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RELIABILITY ANALYSIS OF TRANSIT FUEL SYSTEM IN DIESEL ENGINE

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Abstract
The reliability analysis of a complex system consisting of two subsystems A and B connected in series has been discussed in the present study. Subsystem A is of M-cylinders which consists j-out of –M: G system (more than M - j + 1 cylinders must fail for A to fail), while subsystem B consists of N non-identical units connected in series (the failure of any one causes B to fail), also the system can fail due to critical human error from the normal efficiency state. By applying the supplementary variable technique and Laplace transformations various state probabilities either time dependent or at steady state along with reliability, availability and MTTF have been obtained. At the end, some special cases of the system have been considered.

Keywords: Mean Time to Failure (MTTF), Reliability, Availability, Up State Probabilities, Complex System, Supplementary Variable Technique.

1. INTRODUCTION

In the modern day systems, the most important issue is the reliability and maintainability of the system. User always requires a highly reliable system to use. An unreliable system has higher probability of failure as and when any item of the associated equipment is damaged or destroyed due to some or other cause. Consequently the unreliability of a system may result into the wastage of cost, time of personal as well as national security. Thus, a high degree of reliability is prime requirement in various practical systems, e.g. nuclear system, space system; defense combat system, power plants, fuel system etc. Introducing parallel redundancy and using service facility, as mentioned earlier, usually achieve this high degree of reliability. Usually, the reliability of a system can easily be improved with the redundancy. Sometimes additional paths are included in the system design so that the system continues to function. Redundancies can be classified as active, standby or partial. If n units are connected in such a way that all the units work simultaneously, it is called active redundancy. If an extra but identical component is attached to the system as standby to work at the place of the failed component, the redundancy is known to be standby redundancy. The redundancy where in two or more redundant units are required to perform the function of j-out of –M the system is called the partial redundant. Mostly such models are useful for electrical and mechanical systems.

Fuel system, discussed in the present study helps to transfer the fuel from the fuel tank to the engine and regulate its flow into the cylinder depending on speed and load requirement. Here, a multi-component fuel system in diesel engine, comprised of two subsystems A and B in series has been considered. In this the fuel is injected into the cylinder using an injection pump and an injector. The block diagram (fig. 1) depicts the fuel system in a M-cylinder diesel engine.

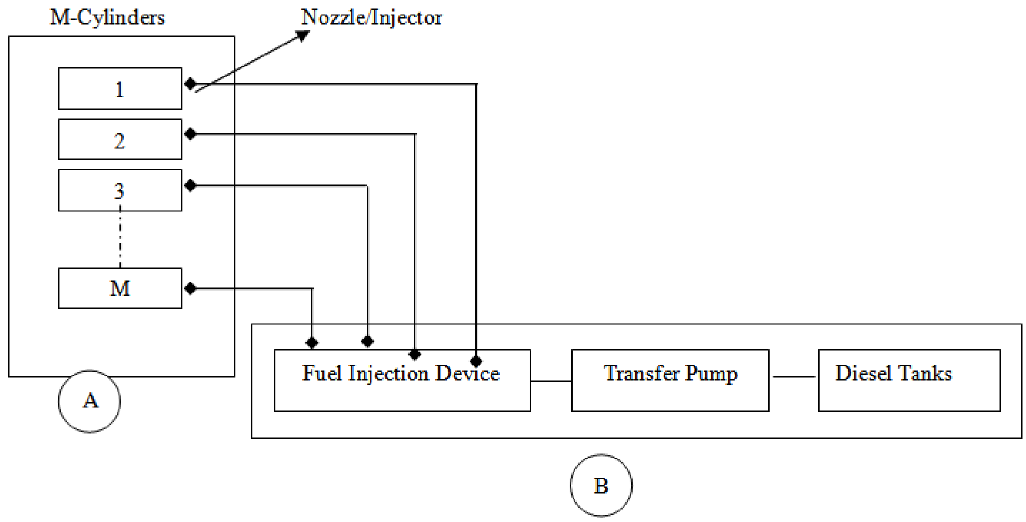

Figure 1: Block diagram of fuel system in a diesel engine

The diesel fuel from the tank is pumped to the fuel injection device. This device consists of small plunger pumps operating to push the fuel through the injectors (nozzles) into the cylinders. A can operates one pump from the fuel injection pumps at the appropriate time i.e., towards the end of the compression stroke. The fuel is pushed through a small hole in the injector, and is atomized as it enters into the cylinder.

Now M-cylinders is j-out of –M: G system (more than M-j + 1 cylinders must fail for A to fail), while subsystem B consists of N non-identical units connected in series (the failure of any one causes B to fail), also the system can fail due to critical human error from the normal efficiency state.

The life times of the active units depend on each other in having simultaneous failure of all the operating units and repair times are distributed quite generally. A failed unit is repaired at a single service channel. Earlier researchers have studied the model for j = 1, 2; under the assumption that the repair of any unit is possible only when the system stops operating and the life times of the active units are mutually independent, and have studied the model for j = 1, 2; with exponential failure distribution under the different repair policies. In the analysis, the analytical behavior under the different repair policies. In the analysis, the analytical behavior of a complex system with parallel redundancy under head of line (first come first served) repair incorporating the concept of reduced efficiency in the class of redundantly connected components and also the concept of critical human error has been examined; for example, a heavy loaded consisting of sixteen wheels connected in parallel redundancy goes to a reduced efficiency state if four wheels (out of sixteen) get burst. Further, though a multi-engine air-craft may continue to fly even after the loss of two engines, yet it will not achieve its usual performance.

The availability and reliability function of the system are obtained simultaneously, using supplementary-variable technique. Explicit expressions for the steady state availability of the system and the mean time to the first system failure are obtained.

2. ASSUMPTIONS

1. Initially all the units are operating.
2. Each unit results in failure after receiving only one ‘shock’, and failure rates are exponential.
3. The system breaks down if more than M - j + 1 unit in subsystem A are simultaneously in a failed state or if any failure in subsystem B occurs or critical human error occurs.
4. A failed unit is repaired at a single service channel.
5. A repaired unit is put into operation immediately.
6. A repaired unit is assumed to behave like new after repair.
7. Repair times are distributed quite generally.
8. The system has three modes viz.; good, degraded and failed state.

9. When the system is in a failed state, the residual good units do not fail.
10. While the system is operating, whenever any unit in A fails, it is sent to the repair station at once where it is repaired on a first come first served basis.

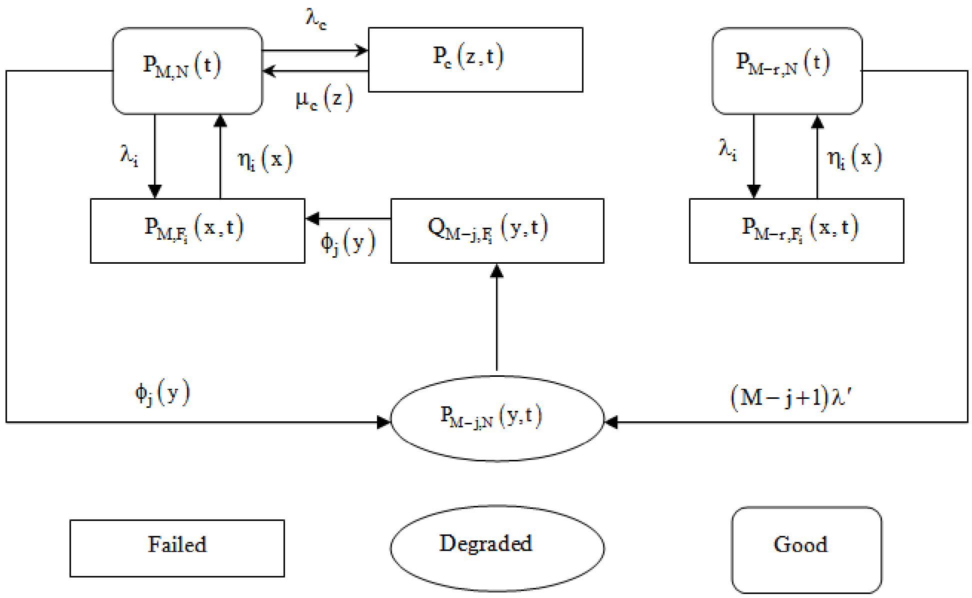

Figure 2: Transition state diagram

3. NOTATIONS

$$\frac{d}{dt} / \frac{\partial}{\partial x} / \frac{\partial}{\partial y} / \frac{\partial}{\partial z} / \frac{\partial}{dt}$$

$\int$ $\qquad$ $\int_0^\infty$ Otherwise stated

M, N: No. of units in subsystem A & B respectively.

$\lambda' / \lambda_i / \lambda_e$: Constant failure rates of any unit of A/ i th –B/ critical human error.

$\eta_i(x) / S_i(x)$: Transition repair rate of the subsystem B / pdf of repair rate.

$\phi_j(y) / S_j(y)$: Transition repair rate of the subsystem A / pdf of repair rate.

$\mu_c(z) / S_c(z)$: Transition repair rate for critical human error / pdf of repair rate.

$\overline{S}_r(s)$: Laplace transform of pdf of repair time $S_r(t)$.

$\overline{f}(s)$: Laplace transform of f(t).

$P_{M,N}(t)$: The prob. that at time ‘t’, the system is operating in the state of normal efficiency.

$P_{M-r,N}(t)$: The prob. that at time ‘t’, the system is in operable state where ‘r’ components of subsystem A have already failed.

$P_{M-j,N}(y,t)\Delta$: The prob. that at time ‘t’, the system is operating in reduced efficiency due to the failure of j components of subsystem A, the elapsed repair time lies in the interval $(y, y+\Delta)$.

$P_{M,F_i}(x,t)\Delta$: The prob. that at time‘t’, the system is in failed state due to the failure of ith-B and all the M units of A are in operable state and the elapsed repair time lies in the interval $(x, x+\Delta)$.

$P_{M-r,F_i}(x,t)\Delta$: The prob. that at time't', the system is in failed state due to the failure of ith-B and M-r units of A are in operable and the elapsed repair time for B lies in the interval $(x, x+\Delta)$, conditioned that no repair is carried out for A.

$P_c(z,t)\Delta$: The prob. that at time't', the system is in failed state due to critical human error and elapsed repair time for this lies in the interval $(z, z+\Delta)$.

$Q_{M-j,F_i}(y,t)\Delta$: The prob. that at time 't', the system is in the failed state due to the failure of ith-B and any j units of A, the elapsed repair time of any j units of subsystem A lies in the interval $(y, y+\Delta)$, whereas subsystem B is awaiting for repair. $1 \le r \le j-1$

$$\lambda = \sum_{i=1}^{N} \lambda_i$$

4. FORMULATION

The analysis crucially depends on the method of supplementary variables, and the supplementary variable x denotes the elapsed time that a unit has been undergoing repair. Viewing the nature of this problem, the following set of difference-differential equations is obtained, and is as follows:

$$(D+\lambda+M\lambda'+\lambda_c)P_{M,N}(t) = \sum_i \int P_{M,F_i}(x,t)\eta_i(x)d_x$$

$$+\int P_c(z,t)\mu_c(z)dz + \int P_{M-j,N}(y,t)\phi_j(y)dy \quad (1)$$

$$\left(D+\lambda+\overline{M-r}\lambda'\right)P_{M-r,N}(t) = (M-r+1)\lambda' P_{M-r+1,N}(t) + \sum_i \int P_{M-r,F_i}(x,t)\eta_i(x)d_x \quad (2)$$

$$\left(D_t + D_y + \lambda + \phi_j(y)\right)P_{M-j,N}(y,t) = 0 \quad (3)$$

$$\left(D_x + D_t + \eta_i(x)\right)P_{M,F_i}(x,t) = 0 \quad (4)$$

$$\left(D_x + D_t + \eta_i(x)\right)P_{M-r,F_i}(x,t) = 0 \quad (5)$$

$$\left(D_y + D_t + \phi_j(y)\right)Q_{M-j,F_i}(y,t) = \lambda_i P_{M-j,N}(y,t) \quad (6)$$

$$\left(D_z + D_t + \mu_c(z)\right)P_c(z,t) = 0 \quad (7)$$

These equations are to be solved under the following boundary and initial conditions:

5. BOUNDARY AND INITIAL CONDITIONS

$$P_{M,F_i}(0,t) = \lambda_i P_{M,N}(t) + \int Q_{M-j,F_i}(y,t)\phi_j(y)dy \quad (8)$$

$$P_{M-r,F_i}(0,t) = \lambda_i P_{M-r,N}(t) \quad (9)$$

$$P_{M-j,N}(0,t) = (M-j+1)\lambda' P_{M-j+1,N}(t) \quad (10)$$

$$Q_{M-j,F_i}(0,t) = 0 \quad (11)$$

$$P_C(0,t) = \lambda_C P_{M,N}(t) \quad (12)$$

$$P_k(0) = \begin{cases} 1, \textit{when}\ \mathrm{k = M, N} \\ \ \ 0, \text{otherwise} \end{cases} \quad (13)$$

Taking Laplace transform of the equations from 1 to 12 and using relevant conditions, the following solutions are obtained:

6. SOLUTIONS:

$$\overline{P}_{M,N}(s)=\frac{1}{A(s)} \tag{14}$$

$$\overline{P}_{M-r,N}(s)=\frac{(\lambda')^r}{A(s)}\prod_{p=0}^{r-1}B(s,p+1) \tag{15}$$

$$\overline{P}_R(s)=\overline{P}_{M-j,N}(s)=\frac{(M-j+1)(\lambda')^j}{A(s)}.\frac{1-\overline{S}_j(s+\lambda)}{(s+\lambda)}.\prod_{p=0}^{j-1}B(s,k) \tag{16}$$

$$\overline{P}_{M,F_i}(s)=\frac{\lambda_i}{A(s)}.\frac{1-\overline{S}_i(s)}{(s)}.\left[1+\frac{M-j+1}{\lambda}(\lambda')^j\left\{\overline{S}_j(s)-\overline{S}_j(s+\lambda)\right\}\prod_{p=0}^{j-1}B(s,k)\right] \tag{17}$$

$$\overline{P}_{M-r,F_i}(s)=\frac{\lambda_i.(\lambda')^r}{A(s)}\frac{1-\overline{S}_i(s)}{(s)}\prod_{p=0}^{r-1}B(s,p+1) \tag{18}$$

$$\overline{Q}_{M-j,F_i}(s)=\frac{\lambda_i(M-j+1)(\lambda')^j}{\lambda.A(s)}.\left[\frac{1-\overline{S}_j(s)}{(s)}-\frac{1-\overline{S}_j(s+\lambda)}{(s+\lambda)}\right].\prod_{p=0}^{j-1}B(s,k) \tag{19}$$

$$\overline{P}_c(s)=\frac{\lambda_c}{A(s)}\cdot\frac{1-\overline{S}_c(s)}{s} \tag{20}$$

Where,

$$B(s,k)=\frac{M-k+1}{s+\lambda+M-k\lambda'-\sum_i\lambda_i\overline{S}_i(s)}$$

And

$$A(s)=s+\lambda+M\lambda'+\lambda_c-\sum_i\lambda_i\overline{S}_i(s)-\lambda_c\overline{S}_c(s)-(\lambda')^j(M-j+1)\prod_{k=1}^{j-1}B(s,k),$$

$$\left[\overline{S}_j(s+\lambda)+\frac{1}{\lambda}\sum_i\lambda_i\overline{S}_i(s)\left\{\overline{S}_j(s)-\overline{S}_j(s+\lambda)\right\}\right]$$

7. EVALUATION OF L.T. OF UP AND DOWN STATE PROBABILITIES

The Laplace transforms of the prob. that the system is in up (i.e. either good or degraded) and down (i.e. failed) state at time‘t’ are as follows:

$$\overline{P}_{up}(s)=\overline{P}_{M,N}(s)+\sum_{r=1}^{j-1}\overline{P}_{M-r,N}(s)+\overline{P}_R(s)$$

$$=\frac{1}{A(s)}\left[1+\sum_{r=1}^{j-1}(\lambda')^r\prod_{p=0}^{r-1}B(s,p+1)+(M-j+1)(\lambda')^j\cdot\frac{1-\overline{S}_j(s+\lambda)}{s+\lambda}\prod_{k=1}^{j-1}B(s,k)\right] \tag{21}$$

$$\overline{P}_{down}(s)=\overline{P}_C(s)+\sum_i\left[\overline{P}_{M,F_i}(s)+\sum_{r=1}^{j-1}\overline{P}_{M-r,F_i(s)}+\overline{Q}_{M-r,F_i}(s)\right] \tag{22}$$

It would be worth noticing that

$$\overline{P}_{up}(s)+\overline{P}_{down}(s)=\frac{1}{s} \tag{23}$$

8. SPECIAL CASES

(A) Assuming that the failure of any two units out of M causes the system to work in reduced efficiency state:

Setting j = 2 in equations (14) to (20), one gets

$$\overline{P}_{M,N}(s)=\frac{1}{C(s)}\left\{s+\lambda+\overline{M-1}\lambda'-\sum_{i=1}^{N}\lambda_i\overline{S_i}(s)\right\} \tag{24}$$

$$\overline{P}_{M-1,N}(s)=\frac{M\lambda'}{C(s)} \tag{25}$$

$$\overline{P}_{M-2,N}(s)=\frac{M(M-1)\lambda'^2}{C(s)}\cdot\frac{1-\overline{S_2}(s+\lambda)}{s+\lambda} \tag{26}$$

$$\overline{P}_{M,F_i}(s)=\frac{\lambda_i}{C(s)}\cdot\frac{1-\overline{S_i}(s)}{s}\left[\left\{s+\lambda+\overline{M-1}\lambda'-\sum_i\lambda_i\overline{S_i}(s)\right\}+\frac{M(M-1)}{\lambda}(\lambda')^2\left\{\overline{S_2}(s)-\overline{S_2}(s+\lambda)\right\}\right] \tag{27}$$

$$\overline{P}_{M-1,F_i}(s)=\frac{M\lambda'\lambda_i}{C(s)}\cdot\frac{1-\overline{S_i}(s)}{s} \tag{28}$$

$$\overline{Q}_{M-2,F_i}(s)=\frac{M(M-1)\lambda'^2\lambda_i}{\lambda C(s)}\cdot\left\{\frac{1-\overline{S_2}(s)}{s}-\frac{1-\overline{S_2}(s+\lambda)}{s+\lambda}\right\} \tag{29}$$

$$\overline{P_c}(s)=\frac{\lambda_c}{C(s)}\left\{s+\lambda+\overline{M-1}\lambda'-\sum_i\lambda_i\overline{S_i}(s)\right\}\frac{1-\overline{S}_c(s)}{s} \tag{30}$$

Where

$$C(s)=\left\{s+\lambda+M\lambda'+\lambda_c-\sum_i\lambda_i\overline{S_i}(s)-\lambda_c\overline{S_c}(s)\right\}\left\{s+\lambda+\overline{M-1}\lambda'-\sum_i\lambda_i\overline{S_i}(s)\right\}-M(M-1)\lambda^2\left[\overline{S_2}(s+\lambda)+\sum_{i=1}^{N}\frac{\lambda_i}{\lambda}\left\{\overline{S_2}(s)-\overline{S_2}(s+\lambda)\right\}\right]$$

(B) Repair follows exponential time distribution:

Setting $\overline{S_i}(s)=\frac{\eta_i}{s+\eta_i},\overline{S_j}(s)=\frac{\phi_j}{s+\phi_j},\overline{S_c}(s)=\frac{\mu_c}{s+\mu_c}$

in equations (14) to (21), one obtains;

$$\overline{P}_{M,N}(s)=\frac{1}{D(s)} \tag{31}$$

$$\overline{P}_{M-r,N}(s)=\frac{(\lambda')^r}{D(s)}\prod_{p=0}^{r-1}E(s,p+1) \tag{32}$$

$$\overline{P}_R(s)=\frac{(\lambda')^j(M-j+1)}{(s+\lambda+\phi_j)D(s)}\prod_{k=0}^{j-1}E(s,k) \tag{33}$$

$$\overline{P}_{M,F_i}(s)=\frac{\lambda_i}{(s+\eta_i)D(s)}\left[1+(M-j+1)(\lambda')^j\prod_{k=1}^{j-1}\frac{E(s,k)\phi_j}{(s+\phi_j)(s+\lambda+\phi_j)}\right] \tag{34}$$

$$\overline{P}_{M-r,F_i}(s)=\frac{\lambda_i(\lambda')^r}{(s+\eta_i)D(s)}\prod_{p=0}^{r-1}E(s,p+1) \tag{35}$$

$$\overline{Q}_{M-j,F_i}(s)=\frac{\lambda_i(\lambda')^j(M-j+1)}{(s+\phi_j)(s+\lambda+\phi_j)D(s)}\prod_{k=1}^{j-1}E(s,k) \tag{36}$$

$$\overline{P}_c(s)=\frac{\lambda_c}{D(s)(s+\mu_c)} \tag{37}$$

$$\overline{P}_{up}(s)=\frac{1}{D(s)}\left[1+\sum_{r=1}^{j-1}(\lambda')\prod_{p=0}^{r-1}E(s,p+1)+\frac{(\lambda')^j(M-j+1)}{s+\lambda+\phi_j}\prod_{k=1}^{j-1}E(s,k)\right] \tag{38}$$

Where

$$E(s,k)=\frac{M-k+1}{s+\lambda+\overline{M-k}\lambda'-\sum_i\frac{\lambda_i\eta_i}{s+\eta_i}}$$

$$D(s)=s+\lambda+M\lambda'+\lambda_c-\sum_i\frac{\lambda_i\eta_i}{s+\eta_i}-\frac{\lambda_c\mu_c}{s+\mu_c}-\frac{(M-j+1)(\lambda')^j\phi_j}{s+\lambda+\phi_j}\prod_{k=1}^{j-1}E(s,k).\left\{1+\sum_i\frac{\lambda_i\eta_i}{(s+\eta_i)(s+\phi_j)}\right\}$$

RELIABILITY ANALYSIS

The Laplace transform of the reliability function when all repairs are zero of the system is given by;

$$\overline{R}(s)=\frac{1}{s+\lambda+M\lambda'+\lambda_c}\left[1-\frac{M\lambda'}{\lambda'+\lambda_c}+\frac{M(M-1)(\lambda')^2}{(M\lambda'+\lambda_c)(\lambda'+\lambda_c)}\right]+\frac{M\lambda'}{(M\lambda'+\lambda_c)(s+\lambda)} \tag{39}$$

MTTF ANALYSIS

Mean time to failure

$$\text{MTTF}=\frac{1}{\lambda+M\lambda'+\lambda_c}\left(1+\frac{M\lambda'}{\lambda}\right) \tag{40}$$

(C) Evaluation of time dependent probabilities when repair follows exponential time distribution and the failure of any two units out of M causes the system to work in reduced efficiency state:

Setting j = 2 in relation (31) through (38), we get

$$\overline{P}_{M,N}(s)=\frac{s+\lambda+\phi}{F(s)}\left\{s+\lambda+(M-1)\lambda'-\sum_i\frac{\lambda_i\eta_i}{s+\eta_i}\right\}$$

(41)

$$\overline{P}_{M-1,N}(s)=\frac{M\lambda'(s+\lambda+\phi)}{F(s)}$$

(42)

$$\overline{P}_{M-2,N}(s)=\frac{M(M-1)\lambda'^2}{F(s)}$$

(43)

$$\overline{P}_{M,F_i}(s)=\frac{\lambda_i}{F(s)(s+\eta_i)}\left\{(s+\lambda+\phi)\left(s+\lambda+(M-1)\lambda'-\sum_i\frac{\lambda_i\eta_i}{s+\eta_i}\right)+\frac{M(M-1)\phi(\lambda')^2}{s+\phi}\right\}$$

(44)

$$\overline{P}_{M-1,F_i}(s)=\frac{M\lambda'\lambda_i(s+\lambda+\phi)}{F(s)(s+\eta_i)}$$

(45)

$$\overline{Q}_{M-2,F_i}(s)=\frac{M(M-1)\lambda'^2\lambda'}{F(s)(s+\phi)}$$

(46)

$$\overline{P_c}(s)=\frac{\lambda_c}{F(s)}\left\{s+\lambda+(M-1)\lambda'-\sum_i\frac{\lambda_i\eta_i}{s+\eta_i}\right\}\frac{(s+\phi+\lambda)}{s+\mu_c}$$

(47)

and

$$\overline{P_{up}}(s)=\frac{(s+\lambda+\phi)}{F(s)}\left\{s+\lambda+(M-1)\lambda'-\sum_i\frac{\lambda_i\eta_i}{s+\eta_i}\right\}.$$

$$\left[1+\frac{M\lambda'}{s+\lambda+(M-1)\lambda'-\sum_i\frac{\lambda_i\eta_i}{s+\eta_i}}+\frac{M(M-1)(\lambda')^2}{(s+\lambda+\phi)\left(s+\lambda+(M-1)\lambda'-\sum_i\frac{\lambda_i\eta_i}{s+\eta_i}\right)}\right]$$

(48)

Where

$$F(s)=(s+\lambda+\phi)\left\{s+\lambda+M\lambda'+\lambda_c-\sum\frac{\lambda_i\eta_i}{s+\eta_i}-\frac{\lambda_c\mu_c}{s+\mu_c}\right\}\left\{s+\lambda+\overline{M-1}\lambda'-\sum\frac{\lambda_i\eta_i}{s+\eta_i}\right\}$$
$$-M(M-1)\lambda'^2\phi\left\{1+\sum_i\frac{\lambda_i\eta_i}{(s+\phi)(s+\eta_i)}\right\}$$

9. ASYMPTOTIC BEHAVIOR

Using Abel's lemma in L.T. viz.

$$\lim_{s\to 0} s\bar{f}(s) = \lim_{t\to\infty} f(t) = f(\text{say})$$

Provided the limit on the right hand side exists, the time independent probabilities are obtained as follows:

$$P_{M,N} = \frac{1}{A} \tag{49}$$

$$P_{M-r,N} = \frac{M}{(M-r)A} \tag{50}$$

$$P_R = P_{M-j,N} = \frac{M\lambda'}{A\lambda}\left\{1-\overline{S_j}(\lambda)\right\} \tag{51}$$

$$P_{M,F_i} = \frac{\lambda_i M_i}{A}\left\{1+\frac{M\lambda'}{\lambda}\left\{1-\overline{S_j}(\lambda)\right\}\right\} \tag{52}$$

$$P_{M-r,F_i} = \frac{\lambda_i M_i}{A}\cdot\frac{M}{M-r} \tag{53}$$

$$Q_{M-j,F_i} = \frac{\lambda_i M\lambda'}{A\lambda}\left\{M_j - \frac{1-\overline{S_j'}(\lambda)}{\lambda}\right\} \tag{54}$$

$$P_c = \frac{\lambda_c M_c}{A} \tag{55}$$

$$P_{up} = Avss = \frac{1}{A}\left[1+\sum_{r=1}^{j-1}\frac{M}{M-r}+\frac{M\lambda'}{\lambda}\left\{1-\overline{S_j'}(\lambda)\right\}\right] \tag{56}$$

Where Avss = point wise availability at $t=\infty$

$$A = 1+\sum_i \lambda_i M_i + \lambda_c M_c + MM_j\lambda' + \frac{M\lambda'}{\lambda}\left\{1-\overline{S_j'}(\lambda)\right\}\cdot\sum_i \lambda_i M_i$$
$$+\left(1-\sum_i \lambda_i M_i\right)\sum_{k=1}^{j-1}\frac{M}{M-k}$$

and $\qquad M_r = -\overline{S_r'}(0)$

10. NUMERICAL COMPUTATIONS

Substituting $M=4, j=2, \lambda=0.1, \lambda'=0.2, \lambda_c=0.1$.

Reliability Function: (for non-repairable system)

$$R(t) = \left\{1-\frac{M\lambda'}{\lambda'+\lambda_c}+\frac{M(M-1)(\lambda')^2}{(M\lambda'+\lambda_c)(\lambda'+\lambda_c)}\right\}e^{-(\lambda+M\lambda'+\lambda_c)t}+\frac{M\lambda'}{(M\lambda'+\lambda_c)}e^{-\lambda t} \tag{57}$$

$$R(t) = \left\{1-0.667M+\frac{4M(M-1)}{3(2M+1)}\right\}e^{-0.2(M+1)t}+\frac{2M}{2M+1}e^{-0.1t} \tag{58}$$

Pointwise Availability: for $\quad \phi=\mu_c=\eta=1$.

$$P_{up}(t) = 0.9161-0.173178e^{-1.04t}+e^{-1.381t}\left\{0.1970\cos(0.4643t)-0.7958\sin(0.4643t)\right\} \tag{59}$$

$$P_{down}(t) = 1-P_{up}(t)$$

Mean Time to Failure:

$$\text{MTTF} = \frac{1}{0.2 + M\lambda'}\{1 + 10M\lambda'\} \tag{60}$$

Steady State Availability: $\phi = \mu_c = \eta = 1.$

$$\text{Avss} = \frac{1 + \frac{M}{M-1} + \frac{M\lambda'}{\lambda}\left(1 + \frac{1}{(\lambda+1)^2}\right)}{1 - \lambda - \lambda_c - M\lambda' + M\lambda'\left(1 + \frac{1}{(\lambda+1)^2}\right) + \frac{M(1+\lambda)}{M-1}} \tag{61}$$

and also $\text{Avss} = \dfrac{1 + \frac{M}{M-1} + 18.2645M\lambda'}{0.8 + 0.82645 + M\lambda' + 1.1 + \frac{M}{M-1}}$ (62)

11. INTERPRETATION

The plots of $P_{up}(t)$ with respect to the time is shown in figure 3. The graph of MTTF varrying for different values of the number of components in subsystem A that is units in subsystem A are 2-out-of-4: G, 2-out-of-8: G, 2-out-of-20: G, 2-out-of-100-G and the failure rate is depicted in figure 4.

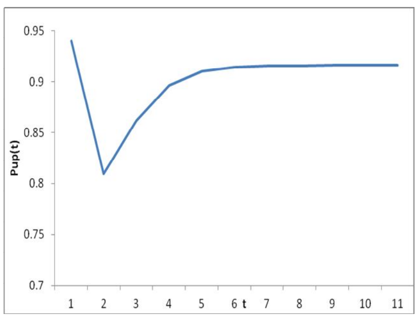

Figure 3: Probability of up state vs time

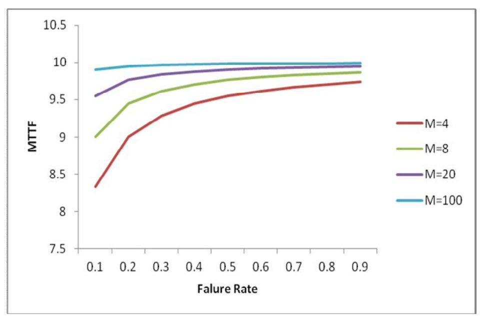

Figure 4: MTTF vs Failure rate for different M

The variation of the reliability function of the parallel transit system with time where units in the subsystem A are 2-out-of-4:G, 2-out-of-6:G, 2-out-of-8:G, 2-out-of-10:G is shown in figure 5. Figure 6 indicates the variation in availability in the steady state as given in equation (62) for different values of M and respect to the ith failure rate.

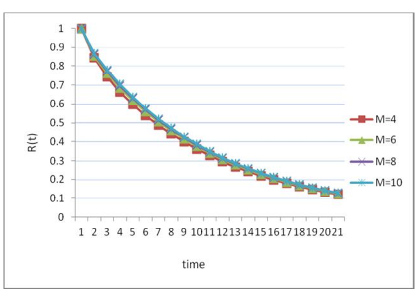

Figure 5: Relaibility vs time

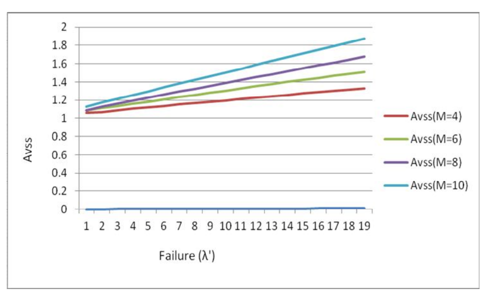

Figure 6: Avss vs Failure Rate

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