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IDENTIFICATION OF UNIT REPLACEMENT BASED ON LONG-RUN REPAIR AVERAGE COST RATE (ACR) BASED ON TRUNCATED EXPONENTIAL FAILURE MODEL

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Abstract

Earlier, authors presented [1] and published [2] investigated failure nature, repair pattern and reliabilities of hydro power generators, using simple statistical tools and simulation techniques respectively. It is identified that analysis of hydro power generators require a special approach by dividing the repair into two categories based on the repair duration. That is i) Minor repairs (Repair hours less than or equal a threshold value T) and ii) Major Repairs (Repair hours greater than a threshold value T).This approach is specially introduced by authors [1, 2] and obtained good fit of **"truncated exponential failure model".** Through this proposed model reliabilities are estimated and conclusions were drawn. In the above work, we have assumed that **the system after repair is 'as good as new'**. On the basic assumption that **the system after repair is not 'as good as new'** and also the successive working times are stochastically decreasing while, the successive repair time's are stochastically increasing and are exposing to exponential truncated failure law. Under these assumptions, an optimal replacement policy T in which we replace the system when the repair time (working time) reaches T. It can be determined that an optimal repair replacement policy T* such that the long run average cost per unit time is minimized. It can also be derived an explicit expression of the long-run average cost and the corresponding optimal replacement policy T* can be determined analytically. Numerical results are provided to support the theoretical results.

1. INTRODUCTION

In modern Industry, millions of rupees are being spent to produce highly quality and reliability products. It requires optimal decisions to the maintenance problems of the systems. Obviously, minimizing the cost, maximizing the profit, and make sure that the reliability of the system shall be maximum. In the early stage, many replacement models were developed under the assumption that the system after repair is "as good as new". This leads to a perfect repair model. But it is not always true for deteriorating systems due to ageing and accumulated wear. In this direction Barlow and Hunter [3] developed a minimal repair model in which the minimal repair does not change the age of the system. authors presented [1] and published [2] investigated failure nature, repair pattern and reliabilities of hydro power generators, using simple statistical tools and simulation techniques respectively. It is identified that analysis of hydro power generators require a special approach by dividing the repair into two categories based on the repair duration. That is i) Minor repairs (Repair hours less than or equal a threshold value T) and ii) Major Repairs (Repair hours greater than a threshold value T).This approach is specially introduced by authors [1, 2] and obtained good fit of **"truncated exponential failure model".** Through this proposed model reliabilities are estimated and conclusions were drawn. In the above work, we have assumed that **the system after repair is 'as good as new'**.

Brown and Proschan [4] proposed an imperfect repair model under which the repair will be perfect repair with probability 'p' and with probability '(1-p)' as a minimal repair. Much research work has been carried out by Block et al [5,6] and others have also worked in this direction. It is reasonable to assume that the successive working times of the deteriorating systems after repair will become shorter and shorter, while the consecutive repair time of the system will become longer and longer. Finally it can't work any longer, neither can it be repaired. To model such a deteriorating repairable system Stadje and Zukerman [9] discussed two maintenance models for repairable systems. Further, they determined optimal maintenance strategies for some age dependent cost functions.

Wang and Zhang [10] discussed the optimal replacement problem for a system with two types of failures. One type of failure is repairable, which is conducted by a repairman when it occurs, and the other is un-repairable, which leads to a replacement of the system at once. The repair of the system is not 'as good as new'. Under these assumptions, two replacement models are considered, one is based on the limiting availability and the other based on the long-run average cost rate of the system. They derived an explicit expression for the limiting availability and the long-run average cost rate of the system under policy N, respectively. By maximizing the limiting availability A(N) and minimizing the long-run average cost rate C(N), They theoretically obtain the optimal replacement policies N* in both the cases. In this direction, K.S.Venkateshan and S.Venmathi [7] estimated an optimal replacement policy T of equipment via simulation for truncated failure distributions under the assumption that the life time of equipment is one year. But in most of the cases the life time of equipment may not be one year. Based on this understanding the present paper study a simple repairable system with one repairman assuming that the system after repair is not 'as good as new' and also the successive working times are stochastically decreasing while, the successive repair time's are stochastically increasing and are exposing to exponential truncated failure law. Under these assumptions, an optimal replacement policy T in which we replace the system when the repair time (working time) reaches T. It can be determined that an optimal repair replacement policy T* such that the long run average cost per unit time is minimized. It can also be derived an explicit expression of the long-run average cost and the corresponding optimal replacement policy T* can be determined analytically. Numerical results are provided to support the theoretical results.

2. THE MODEL

In this section, we consider that a new system is put into operation at time 0.When the system if fails, a repairman is immediately assigned to repair it. However, the system after repair is not as good as new. The duration time 0 to completion of the repair of the first breakdown is called the first cycle. After the system is repaired and put into use again, it enters into second cycle. The second cycle length ends at second failure is completed. The time interval between n-1 th repair and nth repair of the system is called nth cycle. Because the system after repair is not as good as new, the life of the system after each additional repair is stochastically decreasing and the life of the system is finite. An optimal replacement policy T for repairable system using truncated exponential failure law is studied under the following assumptions:

ASSUMPTIONS:

1. Assume that that system is put into operation at time t=0.
2. As soon as the system fails, it is immediately repaired by the repairman
3. Let the system after repair is not 'as good as new'.
4. The time interval between the completion of the $(n-1)^{th}$ repair and the completion of the n^{th} repair of a system.
5. Let X_n and Y_n are all independent, for n=1, 2, 3….. .
6. Let X_n and Y_n be successive working time and the successive repair times of the system and both the processes are exposing to truncated exponential failure law.

7. Let $F_n(X)$ and $G_n(Y)$ be the distribution function of X_n and Y_n respectively, n=1,2,….
8. Let $E(X_n)=\int_0^\infty t\,dF(X_n)=\frac{1}{\lambda}$, and $E(Y_n)=\int_0^\infty t\,dG(Y_n)=\frac{1}{\mu};\lambda,\mu>0$.
9. Assume that repair time(working time) is truncated at $\upsilon \le T$ such that the underlying distribution is good fit to the data sets.
10. Assume that an optimal replacement policy T is applied.
11. Assume that the repair time is less than or equal to υ. i.e., $T \le \upsilon$ then the repair is repairable failure with probability 'p' while, the repair time greater than υ then the repair is un- repairable failure with probability '1-p'.
12. Let C_f be the repairable cost and C_p be un-repairable cost.
13. Assume that failure cost is larger than replacement cost.

Under these assumptions, an explicit expression for the long-run average cost per unit time under the policy T is considered and an optimal solution for T* which minimizes the long-run average cost per unit time, is discussed in the next section.

3. THE LONG-RUN AVERAGE COST RATE UNDER POLICY T

Let T_n (n≥2) be the time between the (n-1)th replacement and the nth replacement of the system under policy T. Clearly {T_1, T_2…. } form a renewal process and the inter arrival time between two consecutive replacements is called renewal cycle.
According to renewal reward theorem Ross [8], the long-run average cost rate under policy T is:

$$C(N)=\frac{\text{The expected cost incurred in a renewal cycle}}{\text{The expected length of the renewal cycle}}. \tag{3.1}$$

$$C(T)=\frac{C_f\int_0^T f(t)\,dt+C_p\int_T^\infty f(t)\,dt}{\int_0^T t\,f(t)\,dt+T\int_T^\infty f(t)\,dt} \tag{3.2}$$

According to the assumption (10), let f(t) be an exponential failure model truncated at υ, the truncated failure model is:

$$f(t)=\frac{\lambda e^{-\lambda t}}{1-e^{-\lambda\upsilon}},\ 0\le t\le\upsilon \tag{3.3}$$

$$=0, \qquad otherwise.$$

Then the distribution function F(x) is given by:

$$F_n(X)=\frac{\lambda(1-e^{-\lambda T})}{1-e^{-\lambda\upsilon}},\lambda>0,\upsilon>0. \tag{3.4}$$

Using the equation (3.3) , the equation (3.2) becomes:

$$C(T)=\frac{C_f\left[1-e^{-\lambda T}\right]+C_p\left[e^{-\lambda T}\right]}{1-e^{-\lambda T}-\lambda T e^{-\lambda T}+T e^{-\lambda T}} \tag{3.5}$$

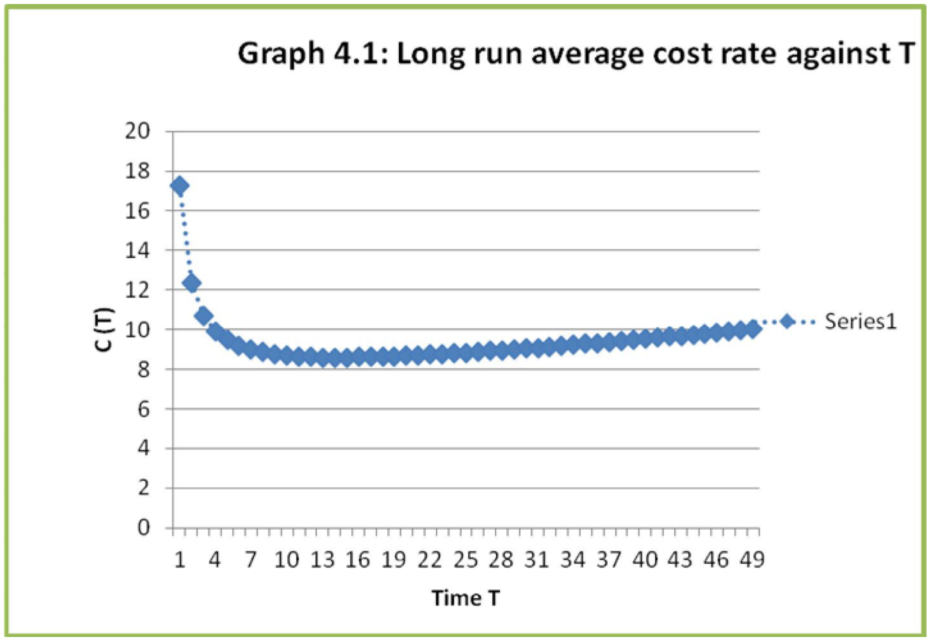
This is the long run average cost rate function.
Using this C (T), the optimal replacement policy T* is determined by numerical methods such that C (T^*) is minimized. The next section provides some numerical results to highlight the obtained theoretical results.

4. NUMERICAL RESULTS AND CONCLUSIONS

For the given hypothetical values of the parameters of λ=0.01281, $C_{f=}$ 600 and $C_{p\ =}$ 6 the long-run average cost per unit time is calculated from the expression (3.4) as follows:

Table 4.1: Values of the long-run average cost rate under policy T.

Time (T)	C(T)	Time (T)	C(T)
1	17.29069	25	8.843584
2	12.33445	26	8.881223
3	10.71189	27	8.920432
4	9.923023	28	8.961093
5	9.467839	29	9.003104
6	9.179679	30	9.046377
7	8.987118	31	9.090838
8	8.854444	32	9.13642
9	8.76182	33	9.183067
10	8.697343	34	9.230729
11	8.653442	35	9.279362
12	8.62507	36	9.328927
13	8.608732	37	9.379392
14	8.601935	38	9.430726
15	8.602851	39	9.482903
16	8.61011	40	9.5359
17	**8.622665**	41	9.589697
18	8.639701	42	9.644276
19	8.660579	43	9.699621
20	8.684785	44	9.755717
21	8.711904	45	9.812553
22	8.7416	46	9.870118
23	8.773594	47	9.928402
24	8.807653	48	9.987398
		49	10.0471



If the underlying distribution is truncated at time T=17 hours, we obtained the following frequency distribution:

Time(in hours)	0 - 2	2-4	4-6	6-8	8-10	10 – 12	12-14	14-16
Number of values	7	15	9	5	1	3	4	3

It is also verified that the statistical hypothesis H_0: "The truncated exponential distribution Truncated at T=17 is a good-fit". It is accepted at 5% L.O.S. The chi-square value and critical value at 5% level of significance are calculated as 6.5714 and 7.8147 respectively. Since we got good fit the parameters of the fitted distribution are **λ=0.01281 , $C_{f=}$ 600 and $C_{p\,=}$ 6.** With these parameters the ACR is 8.6225 at identified optimal truncated time T=17 hours. Thus, the hydro power generator is to be replaced at T=17 because the ACR is minimum. This can be observed in the table and the corresponding graph 4.1. In paper [2] truncated point is determined based on the reliabilities. In this paper, it is identified based on the cost.

Further, earlier works and common sense says that as failure rate "λ" increases the replacement time of the power generator will reduce. That is there exist a negative correlation between λ and T. This concept is verified in the following tables 4.2 and 4.3 by changing **λ =0.075 and λ= 0.095** respectively. The obtained results presented in graphically in 4.2 and 4.3 respectively.

Table 4.2: Long run Average Cost Rate (ACR) values

For the given hypothetical values of the parameters of λ=0.075, $C_{f=}$ 100 and $C_{p\,=}$ 30 the long-run average cost per unit time is calculated from the expression (3.4) as follows:

Time T	C(T)	Time T	C(T)	Time T	C(T)	Time T	C(T)
1	37.67974	14	14.57119	27	21.79356	40	34.56424
2	22.95588	15	14.91811	28	22.57948	41	35.76598
3	18.24459	16	15.30432	29	23.39834	42	36.99546
4	16.04646	17	15.72743	30	24.25023	43	38.2514
5	14.86206	18	16.18576	31	25.13524	44	39.5324
6	14.19202	19	16.67815	32	26.05338	45	40.8369

7	13.82269	20	17.20381	33	27.00458	46	42.16321
8	13.64756	21	17.76224	34	27.98871	47	43.5095
9	13.6078	22	18.35315	35	29.00553	48	44.87378
10	**13.66837**	23	18.97641	36	30.05471	49	46.25399
11	13.80726	24	19.63199	37	31.13581		
12	14.01002	25	20.31996	38	32.24824		
13	14.26692	26	21.04044	39	33.39133		

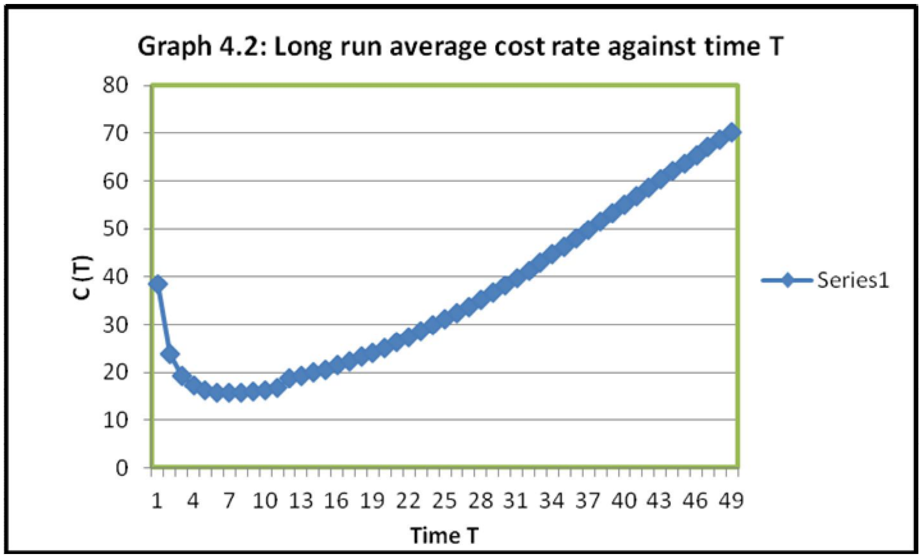
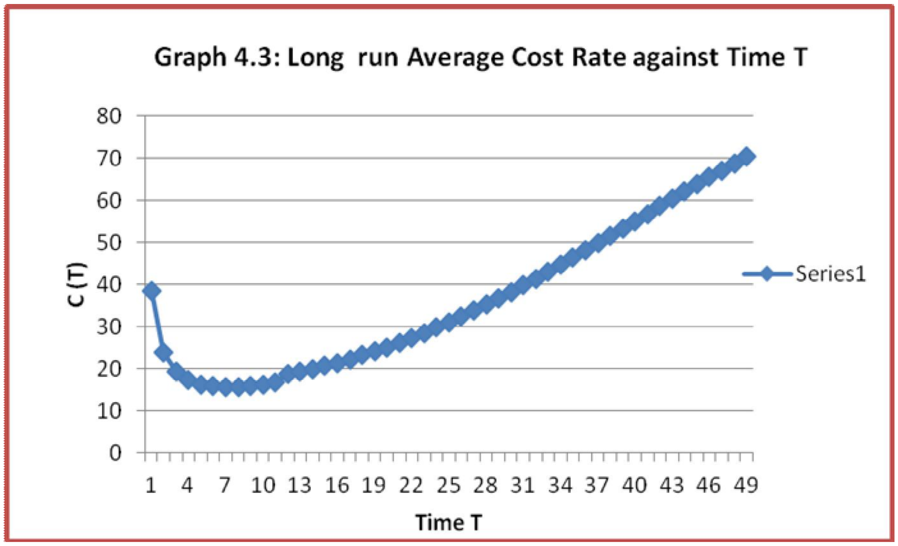


Table 4.3: Long run Average Cost Rate (ACR) values

For the given hypothetical values of the parameters of λ=0.095, $C_{f=}$ 100 and $C_{p\,=}$ 30 the long-run average cost per unit time is calculated from the expression (4.3) as follows:

Time T	C(T)	Time T	C(T)	Time T	C(T)	Time T	C(T)
1	38.37302	13	19.27977	26	32.37745	39	53.30579
2	23.80498	14	19.94068	27	33.75995	40	55.0695
3	19.26182	15	20.66508	28	35.19046	41	56.83258
4	17.24484	16	21.45013	29	36.66706	42	58.58997
5	16.25545	17	22.29389	30	38.18745	43	60.33667
6	15.79505	18	23.19502	31	39.74905	44	62.06779
7	**15.6508**	19	24.15254	32	41.34891	45	63.77855
8	15.71706	20	25.16573	33	42.98375	46	65.46441
9	15.93579	21	26.23398	34	44.64997	47	67.12105
10	16.2728	22	27.35669	35	46.34367	48	68.74445
11	16.70688	23	28.53323	36	48.06065	49	70.33092
12	18.68647	24	29.76285	37	49.79647		
		25	31.04463	38	51.54646		



It is observed that as λ is increases the cost of the repair is also increasing accordingly and the truncated point decreases. Thus, the parameter λ, the failure rate is positively related with cost of the repair and negatively related with truncated time point T.

5. FURTHER SCOPE OF THE WORK

In this paper, we have considered truncated exponential failure model. Similar exercise can also be extended to other truncated laws. However, the work in this direction is progression well.

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