Original Article

Bulletin of Pure and Applied Sciences.
Vol. 37E (Math & Stat.), No.1, 2018.P.254-259
Print version ISSN 0970 6577
Online version ISSN 2320 3226
DOI: 10.5958/2320-3226.2018.00025.5

Evaluation of M.T.T.F. and Variance Of A k-Out of N:G Redundant Complex System Under Waiting For Repair

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Received on 20.04.2018, **Accepted on 17.05.2018**

Abstract

This paper presents the investigations for the evaluation of reliability behavior of a k-out of N:G redundant complex system with r repair facilities incorporating the concept of waiting.

Keywords: Supplementary variable Technique, Laplace transforms, M.T.T.F., complex system.

1. INTRODUCTION

The significant part of the paper is that the system consists of N units in series and continues to work so long as r (r = 1,2,…, k-1) units fail at a time but ceases to operate when k units fail. Failure of the units follow exponential time distribution. Repair upto (k-1)th failure is carried out under exponential law whereas the repair at the kth failure stage follows general distribution. Long run availability, M.T.T.F. and variance of the system have also been computed at the end.

2. ASSUMPTIONS

1. The system consists of N identical units and requires {N, N-1,…,N-k+1} units to operate.
2. Initially the system is good and all N units are in operation.
3. After repair the system works like new.
4. Repair of a failed unit is undertaken at once whereas system waits for repair in failed state.

3. NOTATION

0 initial state (i.e. at t = 0)
i number of failed units, i = 0, 1,…, k-1
$P_i(t)$ probability that the system is in state i at time t
a_i failure rate of i units failed
b constant repair rate of a unit
b_i min (i,r)b
k maximum number of failed units for the system to be considered in failed state
W failed state of the system & under waiting for repair
r number of repair facilities
w waiting rate for repair in failed state of the system
s Laplace transform variable

$\bar{F}(s)$ Laplace transform of the function F(t)

$\phi_k(x)\Delta$ the first order probability that the failed system is repaired in the time interval $(x, x+\Delta)$, conditional that it was not repaired up to time x, with probability density $S_k(x)$

$P_w(x,t)$ probability density function (system is in state W and is under repair; elapsed repair time is x , t).

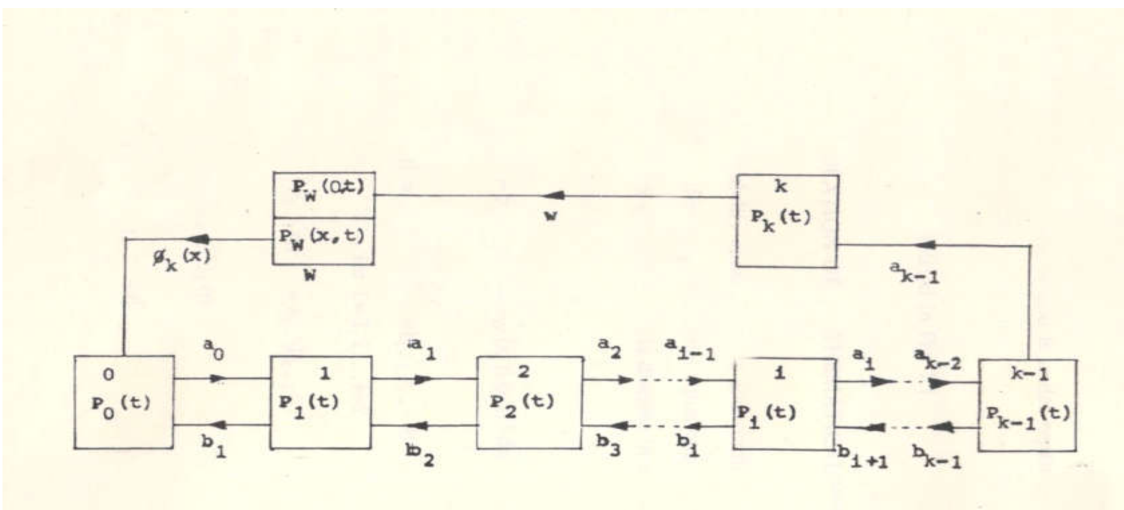

Fig. 1 State transition diagram

4. FORMULATION OF MATHMATICAL MODEL

The following are the forward difference-differential equations governing the stochastic behavior of the complex system which is discrete in space and continuous in time.

$$\left\{\frac{d}{dt}+a_0\right\}P_0(t) = b_1P_1(t)+\int_0^\infty Pw(x,t)\phi_k(x)\,dx \tag{1}$$

$$\left\{\frac{d}{dt}+a_i+b_i\right\}P_i(t)=a_{i-1}P_{i-1}(t)+b_{i+1}P_{i+1}(t) \tag{2}$$

for i = 1, 2, …, k-2

$$\left\{\frac{d}{dt}+b_{k-1}+a_{k-1}\right\}P_{k-1}(t) \;=\; a_{k-2}P_{k-2}(t) \tag{3}$$

$$\left\{\frac{d}{dt}+w\right\}P_{k}(t) \;=\; a_{k-1}P_{k-1}(t) \tag{4}$$

$$\left\{\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\phi_k(x)\right\}Pw(x,t) \;=\; 0 \tag{5}$$

Boundary Condition

$$P_w(0,t)=wP_k(t) \tag{6}$$

Initial Conditions

$P_0(0)=1$ and other state probabilities are zero at time t = 0. (7)

5. SOLUTION OF THE MODEL

Taking Laplace transform of equations (1)-(6) one can obtain the following

$$(s+a_0)\bar{P}_0(s)=1+b_1\bar{P}_1(s)+\int_0^\infty \bar{P}w(x,s)\phi_k(x)\,dx \tag{8}$$

$$(s+a_i+b_i)\bar{P}_i(s)=a_{i-1}\bar{P}_{i-1}(s)+b_{i+1}\bar{P}_{i+1}(s) \tag{9}$$

for i = 1,2…, k – 2

$$(s+b_{k-1}+a_{k-1})\bar{P}_{k-1}(s)=a_{k-2}\bar{P}_{k-2}(s) \tag{10}$$

$$(s+w)\bar{P}_k(s)=a_{k-1}\bar{P}_{k-1}(s) \tag{11}$$

$$\left(s+\frac{\partial}{\partial x}+\phi_k(x)\right)\bar{P}w(x,s)=0 \tag{12}$$

and

$$\bar{P}w(0,s)=w\bar{P}_k(s) \tag{13}$$

Solving equations (8) – (13) one can obtain the following

$$\bar{P}_0(s)=\frac{1}{D(s)} \tag{14}$$

$$\bar{P}_1(s)=\frac{a_0}{C(s)D(s)'} \tag{15}$$

$$\bar{P}_i(s)=\frac{a_{i-1}}{A_i(s)}\bar{P}_{i-1}(s)=\prod_{p=1}^{i-1}\frac{a_{i-p}}{A_{i-p+1}(s)}\bar{P}_1(s) \tag{16}$$

i = 2,3 …, k – 2

$$\bar{P}_{k-1}(s)=\left\{\frac{a_{k-2}}{s+b_{k-1}+a_{k-1}}\prod_{i=1}^{k-3}\frac{a_i}{A_{i+1}(s)}\right\}\bar{P}_1(s) \tag{17}$$

$$\bar{P}_k(s)=\frac{a_{k-1}}{(s+w)}\bar{P}_{k-1}(s) \tag{18}$$

$$\bar{P}_w(s)=w\bar{P}_k\left\{\frac{1-\bar{S}_k(s)}{s}\right\} \tag{19}$$

where,

$$A_i(s)=s+a_i+b_i-\frac{b_{i+1}.a_i}{s+b_{i+1}+a_{i+1}} \tag{20}$$

$$B(s)=b_1+w\overline{S}_k(s)-\frac{a_{k-1}a_{k-2}}{(s+w)(s+b_{k-1}+a_{k-1})}\prod_{i=1}^{k-3}\frac{a_i}{A_{i+1}(s)} \tag{21}$$

$$C(s)=s+a_1+b_1-\frac{a_1b_1}{A_2(s)} \tag{22}$$

$$D(s)=s+a_0-\frac{B(s)a_0}{C(s)} \tag{23}$$

The Laplace transforms of the probability that the system is in up state is as follows :

$$\overline{P}_{up}(s)=\overline{P}_0(s)+\overline{P}_1(s)+......+\overline{P}_{k-1}(s) \tag{24}$$

Therefore,

$$M.T.T.F.=\sum_{i=0}^{k-1}\frac{1}{a_j} \tag{25}$$

Using Abel's Lemma, the Long run availability is given by

$$P_{up}(\infty)=\lim_{s\to\infty} s\overline{P}_{up}(s)$$

$$=\frac{1}{D'(0)}+\frac{a_0}{C(0)D'(0)}+\sum_{i=2}^{k-2}\left[\left\{\prod_{p=1}^{i-1}\frac{a_{i-p}}{A_{i-p+1}(0)}\right\}\frac{a_0}{C(0)D'(0)}\right]$$

$$+\frac{a_{k-2}}{b_{k-1}+a_{k-1}}\prod_{i=1}^{k-3}\frac{a_i}{A_{i+1}(0)}.\frac{a_0}{C(0)D'(0)} \tag{26}$$

where

$$D'(0)=\left\{\frac{d}{ds}D(s)\right\}\text{at } s=0 \tag{27}$$

And the expression for variance of the time to failure is as follows

$$\text{Variance } \sigma=-2.\lim_{s\to 0}\left\{\frac{d}{ds}\overline{P}_{up}(s)\right\}-\left\{\lim_{s\to 0}\overline{P}_{up}(s)\right\}^2$$

$$=2.\lim_{s\to 0}\sum_{j=0}^{k-1}X_j(s)Z_J(s)-(M.T.T.F.)^2$$

$$=2\sum_{j=0}^{k-1}X_j(0)Z_J(0)-\left\{\sum_{p=0}^{k-1}\frac{1}{a_p}\right\}^2 \tag{28}$$

where

$$X_j(s)=\sum_{m=0}^{j}\frac{1}{s+a_m^{'}} \tag{29}$$

$$Z_j(s)=\frac{1}{a_j}\prod_{m=0}^{j}\frac{a_m}{s+a_m} \tag{30}$$

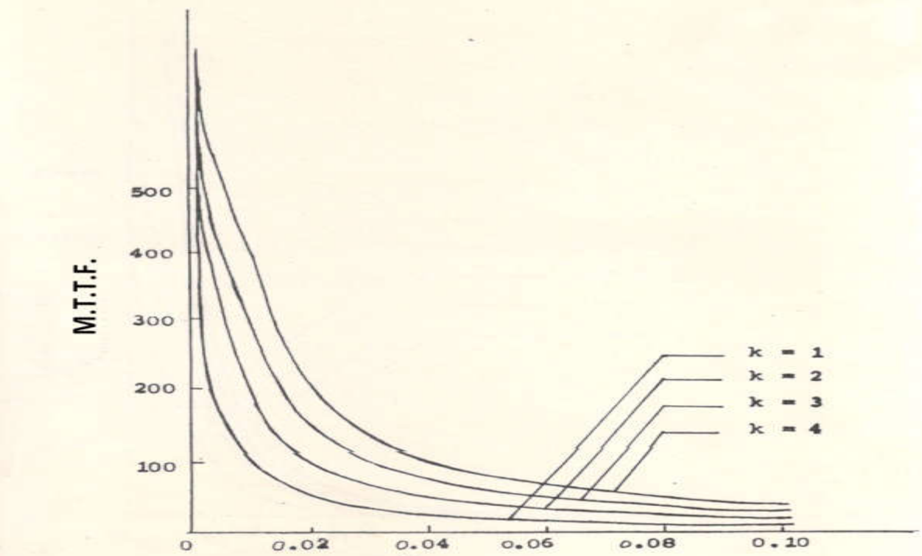

Fig. 2 M.T.T.F. vs failure rate

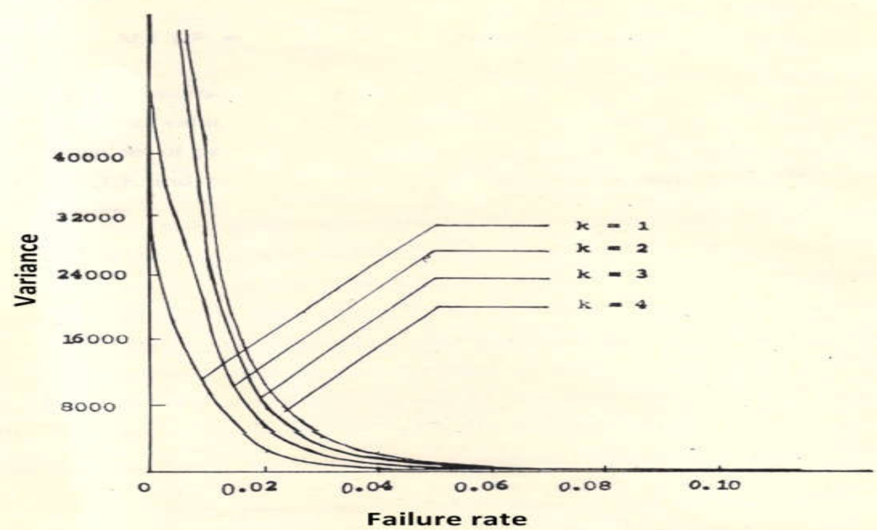

Fig. 3 Variance vs Failure rate

6. PARTICULAR CASE

Setting a_i = a for i = 1, 2, …, k, one can get :

$$\text{M.T.T.F.} = \frac{k}{a} \text{ and variance } \sigma = \frac{k}{a^2} \tag{31}$$

The variations of M.T.T.F. and variance with respect to failure rate (a) for difference values of k are shown in Fig. 2 and Fig. 3 respectively. Thus for a given set of parametric values of $a_1, a_2, ..., a_k$ one can estimate the reliability, M.T.T.F. and variance to forecast the operational behavior of such a complex system.

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