j^{(i)} C u^{(1)^2} + C_5^{(1)^2}\left(b_2^2 - q^2\right) - 2C^2 + \rho^{(1)} j^{(i)} q^2 c^2}{-C_3^{(1)^2} iq} \tag{35}$$

and

$$\lambda_1 = \frac{q}{ib_1},\ \lambda_2 = \frac{b_2}{iq},\ \lambda_3 = \frac{b_3}{iq} \tag{36}$$

The system of equations in $A^{(2)}, B^{(2)}$ and $C^{(2)}$ are similar to the set equations in $A^{(1)}$, $B^{(1)}$ and $C^{(1)}$ except change in super suffix. Hence the solutions $w^{(2)}, \phi_1^{(2)}, \phi_2^{(2)}$ for the upper half space (y>0) are

$$w^{(2)} = M^{(2)} L_5 \exp\left(b_5 y\right) \exp\left[iq\left(x - ct\right)\right] \tag{37}$$

$$\phi_1^{(2)} = \sum_{k=4}^{6} \lambda_k\ L_k \exp\left(b_k y\right) \exp\left[iq\left(x - ct\right)\right] \tag{38}$$

$$\phi_2^{(2)} = \sum_{k=4}^{6} L_k \exp\left(b_k y\right) \exp\left[iq\left(x - ct\right)\right] \tag{39}$$

where L_4, L_5 and L_6 are constants; b_4^2, b_5^2, b_6^2 are roots of the determinant of the coefficients of the equations in $A^{(2)}$, $B^{(2)}$, $C^{(2)}$ by neglecting $\in^{(2)^2}$,

$$M_2 = \frac{j^{(2)} C_4^{(2)^2} + C_5^{(2)^2}\left(b_5^2 - q^2\right) - 2c^2 + \rho^{(2)} j^{(2)} q^2 c^2}{-C_3^{(2)^2}\, iq} \tag{40}$$

$$\lambda_4 = \frac{-q}{ib_4},\qquad \lambda_5 = \frac{-b_5}{iq},\qquad \lambda_6 = \frac{-b_6}{iq} \tag{41}$$

and b_4^2, b_5^2, b_6^2 are respectively equal to the right hand sides of the equations (29) - (31) replacing the super suffix 1 by 2. On the stress free boundary surface the required boundary conditions are

$$w^{(1)} = w^{(2)}$$

$$\phi_1^{(1)} = \phi_1^{(2)}, \qquad \phi_2^{(1)} = \phi_2^{(2)}$$

$$t_{23}^{(1)} = t_{23}^{(2)}, \qquad m_{21}^{(1)} = m_{21}^{(2)}$$

These boundary conditions involves the macro-displacement and micro-rotations. Substituting equations (32) to (34) and (37) to (39) in equations (42) we get

$$M^{(1)} L_2 - M^{(2)} L_5 = 0 \tag{43}$$

$$\lambda_1 L_{14} + \lambda_2 L_2 + \lambda_3 L_3 - \lambda_4 L_4 - \lambda_5 L_5 - \lambda_6 L_6 = 0 \tag{44}$$

$$L_1 + L_2 + L_3 - L_4 - L_5 - L_6 = 0 \tag{45}$$

$$A_2^{(1)} b_2 m^{(1)} L_2 + A_2^{(2)} b_5 m^{(2)} L_5 = 0 \tag{46}$$

$$\left[B_3^{(1)}iq - B_4^{(1)}\lambda_1 b_1\right]L_1 + \left[B_3^{(1)}iq - B_4^{(1)}\lambda_2 b_2\right]L_2$$

$$+\left[B_3^{(1)}iq - B_4^{(1)}\lambda_3 b_3\right]L_3 - \left[B_3^{(2)}iq - B_4^{(2)}\lambda_4 b_4\right]L_4$$

$$-\left[B_3^{(2)}iq + B_4^{(2)}\lambda_5 b_5\right]L_5 - \left[B_3^{(2)}iq + B_4^{(2)}\lambda_6 b_6\right]L_6 = 0 \qquad (47)$$

$$\left[\left(B_3^{(1)} + B_4^{(1)} + B_5^{(1)}\right) b_1 - B_5^{(1)}\lambda_1 iq\right] L_1 + \left[B_3^{(1)} + B_4^{(1)} + B_5^{(1)} \; b_2 - B_5^{(1)}\lambda_2 iq\right] L_2$$

$$+\left[\left(B_3^{(1)} + B_4^{(1)} + B_5^{(1)}\right) b_3 - B_5^{(1)}\lambda_3 iq\right] L_3 + \left[B_3^{(2)} + B_4^{(2)} + B_5^{(2)} \; b_4 - B_5^{(2)}\lambda_4 iq\right] L_4$$

$$+\left[\left(B_3^{(2)} + B_4^{(2)} + B_5^{(2)}\right) b_5 + B_5^{(2)}\lambda_5 \; iq\right] L_5$$

$$+\left[B_3^{(2)} + B_4^{(2)} + B_5^{(2)} \; b_6 + B_5^{(2)}\lambda_6 \; iq\right] L_6 = 0 \qquad (48)$$

A non-vanishing solution of the above system of equations for L_1, L_2, L_3, L_4, L_5 and L_6 exists if an only if the determinant of the coefficients is zero i.e.,

$$\left|b_{ij}\right| = 0 \qquad (49)$$

where

$b_{11} = 0$; $b_{12} = M^{(1)}$; $b_{13} = 0$; $b_{14} = 0$

$b_{15} = -M^{(2)}$; $b_{16} = 0$;

$b_{21} = \lambda_1$; $b_{22} = \lambda_2$; $b_{23} = \lambda_3$;

$b_{24} = -\lambda_4$; $b_{25} = -\lambda_5$; $b_{26} = -\lambda_6$;
$b_{31} = 1$; $b_{32} = 1$; $b_{33} = 1$;

$b_{34} = -1$; $b_{35} = -1$; $b_{36} = -1$;

$b_{41} = 0$; $b_{42} = A_2^{(1)} b_2 M^{(1)}$; $b_{43} = 0$;

$b_{44} = 0$; $b_{45} \equiv A_2^{(2)} b_5 M^{(2)}$; $b_{46} = 0$;

$$b_{51} = B_3^{(1)} iq - B_4^{(1)} \lambda_1 b_1$$

$$b_{52} = B_3^{(1)} iq - B_4^{(1)} \lambda_2 b_2$$

$$b_{53} = B_3^{(1)} iq - B_4^{(1)} \lambda_3 b_3$$

$$b_{54} = -\left[B_3^{(2)} iq + B_4^{(2)} b_4 \lambda_4\right]$$

$$b_{55} = -\left[B_3^{(2)} iq + B_4^{(2)} \lambda_5 b_5\right]$$

$$b_{56} = -\left[B_3^{(2)} iq + B_4^{(2)} \lambda_6 b_6\right]$$

$$b_{61} = \left(B_3^{(1)} + B_4^{(1)} + B_5^{(1)}\right) b_1 - B_5^{(1)} \lambda_1 iq$$

$$b_{62} = \left(B_3^{(1)} + B_4^{(1)} + B_5^{(1)}\right) b_2 - B_5^{(1)} \lambda_2 iq$$

$$b_{63} = \left(B_3^{(1)} + B_4^{(1)} + B_5^{(1)}\right) b_3 + B_5^{(1)} \lambda_3 iq$$

$$b_{64} = \left(B_3^{(2)} + B_4^{(2)} + B_5^{(2)}\right) b_4 + B_5^{(2)} \lambda_4 iq$$

$$b_{65} = \left(B_3^{(2)} + B_4^{(2)} + B_5^{(2)}\right) b_5 + B_5^{(2)} \lambda_5 iq$$

$$b_{66} = \left(B_3^{(2)} + B_4^{(2)} + B_5^{(2)}\right) b_6 + B_5^{(2)} \lambda_6 iq$$

The determinant (49) can be expressed as two factors, hence each factor is equal to zero. Thus we have,

$$M^{(1)} M^{(2)} \left(A_2^{(2)} b_5 + A_2^{(1)} b_2\right) = 0 \qquad (50)$$

and

$$\left|c_{ij}\right| = 0 \;\; (i, j = 1,2,3,4) \qquad (51)$$

where

$c_{11} = \lambda_1$; $c_{12} = \lambda_3$; $c_{13} = -\lambda_4$; $c_{14} = -\lambda_6$

$c_{21} = 1$; $c_{22} = 1$; $c_{23} = -1$; $c_{24} = -1$

$c_{31} = b_{51}$; $c_{32} = b_{53}$; $c_{33} = b_{54}$; $c_{34} = b_{56}$

$c_{41} = b_{61}$; $c_{42} = b_{63}$; $c_{43} = b_{64}$; $c_{44} = b_{66}$

It is interesting to note that the equation (50) gives two additional waves (depends only on micromorphic constants) not encountered in classical elasticity.

As the equation (51) yield an equation in complex form a further discussion on it not initiated.

Now we study the effect of micro-strains in the present problem.

We seek the solution of (18) to (20) as

$$\phi^{(1)}_{(31)} = N_1 \exp\left(-\ell^{(1)} y\right) \exp\left[iq(y-ct)\right]$$

$$\phi^{(1)}_{(32)} = N_2 \exp\left(-\ell^{(1)} y\right) \exp\left[iq(y-ct)\right] \tag{52}$$

$$\phi^{(1)}_{(33)} = N_3 \exp\left(-\ell^{(1)} y\right) \exp\left[iq(y-ct)\right]$$

and $\phi^{(2)}_{31} = N_4 \exp\left(-\ell^{(2)} y\right) \exp\left[iq(y-ct)\right]$

$$\phi^{(2)}_{32} = N_5 \exp\left(-\ell^{(2)} y\right) \exp\left[iq(y-ct)\right]$$

$$\phi^{(3)}_{33} = N_6 \exp\left(-\ell^{(2)} y\right) \exp\left[iq(y-ct)\right] \tag{53}$$

$$l^{(1)^2} = \frac{A_5^{(1)}}{B_2^{(1)}} + \left(1 - \frac{\rho^{(1)} j^{(1)} C^2}{4B_2^{(1)}}\right) q^2 \quad l^{(2)^2} = \frac{A_5^{(2)}}{B_2} + \left(1 - \frac{\rho^{(2)} j^{(2)} C^2}{4B_2^{(2)}}\right) q^2$$

and $N_i \ (i = 1,2,3,4,5,6)$ are constants. The boundary conditions to be satisfied involving the micro-strains are

$$\phi^{(1)}_{31} = \phi^{(2)}_{31}$$

$$\phi^{(1)}_{32} = \phi^{(2)}_{32} \qquad \phi^{(1)}_{33} = \phi^{(2)}_{33}$$

$$t^{(1)}_{2(32)} = t^{(2)}_{2(32)} \tag{54}$$

$$t^{(1)}_{2(33)} = t^{(2)}_{2(33)} \quad t^{(1)}_{2(31)} = t^{(2)}_{2(31)}$$

Substituting equations (6.1.44) in (6.1.48) we get

$N_1 = N_4$, $N_2 = N_5$ and $N_3 = N_6$

Further substituting (6.1.45) and (6.1.49) we obtain the frequency equation.

$$\frac{A_5^{(1)}}{B_2^{(1)}} + \left(1 - \frac{\rho^{(1)} j^{(1)} C^2}{4B_2^{(1)}}\right) q^2 - \frac{A_5^{(2)}}{B_2^{(2)}} - \left(1 - \frac{\rho^{(2)} j^{(2)} C^2}{4B_2^{(2)}}\right) q^2 = 0 \tag{55}$$

This wave depends only on micromorphic constants that is other than elastic constants λ, μ of classical elasticity.

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