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Reliability Behaviour of a Two-State Stand by Redundant Power Plant under Arbitrary Failure Time Distribution

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Abstract

This paper presents the investigations for the evaluation of reliability of the power supply having been carried out with the help of Boolean function orthogonalization algorithm. The motive of the complex system is to supply from MPH to several critical consumers.

Keywords: M.T.T.F., B.F. Technique, Exponential distribution, Power plant.

1. INTRODUCTION

In this paper we evaluate the reliability and M.T.T.F. of a complex system consisting of two classes A and B. Class A is nothing but a main power house (MPH) consisting of three power generators G_1, G_2 and G_3 connected in parallel redundancy, while class B consists of two sub-power houses SPH_1 and SPH_2 connected in standby redundancy. All these units are connected with four switches S_1, S_2, S_3, S_4 and a switching over device (SOD) by some cables $C_i's$.

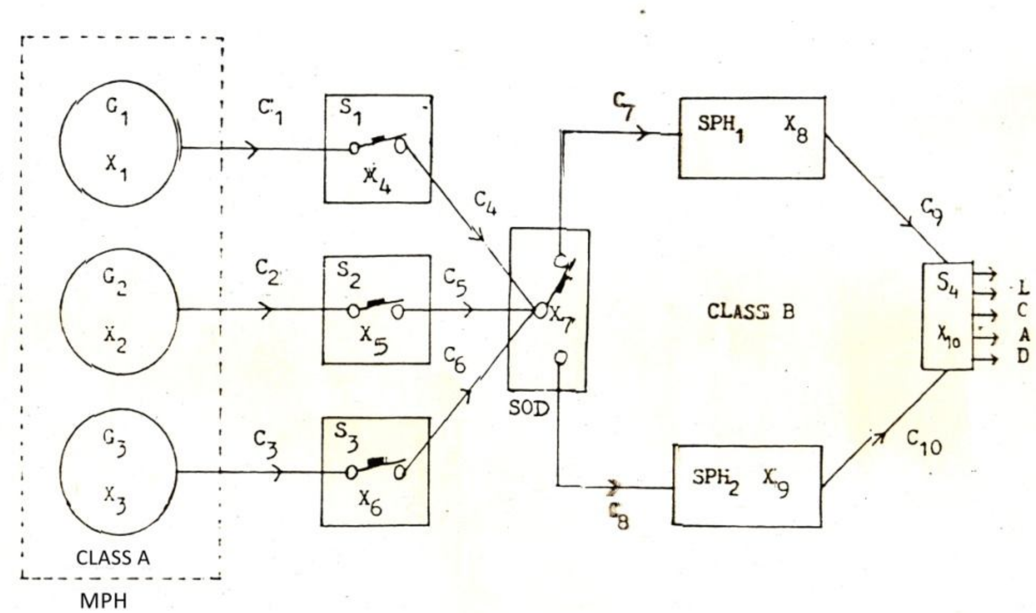

Fig. 1 System Configuration

2. ASSUMPTIONS

The system consists of two classes A and B (with standby redundancy in class B).

1. Class A consists of three-power generators in parallel redundancy.
2. There is no repair facility.
3. The reliabilities of all components are known.
4. Failure times for various components are arbitrary.
5. All the cables are 100% reliable.
6. Switching over device for class B is not 100% reliable.

3. NOTATIONS

x_1, x_2, x_3	states of generators G_1, G_2, G_3
x_4, x_5, x_6	states of switches S_1, S_2, S_3
x_7	state of switching over device SOD
x_8, x_9	states of SPH_1, SPH_2
x_{10}	state of switch S_4
$x_k^{'}$ (k = 1 to 10)	negation of x_k

$$x_i^{'} \ (i=1 to 10) \begin{cases} 0, \text{ in good state} \\ 1, \text{ in bad state} \end{cases}$$

4. FORMULATION OF MATHEMATICAL MODEL

The object of the complex system is to supply power generated by generator G_1, G_2 or G_3 through switch S_4 to critical consumers. By using B.F. technique, the conditions of capability of the successful operation of the complex system in terms of logical matrix can be expressed as :

$$f\left(X_1, X_2, ..., X_{10}\right) = \begin{vmatrix} x_1 & x_4 & x_7 & x_8 & x_{10} & \\ x_2 & x_5 & x_7 & x_8 & x_{10} & \\ x_3 & x_6 & x_7 & x_8 & x_{10} & \\ x_1 & x_4 & x_7 & x_8' & x_9 & x_{10} \\ x_2 & x_5 & x_7 & x_8' & x_9 & x_{10} \\ x_3 & x_6 & x_7 & x_8' & x_9 & x_{10} \end{vmatrix} \quad (1)$$

5. SOLUTION OF THE MODEL

By the application of algebra of logic, equation (1) can be written as

$$f\left(x_1, x_2, ..., x_{10}\right) = \left|x_7 x_{10} g\left(x_1, x_2, ..., x_9\right)\right| \quad (2)$$

where,

$$g\left(x_1, x_2, ..., x_9\right) = \begin{vmatrix} x_1 & x_4 & x_8 & \\ x_2 & x_5 & x_8 & \\ x_3 & x_6 & x_8 & \\ x_1 & x_4 & x_8' & x_9 \\ x_2 & x_5 & x_8' & x_9 \\ x_3 & x_6 & x_8' & x_9 \end{vmatrix} \quad (3)$$

substituting,

$k_i \;=\; x_i \quad x_{i+3} \quad x_8$ (for i = 1 to 3), and

$k_j = x_{j-3} \quad x_j \quad x_8' \quad x_9$ (for j = 4 to 6) in equation (3), one can obtain :

$$g\left(x_1, x_2, ..., x_9\right) = \begin{vmatrix} k_1 & & & & & \\ k_1' & k_2 & & & & \\ k_1' & k_2' & k_3 & & & \\ k_1' & k_2' & k_3' & k_4 & & \\ k_1' & k_2' & k_3' & k_4' & k_5 & \\ k_1' & k_2' & k_3' & k_4' & k_5' & k_6 \end{vmatrix} \quad (4)$$

Using algebra of logic, one can determine the following negations:

$$k_1^{'} = \left| \begin{array}{lll} x_1^{'} & & \\ x_1 & x_4^{'} & \\ x_1 & x_4 & x_8^{'} \end{array} \right| \tag{5}$$

$$k_2^{'} = \left| \begin{array}{lll} x_2^{'} & & \\ x_2 & x_5^{'} & \\ x_2 & x_5 & x_8^{'} \end{array} \right| \tag{6}$$

$$k_3^{'} = \left| \begin{array}{lll} x_3^{'} & & \\ x_3 & x_6^{'} & \\ x_3 & x_6 & x_8^{'} \end{array} \right| \tag{7}$$

$$k_4^{'} = \left| \begin{array}{llll} x_1^{'} & & & \\ x_1 & x_4^{1} & & \\ x_1 & x_4 & x_8 & \\ x_1 & x_4 & x_8^{'} & x_9^{'} \end{array} \right| \tag{8}$$

$$k_5^{'} = \left| \begin{array}{llll} x_2^{'} & & & \\ x_2 & x_5^{1} & & \\ x_2 & x_5 & x_8 & \\ x_2 & x_5 & x_8^{'} & x_9^{'} \end{array} \right| \tag{9}$$

Making use of equations (5) – (9) in equation (4), one can obtain:

$$g\left(x_1, x_2, ..., x_9\right) = \left| \begin{array}{llllllll}
x_1 & x_4 & x_8 & & & & & \\
x_1' & x_2 & x_5 & x_8 & & & & \\
x_1 & x_2 & x_4' & x_5 & x_8 & & & \\
x_1' & x_2' & x_3 & x_6 & x_8 & & & \\
x_1 & x_2' & x_3 & x_4' & x_6 & x_8 & & \\
x_1' & x_2 & x_3 & x_5' & x_6 & x_8 & & \\
x_1 & x_2 & x_3 & x_4' & x_5' & x_6 & x_8 & \\
x_1 & x_2' & x_3' & x_4 & x_8' & x_9 & & \\
x_1 & x_2 & x_3' & x_4 & x_5' & x_8' & x_9 & \\
x_1 & x_2 & x_3' & x_4 & x_5 & x_8' & x_9 & \\
x_1 & x_2' & x_3 & x_4 & x_6' & x_8' & x_9 & \\
x_1 & x_2 & x_3 & x_4 & x_5' & x_6' & x_8' & x_9 \\
x_1 & x_2 & x_3 & x_4 & x_5 & x_6' & x_8' & x_9 \\
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x_1 & x_2 & x_3 & x_4 & x_5' & x_6 & x_8' & x_9 \\
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x_1' & x_2 & x_3 & x_5 & x_6' & x_8' & x_9 & \\
x_1' & x_2 & x_3 & x_5 & x_6 & x_8' & x_9 & \\
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x_1 & x_2 & x_3 & x_4' & x_5 & x_6' & x_8' & x_9 \\
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x_1 & x_2' & x_3 & x_4' & x_6 & x_8' & x_9 & \\
x_1' & x_2 & x_3 & x_5' & x_6 & x_8' & x_9 & \\
x_1 & x_2 & x_3 & x_4' & x_5' & x_6 & x_8' & x_9
\end{array} \right| \tag{10}$$

Finally, the probability of the successful operation (i.e. reliability) of the system is given by :

$$R_s = \Pr\left\{ f = 1 \right\} = \Pr\left(x_7 x_{10} \right).\Pr\left(g = 1 \right) \tag{11}$$

6. PARTICULAR CASES

CASE 1 : If reliability of each component of the complex system is R, equation (11) yields

$$R_s = R^5.\left(R^4 + 3R^3 - 6R^2 - 3R + 6 \right) \tag{12}$$

CASE 2 : When failure rates follow Weibull distributions : Let the failure rate of each component of the complex system be λ , then reliability of the system at an instant t is given by

$$R_{SW}\left(t\right) = e^{-5\lambda t^P}.\left(e^{-4\lambda t^P} + 3e^{-3\lambda t^P} - 6e^{-2\lambda t^P} - 3e^{-\lambda t^P} + 6 \right) \tag{13}$$

where,

P is a positive parameter.

CASE 3 : When failure rates follow Exponential distribution :

$$R_{SE}\left(t\right) = e^{-5\lambda t}.\left(e^{-4\lambda t} + 3e^{-3\lambda t} - 6e^{-2\lambda t} - 3e^{-\lambda t} + 6 \right) \tag{14}$$

$$\text{M.T.T.F.} = \int_0^\infty R_{SE}(t).dt = \frac{0.3289683}{\lambda} \tag{15}$$

7. NUMERICAL COMPUTATION FOR RELIABILITY

Setting λ = 0.1 and p = 2 in equation (13) and (14), one can computes the following Table -1.

Table 1:

Serial Number	Time t	$R_{SE}(t)$	$R_{SW}(t)$
1	0	1.000000	1.000000
2	1	0.767790	0.767790
3	2	0.595098	0.325503
4	3	0.447515	0.044631
5	4	0.325503	0.17363×10^{-2}
6	5	0.228006	0.21297×10^{-4}
7	6	0.146225	0.90063×10^{-7}
8	7	0.099570	0.13687×10^{-9}
9	8	0.068745	0.75922×10^{-13}
10	9	0.044631	0.15459×10^{-16}
11	10	0.028643	0.11572×10^{-20}

8. NUMERICAL COMPUTATION FOR M.T.T.F.

Setting λ = 0.1, 0.2, …, one can compute from equation (15), the following Table- 2.

Table 2:

Serial Number	λ	M.T.T.F.
1	0.0	∞
2	0.1	3.289683
3	0.2	1.644842
4	0.3	1.096561
5	0.4	0.822421
6	0.5	0.657937
7	0.6	0.548281
8	0.7	0.469955
9	0.8	0.411210
10	0.9	0.365520
11	1.0	0.328968

9. INTERPRETATION OF THE RESULTS

Table-1 gives us the reliability of the power supply at an instant t, when failure follows either Weibull or Exponential distribution. An inspection of Fig. 2 "Reliability vs Time" indicates that reliability decreases approximately at a constant rate in case of exponential distribution, while it decreases catastrophically, when failure follows Weibull distribution. A critical examination of the Table-2 and Fig. 3 "M.T.T.F. vs Failure rate" reveals that the Mean time of failure decreases very rapidly in the beginning but later on, it decreases approximately at a uniform rate.

Thus for a given set of failure rates one can determine the reliability behavior of such a standby redundant power plant at any time t.

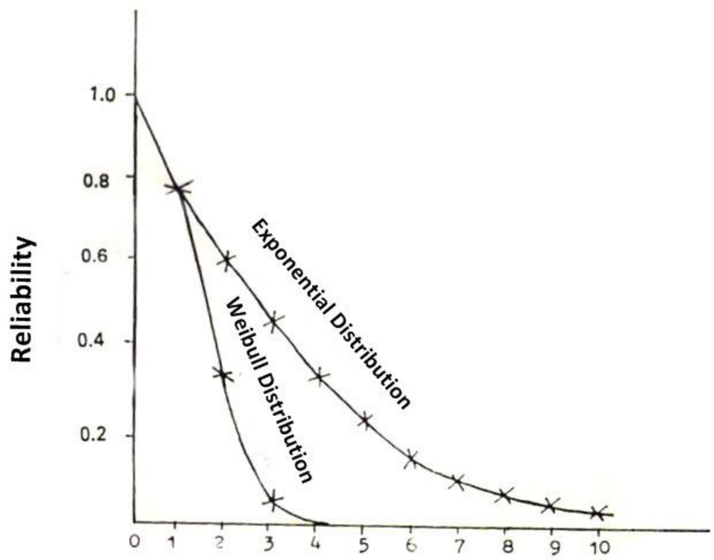

Fig. 2 Reliability vs Time

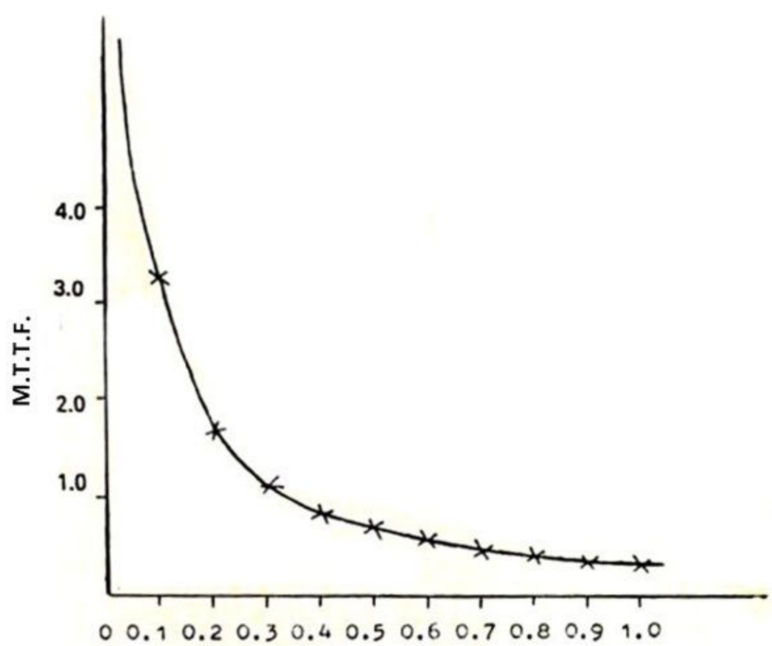

Fig. 3 M.T.T.F. vs failure rate

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