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INVENTORY OPTIMIZATION BY FUZZY LOGIC FOR A PRODUCTION OF SINGLE ITEM WITHOUT SHORTAGES

S. K. Indrajitsingha*, P.N. Samanta and U.K. Misra*****

Author Affiliation:
*DST INSPIRE fellow, P.G. Department of Mathematics, Berhampur University, Berhampur, Bhanja Bihar-760 007, Odisha, India
E-mail: ski.rs.math@buodisha.edu.in
**P.G. Department of Mathematics, Berhampur University, Berhampur,
Bhanja Bihar-760007, Odisha, India
E-mail: pns.math@buodisha.edu.in
**Department of Mathematics, NIST, Golanthara, Palur Hills, Pin-761 008,
Berhampur, Odisha, India
E-mail: umakanta_misra@yahoo.com

Corresponding Author:
S. K. Indrajitsingha, DST INSPIRE fellow, P.G. Department of Mathematics, Berhampur University, Berhampur, Bhanja Bihar-760 007, Odisha, India
E-mail: ski.rs.math@buodisha.edu.in

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Abstract

Here a single item inventory model for deteriorating items without shortages under fuzzy sense is formulated and solved. Fuzziness is introduced by allowing cost components, demand rate, deterioration rate and the production rate. In fuzzy environment, demand, holding cost, set-up cost, deterioration cost, production rate, deterioration rate etc. are assumed to be pentagonal fuzzy numbers. The purpose of this model is to minimize the total cost function in fuzzy scenario. Signed distance method is employed to defuzzify the total cost function per unit time in fuzzy approach. We provide a numerical example to illustrate the theoretical result obtain by the above defuzzification method. Sensitivity analysis of parameters is also carried to detect the effect of change of their values.

Keywords: Fuzzy Inventory, Pentagonal Fuzzy Number, Signed Distance Method, Deterioration, Defuzzification.

AMS Classification No: 03E72

1. INTRODUCTION:

Now-a-days inventory management is a most important factor in production companies. Also inventory fulfills many important functions within the organization. In 1915, the first inventory model was developed by Harris [7] by taking very few parameters. But in actual practice there are many parameters which control the cost of inventory like demand, deterioration rate, holding cost, production rate, set-up cost etc. The above parameters are not realistic in nature though researchers classically applied probability theory to solving the uncertainty (Covert and Philip [2], Ghare and Schrader [6]). However, in certain cases, uncertainties are due to fuzziness, and such situations are dilated in the fuzzy set theory.

In 1965, Zadeh [28] introduced the fuzzy set which is characterized by a membership function assigning each object as a grade of membership ranging between zero and one. It provide a mathematical framework where vague, conceptual phenomena can be rigorously studied. Subsequently this concept was modified and extended for use in different approaches. Since fuzzy logic is an application of the fuzzy set theory particularly used in dealing with process imprecise information with a changed membership functions, researchers started to apply fuzzy set theory in inventory management problems. Fuzzy operation is a process of "Crisp-Fuzzy-Crisp" for a real system in which the original input and the terminal output must be a crisp variable, but the intermediate process is a fuzzy inference process. Kacpryzk and Staniewski [12] developed a model on long-term inventory policy-making through fuzzy-decision. In 1987, K.S Park [17] proposed an economic order quantity model using fuzzy set theory. Yao and Lee [27] developed a fuzzy inventory model with backorder for fuzzy order quantity. Vujosevic et. al. [26] developed an EOQ formula when inventory cost is fuzzy. Gen et. al. [5] introduced a fuzzy inventory control model. Chang [1] applied fuzzy triangular number in production inventory model. Syed and Aziz [25] developed a fuzzy inventory model without shortages in which they defuzzified the total cost function by using signed distance method. De and Rawat [3] developed a fuzzy inventory model without shortages using triangular fuzzy numbers as system parameters. Jaggi et. al. [11] considered a fuzzy inventory model for deteriorating items with time-varying demand and shortages in which parameters are treated as triangular fuzzy numbers. They defuzzified the total cost function by different defuzzified techniques.

Dutta and Kumar [4] considered a fuzzy inventory model for deteriorating items with shortages under fully backlogged condition and defuzzified by the total cost function by different defuzzification methods. Singh and Singh [24] proposed an integrated inventory model from the perspective of a single vendor and multi-buyers for deteriorating items under fuzzy environment and inflation. In that development they are taken all costs and inflation as trapezoidal fuzzy number and graded mean integration representation method is used for defuzzification.

In 2014, Maragatham and Lakshmidevi [14] developed a fuzzy inventory model for deteriorating items with price dependent demand rate in which they consider demand as trapezoidal fuzzy number and defuzzified by applying signed distance method. Ranganathan and Thirunavukarasu [18] considered an inventory control model for constant deterioration and logarithmic demand rate under fuzzy environment in which they defuzzified the total cost function by graded mean integration representation method. Nagar and Surana [16] considered an inventory model for deteriorating items with fluctuating demand using inventory parameters as pentagonal fuzzy numbers. They defuzzified that proposed model by graded mean integration representation method. Kumar and Rajput [13] developed a fuzzy inventory model for deteriorating items with time dependent demand and shortages in partial backlogging in which the demand rate , deterioration rate , backlogging rate are assumed as a triangular fuzzy numbers. They defuzzified the total cost function of that model by signed distance method and centroid method. Ranganathan and Thirunavukarasu [19] consider a fuzzy inventory model under immediate return for deficient items in which they used triangular fuzzy numbers as system parameters. Mishra et. al. [15] presented an inventory control model of deteriorating items where the deteriorating rate, deteriorating cost, carrying cost and shortages are taken as trapezoidal fuzzy numbers.

They defuzzified the model by using graded mean integration representation method. Indrajitsingha et. al. [9] developed a fuzzy economic production quantity model with time dependent demand rate where demand cost and holding cost are taken as triangular fuzzy numbers. They defuzzified the total cost function by using different defuzzification techniques.

In 2016, Indrajitsingha et. al [10] considered a fuzzy inventory model with shortages under fully backlogged where they used signed distance method for defuzzification. Raula et. al. [20] developed a fuzzy inventory model for constant deteriorating items where system parameters are treated as hexagonal fuzzy numbers. They defuzzified the total cost function by using graded mean integration representation method. Indrajitsingha et. al. [8] introduced a fuzzy economic production quantity model with time dependent demand rate in which the system parameters are treated as pentagonal fuzzy numbers. Signed distance method is used for defuzzification of total cost function. Sahoo et. al. [22] consider an inventory model with exponential demand and time-varying deterioration in fuzzy sense in which they used trapezoidal fuzzy numbers as holding cost, deterioration and purchase cost. They defuzzified the total cost function by using graded mean integration representation method. Recently, Sahoo et. al. [21] developed a fuzzy inventory model with time dependent demand rate without shortages where system parameters are treated as pentagonal fuzzy number.

Sen et. al [23] developed a fuzzy inventory model for deteriorating items based on different diffuzification techniques . It is observed that certain absurd values are coming in the production rate and demand. In the present paper we developed a model for deteriorating items by taking pentagonal fuzzy number for system parameters. We determine the total optimum cost by using the Signed Distance Method for defuzzification.

2. DEFINITIONS AND PRELIMINARIES:

In order to establish the model we require the following definitions:

Definition 2.1: (Fuzzy Set) Let X be a space of points with a generic element x of X . Let $\mu : X \rightarrow [0,1]$ be such that for every $x \in X$, $\mu(x)$ is a real number in the interval [0,1], usually called ‘ grade of membership’. We define a fuzzy set $\tilde{A}$ in X as the set of points $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\}$.

Definition 2.2: (Convex Fuzzy Set) A fuzzy set $\tilde{A} = \{(x, \mu_{\tilde{A}}(x))\} \subseteq X$ is called convex fuzzy set if all $\tilde{A}_x$ are convex sets for every $x \in X$. That is, for every pair of elements $x_1, x_2 \in A_\alpha$ and $\alpha \in [0,1]$, $\lambda x_1 + (1-\lambda)x_2 \in A_\alpha$, $\forall \lambda \in [0,1]$. Otherwise the fuzzy set is called non-convex fuzzy set.

Definition 2.3: A fuzzy set $[a_\alpha, b_\alpha]$ where $0 \leq \alpha \leq 1$ and $a < b$ defined on R , is called a fuzzy interval if its membership function is

$$\mu_{[a_\alpha, b_\alpha]} = \begin{cases} \alpha, & a \leq x \leq b \\ 0 & , Otherwise \end{cases}$$

Definition 2.4: A fuzzy number $\tilde{A} = (a, b, c)$ where $a < b < c$ and defined on R, is called triangular fuzzy number if its membership function is

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a}{b-a}, a \le x \le b \\ \frac{c-x}{c-b}, b \le x \le c \\ 0 \quad , Otherwise \end{cases}$$

When $a = b = c$, we have fuzzy points $(c, c, c) = c$. The family of all triangular fuzzy numbers on R is denoted as

$F_N\{(a, b, c): a < b < c \;\forall\; a, b, c \in R\}$

The $\alpha - cut$ of $\tilde{A} = (a, b, c) \in F_N$, $0 \le \alpha \le 1$, is $A(\alpha) = [A_L(\alpha), A_R(\alpha)]$. Where $A_L(\alpha) = a + (b-a)\alpha$ and $A_R(\alpha) = c - (c-b)\alpha$ are the left and right endpoints of $A(\alpha)$.

Definition 2.5: A fuzzy number $\tilde{A} = (a, b, c, d, e)$ where $a < b < c < d < e$ and defined on *R*, is called pentagonal fuzzy number if its membership function is

$$\mu_{\tilde{A}} = \begin{cases} L_1(x) = \frac{x-a}{b-a}, a \le x \le b \\ L_2(x) = \frac{x-b}{c-b}, b \le x \le c \\ 1, \quad x = c \\ R_1(x) = \frac{d-x}{d-c}, c \le x \le d \\ R_2(x) = \frac{e-x}{e-d}, d \le x \le e \\ 0, \quad Otherwise \end{cases}$$

Then $\alpha - cut$ of $\tilde{A} = (a, b, c, d, e)$, $0 \le \alpha \le 1$ is $A(\alpha) = [A_L(\alpha), A_R(\alpha)]$.

Where $A_{L_1}(\alpha) = a + (b-a)\alpha = {L_1}^{-1}(\alpha)$

$A_{L_2}(\alpha) = b + (c-b)\alpha = {L_2}^{-1}(\alpha)$

$A_{R_1}(\alpha) = d - (d-c)\alpha = {R_1}^{-1}(\alpha)$

$A_{R_2}(\alpha) = e - (e-d)\alpha = {R_2}^{-1}(\alpha)$

$$L^{-1}(\alpha) = \frac{{L_1}^{-1}(\alpha) + {L_2}^{-1}(\alpha)}{2} = \frac{a + b + (c-a)\alpha}{2}$$

$$R^{-1}(\alpha) = \frac{{R_1}^{-1}(\alpha) + {R_2}^{-1}(\alpha)}{2} = \frac{d + e - (e-c)\alpha}{2}$$

α -cut of Pentagonal Fuzzy Number:

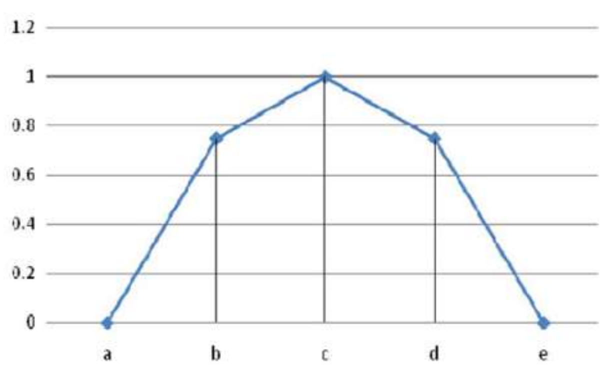


Fig.1 Pentagonal Fuzzy Number

Definition 2.6: If $\tilde{A} = (a, b, c, d, e)$ is a pentagonal fuzzy number then signed distance method of $\tilde{A}$ is defined as

$$d(\tilde{A},0)=\int_0^1 d([A_L(\alpha),A_R(\alpha)],0)=\frac{1}{8(a+2b+2c+2d+e)}$$

Defuzzification: Defuzzification is a process of transforming fuzzy values to crisp values. Defuzzification methods have been widely studied for some years and were applied to fuzzy systems. The major idea behind these methods was to obtain a typical value from a given set according to some specified characters. Defuzzification method provides a correspondence from the set of all fuzzy sets in to the set of all real numbers. Basically, we use the following methods for defuzzification:

- Graded mean integration representation method.
- Signed Distance Method.
- Median rule.
- Centroid method etc.

3. ASSUMPTIONS

Throughout the manuscripts, we make use of the following assumptions:

1. Single inventory will be used.
2. Items are produced and added to the inventory.
3. The lead time is zero.
4. No shortages are allowed.
5. Replenishment is instantaneous.
6. Time horizon is finite.
7. The production rate is proportional to demand rate.
8. The production rate is always greater than demand rate.
9. There is no repair of deteriorated items occurring during the cycle.
10. Neglecting the higher power of θ .

4. NOTATIONS:

S = Set-up cost
θ = Deterioration rate independent of time, $0<\theta\leq 1$
T = Cycle length
P_r = Production rate
H = Holding cost per unit per unit item
d = Deterioration cost per unit per unit time
D = Demand rate is constant
T_1 = Duration of production
$I_0(t)$= Inventory level at time t , $0\leq t\leq T_1$
$I_1(t)$= Inventory level at time t , $T_1\leq t\leq T$
Z = Total cost per unit time
$\tilde{S}$= Fuzzy set-up cost
$\tilde{\theta}$= Fuzzy deterioration
$\tilde{P}_r$= Fuzzy production arte
$\tilde{H}$=Fuzzy holding cost per unit per unit time
$\tilde{d}$= Fuzzy deterioration cost per unit per unit time
$\tilde{D}$= Fuzzy demand rate

$\tilde{Z}$= Total fuzzy inventory cost of Z
$\tilde{Z}_S$ = Defuzzified value of $\tilde{Z}$ by applying signed distance method

5. MATHEMATICAL FORMULATION:

The status of inventory is shown in Fig. 2 as follows

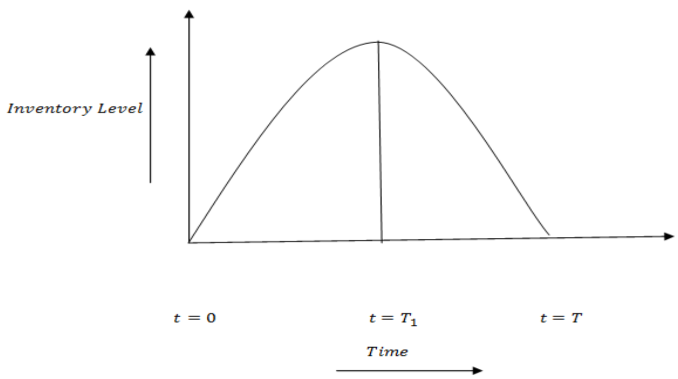


Fig. 2

A machine starts the production at time $t=0$, and initially, inventory level increases up to time T_1 with a constant rate P_r. When the production stopped; after that, the inventory level decreases due to the combined effect of demand and the deterioration and reaches to zero at time $t=T$, where the cycle completes. Hence the rate of change of inventory is governed by the differential equations represented as following:

$$\frac{dI_0(t)}{dt} = P_r - \{D + \theta I_0(t)\}, \quad 0 \le t \le T_1 \qquad (5.1)$$

and

$$\frac{dI_1(t)}{dt} = -\{D + \theta I_1(t)\}, \quad T_1 \le t \le T \qquad (5.2)$$

Solution of (5.1) and (5.2) with the condition $I_0(0) = 0, I_1(T) = 0$ is

$$I_0(t) = \frac{1}{\theta}(P_r - D)\left(1 - e^{-\theta t}\right) \qquad (5.3)$$

and

$$I_1(t) = \frac{D}{\theta\left(e^{\theta(T-t)} - 1\right)} \qquad (5.4)$$

Now, we find T_1 by using $I_0(T_1) = I_1(T_1)$

$$\frac{1}{\theta}(P_r - D)\left(1 - e^{-\theta t}\right) = \frac{D}{\theta\left(e^{\theta(T-t)} - 1\right)}$$

$$\Rightarrow T_1 = \frac{1}{\theta}\log\left\{1 + \frac{D}{P_r}\left(e^{\theta T} - 1\right)\right\} \qquad (5.5)$$

The total cost is calculated by considering set-up cost, holding cost and deterioration cost.

1. Set-up cost $= S$

2. Holding cost per cycle $= H\left[\int_o^{T_1} I_0(t)dt + \int_{T_1}^{T} I_1(t)dt\right]$

$$= \frac{H}{\theta}[P_r T_1 - DT]$$

3. Deterioration cost per cycle $= d\left[\int_o^{T_1} \theta I_0(t)dt + \int_{T_1}^{T} \theta I_1(t)dt\right]$

$$= d\theta\left(\frac{P_r T_1 - DT}{\theta}\right)$$
$$= d(P_r T_1 - DT)$$

Total cost of the system per unit time is given by

$$Z = \frac{1}{T}[Set - up\ cost + Holding\ cost + Deterioration\ cost] \quad (5.6)$$
$$= \frac{S}{T} + \frac{H}{\theta T}(P_r T_1 - DT) + \frac{d}{T}(P_r T_1 - DT)$$
$$= \frac{S}{T} + \frac{(P_r T_1 - DT)}{\theta T}(H + d\theta)$$

Using (5.5) and the assumption (x) we get

$$= \frac{S}{T} + DT\frac{(H + d\theta)}{2}\left(1 - \frac{D}{P_r}\right)$$
$$= \frac{1}{T}\left[S + \frac{1}{2}(H + d\theta)DT^2 - \frac{1}{2}(H + d\theta)\frac{D^2T^2}{2}\right]$$

6. FUZZY MODEL

We consider the model in fuzzy approach. Due to vagueness, it is not easy to define all the parameters precisely. Accordingly we assume the parameters S, H, d, θ, P_r and D in fuzzy environment.

Suppose $\tilde{S} = (S_1, S_2, S_3, S_4, S_5,)$, $\tilde{H} = (h_1, h_2, h_3, h_4, h_5)$, $\tilde{d} = (d_1, d_2, d_3, d_4, d_5)$, $\tilde{\theta} = (\theta_1, \theta_2, \theta_3, \theta_4, \theta_5)$, $\tilde{P}_r = (P_1, P_2, P_3, P_4, P_5)$ and $\tilde{D} = (D_1, D_2, D_3, D_4, D_5)$ are as pentagonal fuzzy number.

Total cost of the system per unit time in fuzzy environment is given by

$$\tilde{Z} = \frac{1}{T}\left[\tilde{S} + \frac{1}{2}\left(\tilde{H} + \tilde{d}\tilde{\theta}\right)\tilde{D}T^2 - \frac{1}{2}\left(\tilde{H} + \tilde{d}\tilde{\theta}\right)\frac{\tilde{D}^2T^2}{\tilde{P}_r}\right] \quad (5.7)$$

We defuzzify the fuzzy total cost $\tilde{Z}$ by Signed Distance Method. Thus the defuzzified value of $\tilde{Z}_S$ is

$$\tilde{Z}_S = \frac{1}{8}[\tilde{Z}_{S1} + 2\tilde{Z}_{S2} + 2\tilde{Z}_{S3} + 2\tilde{Z}_{S4} + \tilde{Z}_{S5}]$$

Where

$$\tilde{Z}_{S1} = \frac{1}{T}\left[S_1 + \frac{1}{2}(h_1 + d_1\theta_1)D_1T^2 - \frac{1}{2}(h_1 + d_1\theta_1)\frac{{D_1}^2T^2}{2}\right]$$
$$\tilde{Z}_{S2} = \frac{1}{T}\left[S_2 + \frac{1}{2}(h_2 + d_2\theta_2)D_2T^2 - \frac{1}{2}(h_2 + d_2\theta_2)\frac{{D_2}^2T^2}{2}\right]$$

$$\tilde{Z}_{S3}=\frac{1}{T}\left[S_3+\frac{1}{2}(h_3+d_3\theta_3)D_3T^2-\frac{1}{2}(h_3+d_3\theta_3)\frac{{D_3}^2T^2}{2}\right]$$
$$\tilde{Z}_{S4}=\frac{1}{T}\left[S_4+\frac{1}{2}(h_4+d_4\theta_4)D_4T^2-\frac{1}{2}(h_4+d_4\theta_4)\frac{{D_4}^2T^2}{2}\right]$$
$$\tilde{Z}_{S5}=\frac{1}{T}\left[S_5+\frac{1}{2}(h_5+d_5\theta_5)D_5T^2-\frac{1}{2}(h_5+d_5\theta_5)\frac{{D_5}^2T^2}{2}\right]$$

To minimize the total cost function per unit time $\tilde{Z}_S$, the optimum value of T can be obtained by solving the differential equation

$$\frac{d\tilde{Z}_S}{dT}=0\ ,\ \frac{d^2\tilde{Z}_S}{dT^2}>0$$

i.e. $$\frac{d\tilde{Z}_S}{dT}=\frac{1}{8}\left[\frac{d}{dT}\tilde{Z}_{S1}+2\frac{d}{dT}\tilde{Z}_{S2}+2\frac{d}{dT}\tilde{Z}_{S3}+2\frac{d}{dT}\tilde{Z}_{S4}+\frac{d}{dT}\tilde{Z}_{S5}\right]=0$$

Now

$$\frac{d\tilde{Z}_{S1}}{dT}=-\frac{S_1}{T^2}+\left[\frac{1}{2}(h_1+d_1\theta_1)D_1-\frac{1}{2}(h_1+d_1\theta_1)\frac{{D_1}^2}{P_1}\right]$$
$$\frac{d\tilde{Z}_{S2}}{dT}=-\frac{S_2}{T^2}+\left[\frac{1}{2}(h_2+d_2\theta_2)D_2-\frac{1}{2}(h_2+d_2\theta_2)\frac{{D_2}^2}{P_2}\right]$$
$$\frac{d\tilde{Z}_{S3}}{dT}=-\frac{S_3}{T^2}+\left[\frac{1}{2}(h_3+d_3\theta_3)D_3-\frac{1}{2}(h_3+d_3\theta_3)\frac{{D_3}^2}{P_3}\right]$$
$$\frac{d\tilde{Z}_{S4}}{dT}=-\frac{S_4}{T^2}+\left[\frac{1}{2}(h_4+d_4\theta_4)D_4-\frac{1}{2}(h_4+d_4\theta_4)\frac{{D_4}^2}{P_4}\right]$$
$$\frac{d\tilde{Z}_{S5}}{dT}=-\frac{S_5}{T^2}+\left[\frac{1}{2}(h_5+d_5\theta_5)D_5-\frac{1}{2}(h_5+d_5\theta_5)\frac{{D_5}^2}{P_5}\right]$$

Thus

$$\frac{d\tilde{Z}_S}{dT}=\frac{1}{8}\left[\begin{array}{c}-\frac{S_1+2S_2+4S_3+2S_4+S_5}{T^2}+\left\{\begin{array}{c}\frac{1}{2}(h_1+d_1\theta_1)D_1+(h_2+d_2\theta_2)D_2\\+(h_3+d_3O_3)D_3+(h_4+d_4O_4)D_4\\+\frac{1}{2}(h_5+d_5\theta_5)D_5\end{array}\right\}-\\ \left\{\begin{array}{c}\frac{1}{2}(h_1+d_1\theta_1)\frac{{D_1}^2}{P_1}+(h_2+d_2\theta_2)\frac{{D_2}^2}{P_2}+(h_3+d_3\theta_3)\frac{{D_3}^2}{P_3}\\+(h_4+d_4\theta_4)\frac{{D_4}^2}{P_4}+\frac{1}{2}(h_5+d_5\theta_5)\frac{{D_5}^2}{P_5}\end{array}\right\}\end{array}\right]$$

Since $\frac{d\tilde{Z}_S}{dT}=0$,

we get

$$T^2=\frac{2(S_1+2S_2+2S_3+2S_4+S_5)}{\left\{\begin{array}{c}(h_1+d_1\theta_1)D_1\left(1-\frac{D_1}{P_1}\right)+2(h_2+d_2\theta_2)D_2\left(1-\frac{D_2}{P_2}\right)+2(h_3+d_3\theta_3)D_3\left(1-\frac{D_3}{P_3}\right)\\+2(h_4+d_4\theta_4)D_4\left(1-\frac{D_4}{P_4}\right)+(h_5+d_5\theta_5)D_5\left(1-\frac{D_5}{P_5}\right)\end{array}\right\}}$$

$$T = \sqrt{\frac{2(S_1 + 2S_2 + 2S_3 + 2S_4 + S_5)}{\left\{\begin{array}{c}(h_1 + d_1\theta_1)D_1\left(1 - \frac{D_1}{P_1}\right) + 2(h_2 + d_2\theta_2)D_2\left(1 - \frac{D_2}{P_2}\right) + 2(h_3 + d_3\theta_3)D_3\left(1 - \frac{D_3}{P_3}\right) \\ +2(h_4 + d_4\theta_4)D_4\left(1 - \frac{D_4}{P_4}\right) + (h_5 + d_5\theta_5)D_5\left(1 - \frac{D_5}{P_5}\right)\end{array}\right\}}}$$

For optimum value, we have to show that $\frac{d^2\tilde{Z}_S}{dT^2} > 0$.

Now

$$\frac{d^2\tilde{Z}_S}{dT^2} = \frac{1}{8}\left[\frac{d^2\tilde{Z}_{S1}}{dT^2} + 2\frac{d^2\tilde{Z}_{S2}}{dT^2} + 2\frac{d^2\tilde{Z}_{S3}}{dT^2} + 2\frac{d^2\tilde{Z}_{S4}}{dT^2} + \frac{d^2\tilde{Z}_{S5}}{dT^2}\right]$$
$$= \frac{1}{6T^3}[S_1 + 2S_2 + 2S_3 + 2S_4 + S_5]$$

Take $\frac{D_1}{P_1} < 1$, $\frac{D_2}{P_2} < 1$, $\frac{D_3}{P_3} < 1$, $\frac{D_4}{P_4} < 1$ and $\frac{D_5}{P_5} < 1$, then in each case, T exists . Then for each value of T we can see $\frac{d^2\tilde{Z}_S}{dT^2} > 0$. Hence we got defuzzified value of total cost function by signed distance i.e. dT_{cg} is minimum.

7. NUMERICAL EXAMPLE:

First, we represent the case of vague value as the type of pentagonal fuzzy number. Consider the inventory system with the following parametric values. Suppose $\tilde{S} = (40, 42, 44, 46, 48)$, $\tilde{H} = (7, 8, 9, 10, 11)$, $\tilde{\theta} = (0.008, 0.010, 0.012, 0.014, 0.016)$, $\tilde{P}_r = (500, 520, 540, 560, 580)$, $\tilde{D} = (400, 420, 440, 460, 480)$, $\tilde{d} = (1.2, 1.3, 1.4, 1.5, 1.6)$ are all pentagonal fuzzy number and

We, solve this by using Matlab R2011b software. The solution of fuzzy total cost $\tilde{Z}_S = 254.3951$ with cycle time T=0.3459

8. SENSITIVITY ANALYSIS:

A sensitivity analysis is carried out to study the effect of changes in parameters S, $\tilde{H}$, $\tilde{\theta}$, $\tilde{D}$ and $\tilde{d}$.

We use Matlab R2011b software for calculation of the total cost and plotting the graphs. In this analysis we change the value of a specific parameter keeping all other parameter remains constant.

Table 1: Sensitivity analysis for $\tilde{S}$

$\tilde{S}$	Time (Yrs)	Total Cost
(36, 38, 40, 42, 44)	0.3298	242.5562
(38, 40, 42, 44, 46)	0.3380	248.5461
(40, 42, 44, 46, 48)	0.3459	254.3951
(42, 44, 46, 48, 50)	0.3537	260.1125
(44, 46, 48, 50, 52)	0.3613	265.7070

Table 2: Sensitivity analysis for $\tilde{H}$

$\tilde{H}$	Time (Yrs)	Total Cost
(5, 6, 7, 8, 9)	0.3920	224.4612
(6, 7, 8, 9,10)	0.3668	239.8955
(7, 8, 9, 10, 11)	0.3459	254.3951
(8, 9, 10, 11, 12)	0.3282	268.1116
(9, 10, 11, 12, 13)	0.3130	281.3333

Table 3: Sensitivity analysis for $\tilde{\theta}$

$\tilde{\theta}$	Time (Yrs)	Total Cost
(0.004, 0.006, 0.008, 0.01, 0.012)	0.3460	254.3163
(0.006, 0.008, 0.01, 0.012, 0.014)	0.3460	254.3556
(0.008, 0.01, 0.012, 0.014, 0.016)	0.3459	254.3951
(0.01, 0.012, 0.014, 0.016, 0.018)	0.3459	254.4346
(0.012, 0.014, 0.016, 0.018, 0.02)	0.3458	254.4740

Table 4: Sensitivity analysis for $\tilde{d}$

$\tilde{d}$	Time (Yrs)	Total Cost
(1.0, 1.1, 1.2, 1.3, 1.4)	0.3460	254.3612
(1.1, 1.2, 1.3, 1.4, 1.5)	0.3459	254.3781
(1.2, 1.3, 1.4, 1.5, 1.6)	0.3459	254.3951
(1.3, 1.4, 1.5, 1.6, 1.7)	0.3459	254.4121
(1.4, 1.5, 1.6, 1.7, 1.8)	0.3459	254.4290

Table 5: Sensitivity analysis for $\tilde{P}_r$

$\tilde{P}_r$	Time (Yrs)	Total Cost
(460, 480, 500, 520, 540)	0.4298	204.7118
(480, 500, 520, 540, 560)	0.3796	231.8522
(500, 520, 540, 560, 580)	0.3459	254.3951
(520, 540, 560, 580, 600)	0.3215	273.6771
(540, 560, 580, 600, 620)	0.3029	287.9853

Table 6: Sensitivity analysis for $\tilde{D}$

$\tilde{D}$	Time (Yrs)	Total Cost
(360, 380, 400, 420, 440)	0.3065	287.0693
(380, 400, 420, 440, 460)	0.3232	272.3016
(400, 420, 440, 460, 480)	0.3459	254.3951
(420, 440, 460, 480, 500)	0.3783	232.6260
(440, 460, 480, 500, 520)	0.4277	205.7722

Fig.3 Sensitivity analysis for Set-up cost

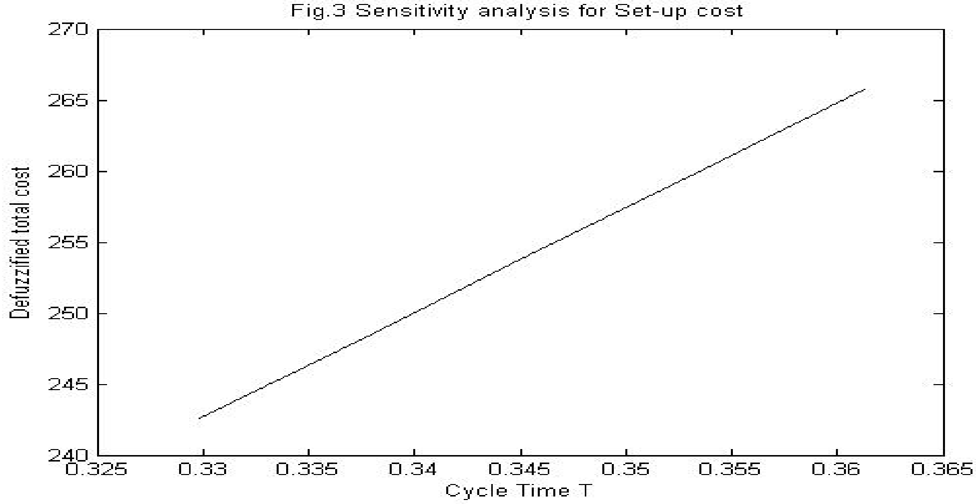


Fig.4 Sensitivity analysis for Holding cost

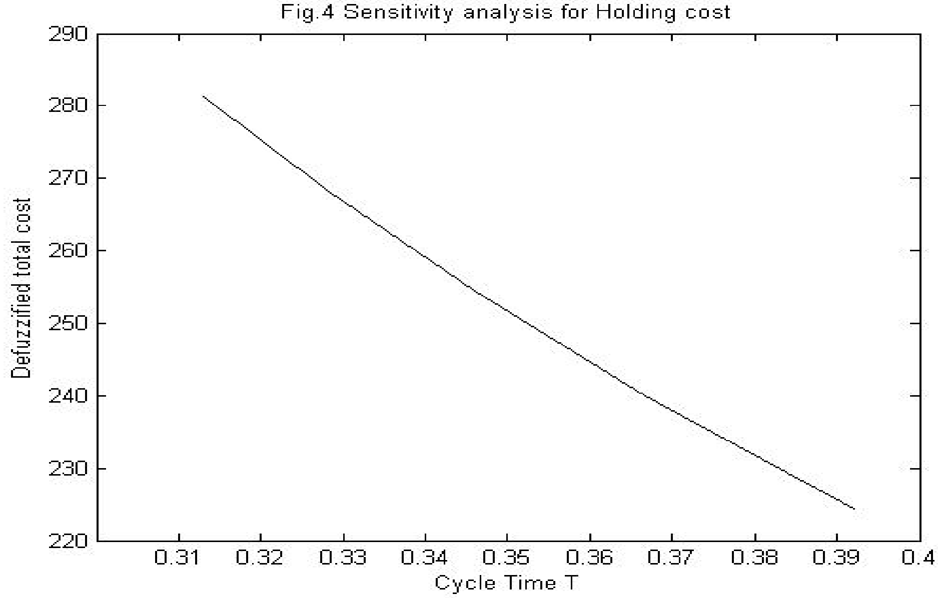


Fig.5 Sensitivity analysis for Deterioration rate

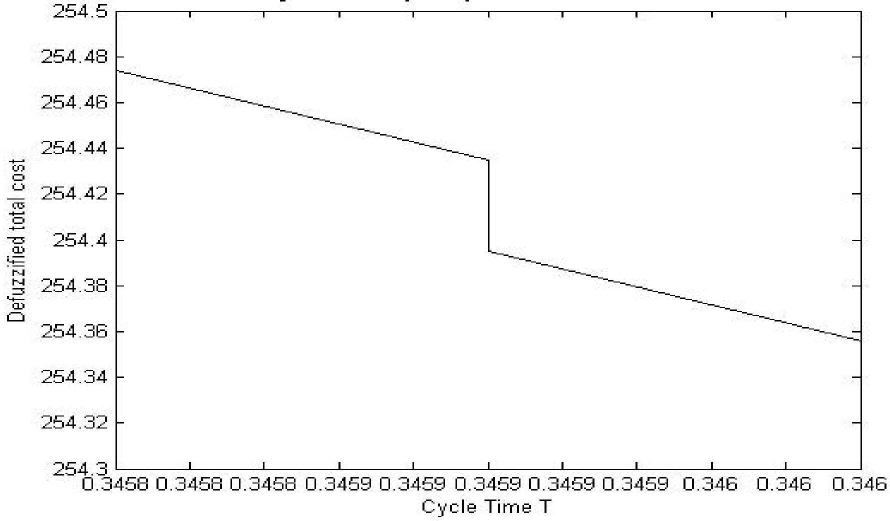


Fig.6 Sensitivity analysis for Deterioration cost

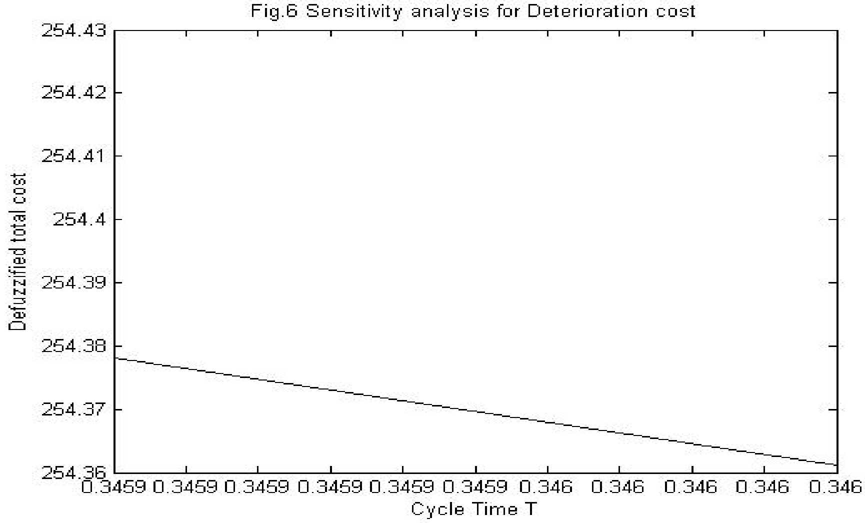
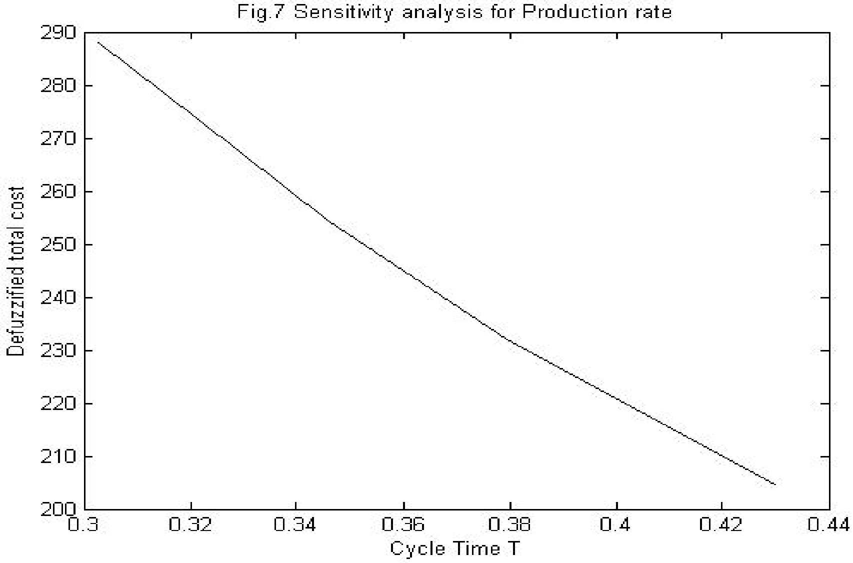
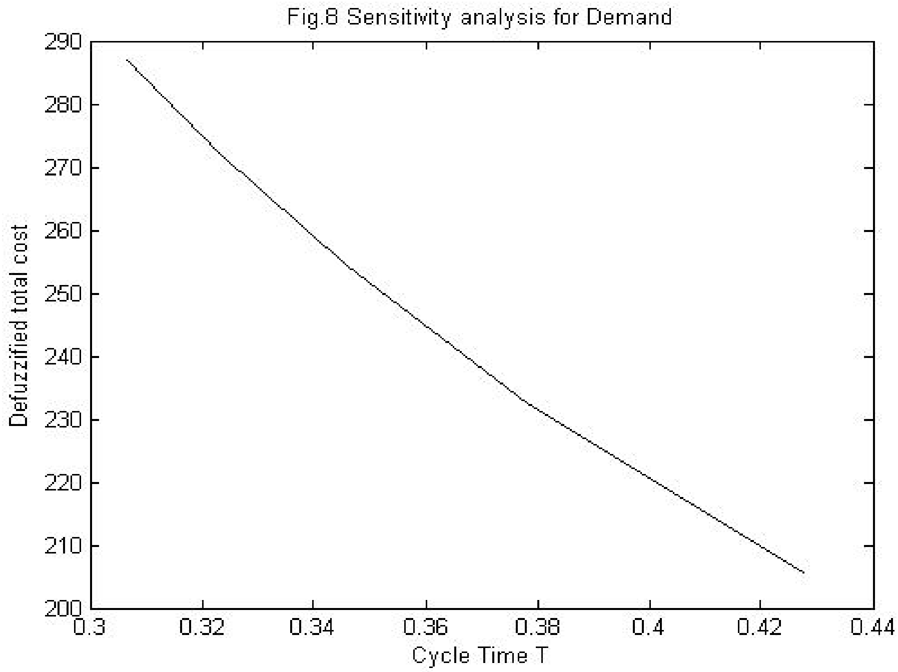


Fig.7 Sensitivity analysis for Production rate





All the above observations can be sum up as follows:

i) In table 1 & fig. 3 Shows that, with increase of the value of the parameter of $\tilde{S}$ keeping all other parameters unchanged, the total cost $\tilde{Z}_S$ increases with the increase of cycle time T .

ii) In table 2, fig. 4, Shows that, with increase of the value of the parameter of $\tilde{H}$, keeping all other parameters unchanged , the total cost $\tilde{Z}_S$ increases with the decrease of cycle time T .

iii) In table 3, fig.5, Shows that, with increase of the value of the parameter of $\tilde{\theta}$, keeping all other parameters unchanged, the total cost $\tilde{Z}_S$ increases slowly with the decrease of cycle time T .

iv) In table 4, fig. 6, Shows that, with increase of the value of the parameter of $\tilde{d}$, keeping all other parameters unchanged , the total cost $\tilde{Z}_S$ increases slowly with the decrease of cycle time T .

v) In table 5, fig. 7, Shows that, with increase of the value of the parameter of $\tilde{P}_r$, keeping all other parameters unchanged, the total cost $\tilde{Z}_S$ increases with the decrease of cycle time T .

vi) In table 6, fig. 8, Shows that, with increase of the value of the parameter of $\tilde{D}$, keeping all other parameters unchanged, the total cost $\tilde{Z}_S$ decreases with the increase of cycle time T .

9. CONCLUSION:

In this paper we developed a production fuzzy inventory model of deteriorating items. This model has been developed for single item without shortages. The set-up cost, holding cost, deterioration cost, deteriorating rate, production rate and demand are represented by pentagonal fuzzy numbers. The optimum result of fuzzy model is defuzzified by using signed distance method. A sensitivity analysis is also carried out to know the behavior of changes in parameters. In this fuzzy inventory model, we have not got any absurd values. So the decision maker can plan for apply this model to get the optimum value of total cost, and for other related parameters, after analyzing the result.

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REFERENCES

1. Chang S.C., Fuzzy production inventory for fuzzy product quality with triangular fuzzy number, Fuzzy set and Systems, 1999, 107(1), 37-57. DOI: 10.1016/S0165-0114(97)00350-3
2. Covert R.P., Philip G.C., An EOQ model for items with Weibull distribution deterioration, AIIE Transactions,1973, 5, 323-326.
3. De P.K., Rawat A., A fuzzy inventory model without shortages using triangular fuzzy number, Fuzzy Information and Engineering, 2011, 1, 59-68.
4. Dutta D., Kumar P., Fuzzy inventory model for deteriorating items with shortages under fully backlogged condition, Int. J. of Soft Computing & Engg., 2013, 3(2), 393-398.
5. Gen M., Tsujimura Y., Zheng D., An application of fuzzy set theory to inventory control models, Computers and Industiral Engineering, 1997, 33(3), 553-556. DOI: 10.1016/S0360-8352(97)00191-5
6. Ghare P.M., Schrader S.F., A model for exponentially decaying inventory, Journal of Industrial Engineering, 1963, 14, 238-243.
7. Harris F., Operations and Cost, AW Shaw Co., Chicago, 1915.
8. Indrajitsingha S.K., Routray S.S., Paikray S.K., Misra U., Fuzzy economic production quantity model with time dependent demand rate, LogForum, 2016, 12(3), 193-198. DOI: 10.17270/J.LOG.2016.3.1
9. Indrajitsingha S.K., Samanta P.N., Misra U.K., An economic production quantity model with time dependent demand under fuzzy environment, Int. J. of Modern Sc & Engg. Tech., 2015, 2(12), 1-8.
10. Indrajitsingha S.K., Samanta P.N., Misra U.K., Fuzzy inventory model with shortages under fully backlogged using Signed distance method, Int. J. Res. in Appl. Sc & Engg. Tech., 2016, 4(1), 197-203.
11. Jaggi C.K., Pareek S., Sharma A., Nidhi, Fuzzy inventory model for deteriorating items with time-varying demand and shortages, American Journal of Operational Research, 2012,2(6), 81-92, DOI: 10.5923/j.ajor.20120206.01
12. Kacpryzk J., Staniewski P., Long-term inventory policy-making through fuzzy-decision making models, Fuzzy Sets and Systems, 1982, 8, 117-132.

13. Kumar S., Rajput U.S., Fuzzy inventory model for deteriorating items with time dependent demand and partial backlogging, Applied Mathematics, 2015, 6, 496-509, http://dx.doi.org/10.4236/am.2015.63047
14. Maragatham M., Lakshmidevi P.K., A fuzzy inventory model for deteriorating items with price dependent demand, Int. J. of Fuzzy Mathematical Archive, 2014, 5(1), 39-47.
15. Mishra S., Barik S., Paikray S.K., Misra U.K., Indrajitsingha S.K., An inventory control model of deteriorating items in fuzzy environment, Global J. of Pure & Applied Mathematics, 2015, 11(3), 1301-1312.
16. Nagar H., Surana P., Fuzzy inventory model for deteriorating items with fluctuating demand and using inventory parameters as pentagonal fuzzy numbers, J. of Comp. & Math. Sc., 2015, 6(2), 55-66.
17. Park K.S, Fuzzy-set theoretic interpretation of economic order quantity, IEEE Transactions on systems, Man, and Cybernatics SMC-17, 1987, 1082-1084.
18. Ranganathan V., Thirunavukarasu P., An inventory control model for constant deterioration in fuzzy environment, International Journal of Fuzzy Mathematics and Systems, 2014, 4(1), 17-26.
19. Ranganathan V., Thirunavukarasu P., Fuzzy inventory model under immediate return for deficient items, International Journal of Mathematical Analysis, 2015, 307-3016.
20. Raula P., Indrajitsingha S.K., Samanta P.N., Misra U.K., A fuzzy inventory model for constant deteriorating item by using GMIR method in which inventory parameters treated as HFN, Open J. of Appl. & Theoretical Math., 2016, 2(1), 13-20.
21. Sahoo A.K., Samanta P.N., Indrajitsingha S.K., Misra U.K., Fuzzy inventory model with time dependent demand rate and without shortages using pentagonal fuzzy numbers, J. of Global Economis, Management and Business Research, 2016, 7(4), 313-318.
22. Sahoo N.K., Mohanty B.S., Tripathy P.K., Fuzzy inventory model with exponential demand and time varying deterioration, Global J. of Pure & Applied Mathematics, 2016, 12(3), 2573-2589.
23. Sen N., Nath B.K., Saha S., A fuzzy inventory model for deteriorating items based on different defuzzification techniques, American J. of Math. & Stat., 2016, 6(3), 128-137, doi:10.5923/j.ajms.20160603.07
24. Singh C., Singh S.R., Vendor-Buyers relationship model for deteriorating items with shortages, fuzzy trapezoidal cost and inflation, Yugoslav J. of Operations Research, 2013, 23(1), 73-85, doi:10.2298/YJOR110421022S
25. Syed J.K., Aziz L.A., Fuzzy inventory model without shortage using signed distance method, Applied Mathematics & Informatics Sciences, 2007, 1(2), 203-209.
26. Vujosevic M., Petrovic D., Petrovic R., EOQ formula when inventory cost is fuzzy, International Journal of Production economics, 1996, 45(1-3), 499-504. DOI: 10.1016/0925-5273(95)00149-2
27. Yao J.S., Lee H.M., Fuzzy inventory with backorder for fuzzy order quantity, Information Sciences, 1996, 93(3-4), 283-319. DOI: 10.1016/0020-0255(96)00074-6
28. Zadeh L.A., Fuzzy Sets, Information Control, 1965, 8(3), 338-353.