Original Article

Bulletin of Pure and Applied Sciences.
Vol. 37E (Math & Stat.), No.1, 2018.P.157-170
Print version ISSN 0970 6577
Online version ISSN 2320 3226
DOI: 10.5958/2320-3226.2018.00016.4

Strike-Slip Fault Model with Linear and Parabolic Dislocations with Width-Depth Ratio

Dinesh Kumar Madan[1,*], Aanchal Gaba[2], Jyoti Singh Raghav[2]

Author Affiliation:
[1]Department of Mathematics, Ch.Bansi Lal University, Bhiwani, Haryana, India-127021.
[2]Department of Mathematics, Mewar University, Chittorgarh, Rajasthan, India-312901.

***Corresponding Author:**
Dinesh Kumar Madan, Associate Professor, Department of Mathematics,Ch. Bansilal University, Bhiwani, Haryana, India-127021.

E-mail: dk_madaan@rediffmail.com, aanchal.gaba54@gmail.com

Received on 07.03.2018, **Accepted on 25.04.2018**

Abstract

This paper tackles the problem of static deformation of an orthotropic elastic layer of uniform thickness overline an orthotropic elastic half-space. This is caused by a very long vertical strike-slip fault in the layer with rigid surface and this problem is solved analytically. The effect of ratio of width and depth of the faults has been studied for parabolic and linear slip profiles by varying the horizontal dimensionless displacements and by depicting contour maps for the stresses.

Keywords: Linear Slip, Orthotropic, Parabolic Slip, Strike-Slip, Width-Depth Ratio.

1. INTRODUCTION

To study the deformation field around the fault, a static deformation model in anti-plane configuration is generally used. Many researchers have obtained the analytical solutions for displacements and stresses in isotropic and anisotropic elastic medium due to strike slip and dip-slip dislocation with uniform slip. In case of long faults, the use of two-dimensional approximation is justified and consequently the algebra is simplified to a great extent and one gets a closed form analytical solutions.

Okada (1992) [9] obtained a complete set of analytical solutions for rectangular faults with uniform slip in homogeneous elastic half space. A number of solutions are also available in isotropic elastic layered media but these are generally assumed to be a constant slip along the fault. Dziewonski and

Anderson (1981) [4] have established that the upper part of the earth is orthotropic instead of isotropic which has proved to be a better approximation to study the deformation field due to fault.

Garg et al (1996) [5] obtained the integral representation of two-dimensional seismic sources carrying the anti-plane strain deformation of an orthotropic elastic medium. To study the coseismic deformation a two-dimensional fault model is very useful. The strike-slip faults are typically steep or vertical and many strike-slip faults are idealized as being vertical cuts going presumably, all the way to core-mantle boundary. In truth, the geometry of apparently vertical strike-slip faults is quite variable with depth. Elastic deformability combined with sample asymmetry, i.e., when there are uniform changes in the shear stress, non-uniform slip can be caused by the mechanical environment of all the non-identical points on the surface. Uniform slip is not ensured even if a sample has perfect symmetry.

Ruina et al. (1986) [10] studied that uniform slip could be unstable to small spatial disturbance which leads to spatially non-uniform slip. In a uniform isotropic elastic half-space, Singh et al (1994) [11] derived the closed form analytical expressions for displacements due to non-uniform slip along a long vertical strike-slip and dip-slip fault.

Madan et al. (2005) [7] have obtained static deformation field due to various non-uniform slip profiles along a long strike-slip fault in an orthotropic elastic half-space.

Chugh et al. (2011)[2] obtained the closed form analytical expression for the deformation field due to a non-uniform slip along the strike-slip fault situated in an orthotropic elastic layer lying over an orthotropic elastic half-space. They considered the surface as a traction free surface. It has been observed that the earthquake source lies in the Paleozoic sedimentary rocks in Enola, Arkansas, USA, (Crampin, 1994 [3]). Such sedimentary rocks may be represented by layer and the bottom of the layer may be taken as half-space. In engineering, the elastic layer represents an elastic plate whereas in geophysics it represents a lithosphere.

Amano (1981) [1] discussed that since the earth is surrounded by a free surface except for the influence of the atmosphere, the phenomenon resulting in the interior of the earth must be processes closely connected with some state of internal stress. Modeling of deformation and stress due to fault motion with stress free boundary does not include crustal rigidity of layering and lateral variations. Seismic surveys and geological data indicate that the rigidity of the surface is likely to affect the magnitude and pattern of deformation and stress. In the present paper we have considered the surface of the orthotropic elastic layer as rigid lying over an orthotropic elastic half-space. The closed form analytical expressions for the stresses and displacements at the surface are obtained for parabolic and linear slip profiles. To study the deformation field due to non-uniform slip the width and depth plays an important role. In the present problem at different values of width-depth (w / d) ratio, the variations of displacement and stresses are depicted graphically. Contour maps showing the stress distribution due to linear and parabolic slips are also presented.

2. LINE SOURCE IN THE LAYER

We consider a vertical strike-slip fault with non-uniform slip situated in orthotropic elastic layer lying over an orthotropic elastic half space with rigid surface at z=0 where the displacement is zero.We consider **d** as depth of the fault and **w**as the width of the fault. Let H be the width of orthotropic elastic layer. The problem is two-dimensional anti-plane strain problem in yz-plane by considering z axis vertically downwards. As given in Garg et al (1996)when line source lies in the layer, the suitable expressions for the horizontal displacements u,parallel to the line sourceto x-axis, passing through the points (β_2, β_3)in the layer are:

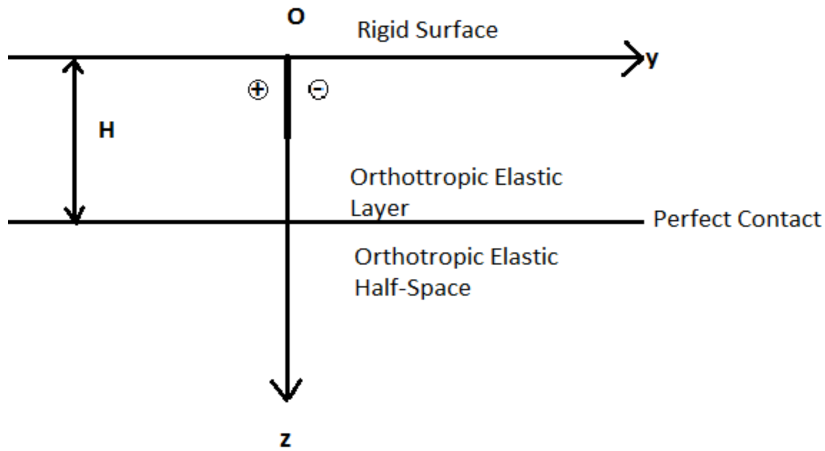


Figure 1: Geometry of a long vertical strike-slip fault ($\oplus$ and $\ominus$ indicate the displacements in the positive negative y-direction respectively)

$$u^{(I)} = u_0 + \int_{-\infty}^{\infty} [\{A_1 \sin k(y-\beta_2) + B_1 \cos k(y-\beta_2)\} e^{-\alpha_1 kz} + \{C_1 \sin k(y-\beta_2) + D_1 \cos k(y-\beta_2)\} e^{\alpha_1 kz}] dk \quad [1]$$

$$u^{(II)} = \int_{-\infty}^{\infty} \{A_2 \sin k(y-\beta_2) + B_2 \cos k(y-\beta_2)\} e^{-\alpha_2 kz}\, dk \quad [2]$$

where

$$u_0 = \int_0^{\infty} [A_0 \sin k(y-\beta_2) + B_0 \cos k(y-\beta_2)]\, e^{-k\alpha|z-\beta_3|} dk \quad [3]$$

The superscript (I) is used to indicate the layer and (II) for the half-space. The constants $A_1, A_2, B_1 B_2, C_1 \& D_1$ are to be determined from the boundary condition:

The bounding surface z=0 is a horizontal plane and is a plane of elastic symmetry. It is further assumed that the bounding plane z=0is rigid, so that

$$u(z=0) = 0 \quad [4a]$$

At the interface when z=H is welded (perfectly bounded), the boundary conditions are that all the components of the displacement and stresses are continuous across the surface at z=H, i.e.

$$u(z=H^-) = u(z=H^+) \quad [4b]$$

$$\tau_{13}(z=H^-) = \tau_{13}(z=H^-) \quad [4c]$$

Here $\tau_{13} = c\alpha^2 \frac{\partial u}{\partial y}, \tau_{13} = c\frac{\partial u}{\partial z}$ [5]

represents the shear stresses in an orthotropic elastic medium,

where $c\alpha^2 = c_{66}$ and $c = c_{55}$ [6]

The values of α and c depend upon the elastic constants. We assume that α and c are positive real numbers.

By using boundary conditions [4a]-[4c] in Eqs.[1] and [2], we obtain the following values of elastic constants.

$$\left.\begin{aligned} A_1 &= -A_0^I S_2\left(e^{-k\alpha_1\beta_3} + S_1 e^{-k\alpha_1(2H-\beta_3)}\right)\\ B_1 &= -B_0^I S_2\left(e^{-k\alpha_1\beta_3} - S_1 e^{-k\alpha_1(2H-\beta_3)}\right)\\ C_1 &= A_0^I S_2 S_1\left(e^{-k(2H-\beta_3)} + e^{-k(2H+\beta_3)}\right)\\ D_1 &= B_0^I S_2 S_1\left(e^{-k(2H+\beta_3)} - e^{-k(2H-\beta_3)}\right)\\ A_2 &= A_0^I S_2(1+S_1)\left(e^{k\alpha_1\beta_3} - e^{-k\alpha_1\beta_3}\right)e^{-k(\alpha_1-\alpha_2)H}\\ B_2 &= B_0^I S_2(1+S_1)\left(e^{-k\alpha_1\beta_3} - e^{+k\alpha_1\beta_3}\right)e^{-k(\alpha_1-\alpha_2)H} \end{aligned}\right\} \quad [7]$$

where

$$S_1 = \frac{c_1\alpha_1 - c_2\alpha_2}{c_1\alpha_1 + c_2\alpha_2},\; S_2 = \frac{1}{1 - S_1 e^{-2Hk\alpha_2}} \quad [8]$$

The substitution of the values of $A_1, A_2, B_1 B_2, C_1$ and D_1 from Eq. [7] in Eqs. [1] and [2] gives the integral expressions for the displacements. These integrals are then evaluated analytically to obtain the following expressions:

$$u^I = \frac{(y-\beta_1)A_0 + \alpha_1|z-\beta_3|B_0}{(y-\beta_1)^2 + (\alpha_1|z-\beta_3|)^2} - \sum_{n=0}^{\infty} S_1^n\left\{\frac{(y-\beta_2)A_0^I + \alpha_1(2nH+\beta_3+z)B_0^I}{(y-\beta_2)^2 + \left(\alpha_1(2nH+\beta_3+z)\right)^2}\right\} + \sum_{n=1}^{\infty} S_1^n\left\{\frac{-(y-\beta_2)A_0^I + \alpha_1(2nH-\beta_3+z)B_0^I}{(y-\beta_2)^2 + \left(\alpha_1(2nH-\beta_3+z)\right)^2} + \frac{(y-\beta_2)A_0^I + \alpha_1(2nH+\beta_3-z)B_0^I}{(y-\beta_2)^2 + \left(\alpha_1(2nH+\beta_3-z)\right)^2} - \frac{(y-\beta_2)A_0^I + \alpha_1(2nH-\beta_3-z)B_0^I}{(y-\beta_2)^2 + \left(\alpha_1(2nH-\beta_3-z)\right)^2}\right\} \quad [9]$$

$$u^{II} = -\sum_{n=0}^{\infty}(1+S_1)S_1^n\left\{\frac{(y-\beta_2)A_0^I - (\alpha_1(2nH+\beta_3+H)+\alpha_2(z-H))B_0^I}{(y-\beta_2)^2 + \left(\alpha_1(2nH+\beta_3+H)+\alpha_2(z-H)\right)^2} + \frac{(y-\beta_2)A_0^I - (\alpha_1(2nH-\beta_3+H)+\alpha_2(z-H))B_0^I}{(y-\beta_2)^2 + (\alpha_1(2nH-\beta_3+H)+\alpha_2(z-H))^2}\right\} \quad [10]$$

From the Eqs. [9] and [10] following Maryuma (1966) [8] and Garg et al (1996) [5] we obtain the following expressions for displacements for perfect contact due to very long vertical strike slip fault with displacement discontinuity b(h) parallel to the fault in the x-direction situated in the layer at a point $(0, h)$ as:

$$u^I = \frac{\alpha_1}{2\pi}\int_0^L b(h)\left[\sum_{n=0}^{\infty} S_1^n\left\{\frac{y}{y^2 + \left(\alpha_1(2nH+h+z)\right)^2} + \frac{y}{y^2 + \left(\alpha_1(2nH+h-z)\right)^2}\right\} + \sum_{n=1}^{\infty} S_1^n\left\{\frac{y}{y^2 + \left(\alpha_1(2nH \pm +z)\right)^2} - \frac{y}{y^2 + \left(\alpha_1(2nH-\beta_3-z)\right)^2}\right\}\right]dh \quad [11]$$

$for\ 0 \le z \le H\ and\ for\ z > H$

$$u^{II} = \frac{\alpha_1}{2\pi}\int_0^L b(h)\left[\sum_{n=0}^{\infty}(1+S_1)S_1^n\left\{\frac{y}{y^2 + \left(\alpha_1(2nH+\beta_3+H)+\alpha_2(z-H)\right)^2} + \frac{y}{y^2 + \left(\alpha_1(2nH-\beta_3+H)+\alpha_2(z-H)\right)^2}\right\}\right] \quad [12]$$

where L denotes the fault stresses.

Let the parabolic and linear slip profiles along the fault be varying accordingly as:

$$\left.\begin{aligned} Parabolic\ slip: b(h) &= b_0\left(1-\frac{h^2}{d^2}\right) \\ Linear\ slip: b(h) &= b_0\left(1-\frac{h}{d}\right) \end{aligned}\right\} \quad [13]$$

Where $0 < h < w < d$

By substituting the value of $b(h)$ from Eq. [13] for parabolic and linear slips along vertical stress-slip fault in orthotropic elastic medium in the Eq. [11]–[12] and then integrating over the limits $0 < h < w$ and using Eq. [5],we obtained the following closed-form expressions for the dimensionless displacements and stresses at any point (y, z).

Parabolic slip

for $0 < z < H$

$$u^I = \frac{b_0}{2\pi\alpha_1}\Big[\sum_{n=0}^{\infty} S_1^n \Big\{\alpha_1\left(1+\frac{Y^2}{\alpha_1^2}-(2n\gamma+Z)^2\right)\left(tan^{-1}\frac{(2n\gamma+Z+R)\alpha_1}{Y}-tan^{-1}\frac{(2n\gamma+Z)\alpha_1}{Y}\right)+\alpha_1 Y(2n\gamma+Z)log\left(\frac{Y^2+(2n\gamma+Z+R)^2\alpha_1^2}{Y^2+(2n\gamma+Z)^2\alpha_1^2}\right)-\alpha_1\left(1+\frac{Y^2}{\alpha_1^2}-(2n\gamma-Z)^2\right)\left(tan^{-1}\frac{(2n\gamma-Z+R)\alpha_1}{Y}-tan^{-1}\frac{(2n\gamma-Z)\alpha_1}{Y}\right)-\alpha_1 Y(2n\gamma+Z)log\left(\frac{Y^2+(2n\gamma-Z+R)^2\alpha_1^2}{Y^2+(2n\gamma-Z)^2\alpha_1^2}\right)\Big\}+\sum_{n=1}^{\infty} S_1^n\Big\{\alpha_1\left(1+\frac{Y^2}{\alpha_1^2}-(2n\gamma+Z)^2\right)\left(tan^{-1}\frac{(2n\gamma+Z-R)\alpha_1}{Y}-tan^{-1}\frac{(2n\gamma+Z)\alpha_1}{Y}\right)-\alpha_1 Y(2n\gamma+Z)log\left(\frac{Y^2+(2n\gamma+Z-R)^2\alpha_1^2}{Y^2+(2n\gamma+Z)^2\alpha_1^2}\right)+\alpha_1\left(1+\frac{Y^2}{\alpha_1^2}-(2n\gamma-Z)^2\right)\left(tan^{-1}\frac{(2n\gamma-Z-R)\alpha_1}{R}-tan^{-1}\frac{(2n\gamma-Z)\alpha_1}{Y}\right)+\alpha_1 Y(2n\gamma-Z)log\left(\frac{Y^2+(2n\gamma-Z-R)^2\alpha_1^2}{Y^2+(2n\gamma-Z)^2\alpha_1^2}\right)\Big\}\Big] \quad [14]$$

and for $z > H$

$$u^{II} = \frac{b_0}{2\pi\alpha_1}\left[\sum_{n=0}^{\infty}(1+S_1)S_1^n\left\{\alpha_1\left(1+\frac{Y^2}{\alpha_1^2}-\frac{[\alpha_1(2n\gamma+Z)+\alpha_2(Z-\gamma)]^2}{\alpha_1^2}\right)\times\left(tan^{-1}\frac{(2n\gamma+Z+R)\alpha_1+\alpha_2(Z-\gamma)}{Y}-tan^{-1}\frac{(2n\gamma+Z-R)\alpha_1+\alpha_2(Z-\gamma)}{Y}\right)-\frac{Y\{\alpha_1(2n\gamma+Z)+\alpha_2(Z-\gamma)\}}{\alpha_1}log\left(\frac{Y^2+[\alpha_1(2n\gamma+Z-R)+\alpha_2(Z-\gamma)]^2}{Y^2+[\alpha_1(2n\gamma+Z+R)+\alpha_2(Z-\gamma)]^2}\right)+2Y\right\}\right] \quad [15]$$

where $Y=\frac{y}{d}$, $Z=\frac{z}{d}$, $\gamma=\frac{H}{d}$ and $R=\frac{w}{d}$ is the width-depth ratio of the fault.

$$
\begin{aligned}
\tau_{12} = \frac{b_0 c_1 \alpha_1}{2\pi}\Bigg[& \sum_{n=0}^{\infty} S_1^n \Bigg\{ \alpha_1^2 Y \left(1 + \frac{Y^2}{\alpha_1^2} - (2n\gamma + Z)^2\right)\left(\frac{(2n\gamma + Z)\alpha_1}{Y^2 + (2n\gamma + Z)^2\alpha_1^2} - \frac{(2n\gamma + Z + R)\alpha_1}{Y^2 + (2n\gamma + Z + R)^2\alpha_1^2}\right) \\
& + \frac{2Y}{\alpha_1}\left(tan^{-1}\frac{(2n\gamma + Z + R)\alpha_1}{Y} - tan^{-1}\frac{(2n\gamma + Z)\alpha_1}{Y}\right) - \alpha_1 Y^2 R(2n\gamma \\
& + Z)\left(\frac{4n\gamma + 2Z + R}{(Y^2 + (2n\gamma + Z + R)^2\alpha_1^2)(Y^2 + (2n\gamma + Z)^2\alpha_1^2)}\right) \\
& + (2n\alpha_1\gamma + Z) log\left(\frac{Y^2 + (2n\gamma + Z + R)^2\alpha_1^2}{Y^2 + (2n\gamma + Z)^2\alpha_1^2}\right) \\
& - \alpha_1^2 Y\left(1 + \frac{Y^2}{\alpha_1^2} - (2n\gamma + Z)^2\right)\left(\frac{(2n\gamma - Z)\alpha_1}{Y^2 + (2n\gamma - Z)^2\alpha_1^2} - \frac{(2n\gamma - Z + R)\alpha_1}{Y^2 + (2n\gamma - Z + R)^2\alpha_1^2}\right) \\
& - \frac{2Y}{\alpha_1}\left(tan^{-1}\frac{(2n\gamma - Z + R)\alpha_1}{Y} - tan^{-1}\frac{(2n\gamma - Z)\alpha_1}{Y}\right) \\
& + \alpha_1 Y^2 R(2n\gamma - Z)\left(\frac{4n\gamma - 2Z + R}{(Y^2 + (2n\gamma - Z - R)^2\alpha_1^2)(Y^2 + (2n\gamma + Z)^2\alpha_1^2)}\right) \\
& - \alpha_1(2n\gamma - Z) log\left(\frac{Y^2 + (2n\gamma - Z - R)^2\alpha_1^2}{Y^2 + (2n\gamma - Z)^2\alpha_1^2}\right)\Bigg\} \\
& + \sum_{n=1}^{\infty} S_1^n \Bigg\{ \alpha_1^2 Y\left(1 + \frac{Y^2}{\alpha_1^2} - (2n\gamma + \quad Z)^2\right)\Bigg(\frac{(2n\gamma + Z)\alpha_1}{Y^2 + (2n\gamma + Z)^2\alpha_1^2} \\
& - \quad \frac{(2n\gamma + Z - R)\alpha_1}{Y^2 + (2n\gamma + Z - R)^2\alpha_1^2}\Bigg) \\
& + \frac{2Y}{\alpha_1}\left(tan^{-1}\frac{(2n\gamma + Z - R)\alpha_1}{Y} - tan^{-1}\frac{(2n\gamma + Z)\alpha_1}{Y}\right) + \alpha_1 Y^2 R(2n\gamma \\
& + Z)\left(\frac{4n\gamma + 2Z - R}{(Y^2 + (2n\gamma + Z - R)^2\alpha_1^2)(Y^2 + (2n\gamma + Z)^2\alpha_1^2)}\right) \\
& + (2n\alpha_1\gamma + Z) log\left(\frac{Y^2 + (2n\gamma + Z - R)^2\alpha_1^2}{Y^2 + (2n\gamma + Z)^2\alpha_1^2}\right) \\
& - \alpha_1^2 Y\left(1 + \frac{Y^2}{\alpha_1^2} - (2n\gamma - Z)^2\right)\left(\frac{(2n\gamma - Z)\alpha_1}{Y^2 + (2n\gamma - Z)^2\alpha_1^2} - \frac{(2n\gamma - Z - R)\alpha_1}{Y^2 + (2n\gamma - Z - R)^2\alpha_1^2}\right) \\
& + \frac{2Y}{\alpha_1}\left(tan^{-1}\frac{(2n\gamma - Z - R)\alpha_1}{Y} - tan^{-1}\frac{(2n\gamma - Z)\alpha_1}{Y}\right) \\
& + \alpha_1 Y^2 R(2n\gamma - Z)\left(\frac{4n\gamma - 2Z - R}{(Y^2 + (2n\gamma - Z - R)^2\alpha_1^2)(Y^2 + (2n\gamma + Z)^2\alpha_1^2)}\right) \\
& - \alpha_1(2n\gamma - Z) log\left(\frac{Y^2 + (2n\gamma - Z - R)^2\alpha_1^2}{Y^2 + (2n\gamma - Z)^2\alpha_1^2}\right)\Bigg\}\Bigg] \qquad [16]
\end{aligned}
$$

$$
\begin{aligned}
&\tau_{13}\\
&= \frac{b_0 c_1}{2\pi}\left[\sum_{n=0}^{\infty} S_1^n \left\{\alpha_1 Y\left(1+\frac{Y^2}{\alpha_1^2}-(2n\gamma+Z)^2\right)\left(\frac{1}{Y^2+(2n\gamma+Z+R)^2\alpha_1^2}-\frac{1}{Y^2+(2n\gamma+Z)^2\alpha_1^2}\right)\right.\right.\\
&- 2(2n\gamma+Z)\left(tan^{-1}\frac{(2n\gamma+Z+R)\alpha_1}{Y}-tan^{-1}\frac{(2n\gamma+Z)\alpha_1}{Y}\right)\\
&+ 2\alpha_1^2 RY(2n\gamma+Z)\left(\frac{Y^2-\alpha_1^2(2n\gamma+Z)(2n\gamma+Z+R)}{(Y^2+(2n\gamma+Z+R)^2\alpha_1^2)(Y^2+(2n\gamma+Z)^2\alpha_1^2)}\right)+Ylog\left(\frac{Y^2+(2n\gamma+Z+R)^2\alpha_1^2}{Y^2+(2n\gamma+Z)^2\alpha_1^2}\right)\\
&- \alpha_1 Y\left(1+\frac{Y^2}{\alpha_1^2}-(2n\gamma+Z)^2\right)\left(\frac{1}{Y^2+(2n\gamma-Z+R)^2\alpha_1^2}-\frac{1}{Y^2+(2n\gamma-Z)^2\alpha_1^2}\right)\\
&- 2(2n\gamma-Z)\left(tan^{-1}\frac{(2n\gamma-Z+R)\alpha_1}{Y}-tan^{-1}\frac{(2n\gamma-Z)\alpha_1}{Y}\right)\\
&+ 2\alpha_1^2 YR(2n\gamma+Z)\left(\frac{Y^2-\alpha_1^2(2n\gamma-Z)(2n\gamma-Z+R)}{(Y^2+(2n\gamma-Z+R)^2\alpha_1^2)(Y^2+(2n\gamma-Z)^2\alpha_1^2)}\right)\\
&- \left. Ylog\left(\frac{Y^2+(2n\gamma-Z+R)^2\alpha_1^2}{Y^2+(2n\gamma-Z)^2\alpha_1^2}\right)\right\}\\
&+ \sum_{n=1}^{\infty} S_1^n \left\{\alpha_1 Y\left(1+\frac{Y^2}{\alpha_1^2}-(2n\gamma+Z)^2\right)\left(\frac{1}{Y^2+(2n\gamma+Z-R)^2\alpha_1^2}-\frac{1}{Y^2+(2n\gamma+Z)^2\alpha_1^2}\right)\right.\\
&- 2(2n\gamma+Z)\left(tan^{-1}\frac{(2n\gamma+Z-R)\alpha_1}{Y}-tan^{-1}\frac{(2n\gamma+Z)\alpha_1}{Y}\right)\\
&- 2\alpha_1^2 RY(2n\gamma+Z)\left(\frac{Y^2-\alpha_1^2(2n\gamma+Z)(2n\gamma+Z-R)}{(Y^2+(2n\gamma+Z-R)^2\alpha_1^2)(Y^2+(2n\gamma+Z)^2\alpha_1^2)}\right)+Ylog\left(\frac{Y^2+(2n\gamma+Z-R)^2\alpha_1^2}{Y^2+(2n\gamma+Z)^2\alpha_1^2}\right)\\
&- \alpha_1 Y\left(1+\frac{Y^2}{\alpha_1^2}-(2n\gamma-Z)^2\right)\left(\frac{1}{Y^2+(2n\gamma-Z-R)^2\alpha_1^2}-\frac{1}{Y^2+(2n\gamma-Z)^2\alpha_1^2}\right)\\
&- 2(2n\gamma-Z)\left(tan^{-1}\frac{(2n\gamma-Z-R)\alpha_1}{Y}-tan^{-1}\frac{(2n\gamma-Z)\alpha_1}{Y}\right)\\
&- 2\alpha_1^2 YR(2n\gamma+Z)\left(\frac{Y^2-\alpha_1^2(2n\gamma-Z)(2n\gamma-Z-R)}{(Y^2+(2n\gamma-Z-R)^2\alpha_1^2)(Y^2+(2n\gamma-Z)^2\alpha_1^2)}\right)\\
&- \left.\left. Ylog\left(\frac{Y^2+(2n\gamma-Z-R)^2\alpha_1^2}{Y^2+(2n\gamma-Z)^2\alpha_1^2}\right)\right\}\right] \qquad [17]
\end{aligned}
$$

***L*inear slip**

$for\ 0 \le z < H$

$$u^I = \frac{b_0}{2\pi\alpha_1}\Bigg[\sum_{n=0}^{\infty} S_1^n\Bigg\{\alpha_1(1+2n\gamma+Z)\left(tan^{-1}\frac{(2n\gamma+Z+R)\alpha_1}{Y} - tan^{-1}\frac{(2n\gamma+Z)\alpha_1}{Y}\right)$$
$$-\frac{Y}{2}log\left(\frac{Y^2+(2n\gamma+Z+R)^2\alpha_1^2}{Y^2+(2n\gamma+Z)^2\alpha_1^2}\right) - \alpha_1(1+2n\gamma-Z)\left(tan^{-1}\frac{(2n\gamma-Z+R)\alpha_1}{Y}\right.$$
$$\left.-tan^{-1}\frac{\alpha_1(2n\gamma-Z)}{Y}\right) + \frac{Y}{2}log\left(\frac{Y^2+(2n\gamma-Z+R)^2\alpha_1^2}{Y^2+(2n\gamma-Z)^2\alpha_1^2}\right)\Bigg\}$$
$$+\sum_{n=1}^{\infty} S_1^n\Bigg\{\alpha_1(1-2n\gamma-Z)\left(tan^{-1}\frac{(2n\gamma+Z-R)\alpha_1}{Y} - tan^{-1}\frac{(2n\gamma+Z)\alpha_1}{Y}\right)$$
$$-\frac{Y}{2}log\left(\frac{Y^2+(2n\gamma+Z-R)^2\alpha_1^2}{Y^2+(2n\gamma+Z)^2\alpha_1^2}\right) - \alpha_1(1-2n\gamma-Z)\left(tan^{-1}\frac{(2n\gamma-Z-R)\alpha_1}{Y}\right.$$
$$\left.-tan^{-1}\frac{\alpha_1(2n\gamma-Z)}{Y}\right) + \frac{Y}{2}log\left(\frac{Y^2+(2n\gamma-Z-R)^2\alpha_1^2}{Y^2+(2n\gamma-Z)^2\alpha_1^2}\right)\Bigg\}\Bigg] \quad [18]$$

and for $z > H$

$$u^{II} = \frac{b_0}{2\pi\alpha_1}\Bigg[\sum_{n=0}^{\infty}(1+S_1)S_1^n\Bigg\{\alpha_1\left(1+\frac{\alpha_1(2n\gamma+Z)+\alpha_2(Z-\gamma)}{\alpha_1}\right)\times\left(tan^{-1}\frac{(2n\gamma+Z+R)\alpha_1+\alpha_2(Z-\gamma)}{Y}\right)$$
$$-\left(1-\frac{\alpha_1(2n\gamma+Z)+\alpha_2(Z-\gamma)}{\alpha_1}\right)\left(tan^{-1}\frac{(2n\gamma+Z-R)\alpha_1+\alpha_2(Z-\gamma)}{y}\right)$$
$$-2[\alpha_1(2n\gamma+Z)+\alpha_1(Z-\gamma]\left(tan^{-1}\frac{\alpha_1(2n\gamma+Z)+\alpha_2(Z-\gamma)}{y}\right)$$
$$-\frac{Y}{2}\log([\alpha_1(2n\gamma+Z+R)+\alpha_2(Z-\gamma)]^2+Y^2)/([\alpha_1(2n\gamma+Z-R)+\alpha_2(Z-\gamma)]^2$$
$$+Y^2)\Bigg\}\Bigg] \quad [19]$$

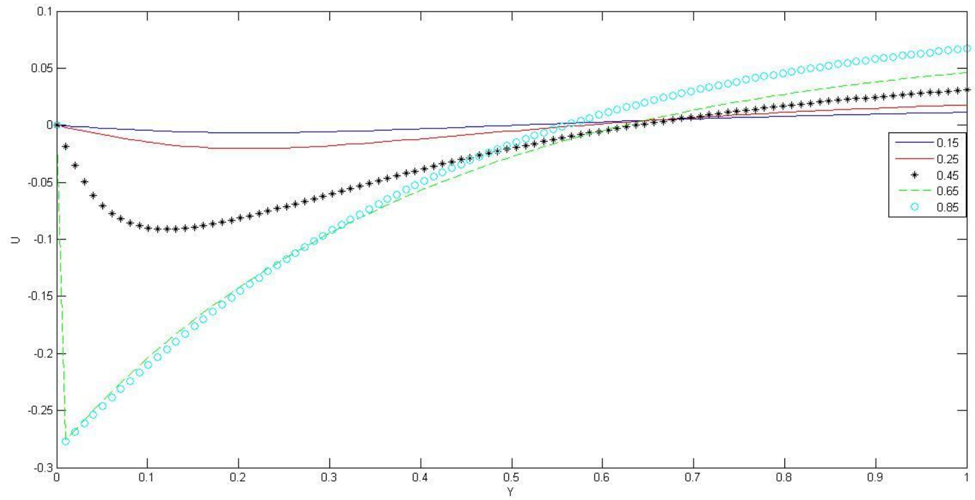
$$\tau_{12} = \frac{c_1\alpha_1 b_0}{2\pi}\Bigg[\sum_{n=0}^{\infty} S_1^n\Bigg\{\alpha_1(1+2n\gamma+Z)\left(\frac{(2n\gamma+Z)\alpha_1}{Y^2+(2n\gamma+Z)^2{\alpha_1}^2} - \frac{(2n\gamma+Z+R)\alpha_1}{Y^2+(2n\gamma+Z+R)^2{\alpha_1}^2}\right)$$
$$- Y^2R\frac{(4n\gamma+2Z+R)}{(Y^2+(2n\gamma+Z)^2{\alpha_1}^2)(Y^2+(2n\gamma+Z+R)^2{\alpha_1}^2)} + \frac{1}{2}\log\left(\frac{Y^2+(2n\gamma+Z+R)^2{\alpha_1}^2}{Y^2+(2n\gamma+Z)^2{\alpha_1}^2}\right)$$
$$-\alpha_1(1+2n\gamma-Z)\left(\frac{(2n\gamma-Z)\alpha_1}{Y^2+(2n\gamma-Z)^2{\alpha_1}^2} - \frac{(2n\gamma-Z+w)\alpha_1}{Y^2+(2n\gamma-Z+R)^2{\alpha_1}^2}\right)$$
$$-Y^2R\frac{(4n\gamma-2Z+R)}{(Y^2+(2n\gamma-Z)^2{\alpha_1}^2)(Y^2+(2n\gamma-Z+R)^2{\alpha_1}^2)} + \frac{1}{2}\log\left(\frac{Y^2+(2n\gamma-Z+R)^2{\alpha_1}^2}{Y^2+(2n\gamma-Z)^2{\alpha_1}^2}\right)\Bigg\}$$
$$+\sum_{n=1}^{\infty} S_1^n\Bigg\{\alpha_1(1-2n\gamma-Z)\left(\frac{(2n\gamma+Z)\alpha_1}{Y^2+(2n\gamma+Z)^2{\alpha_1}^2} - \frac{(2n\gamma+Z-R)\alpha_1}{Y^2+(2n\gamma+Z-R)^2{\alpha_1}^2}\right)$$
$$+ Y^2R\frac{(4n\gamma+2Z-R)}{(Y^2+(2n\gamma+Z)^2{\alpha_1}^2)(Y^2+(2n\gamma+Z-R)^2{\alpha_1}^2)} + \frac{1}{2}\log\left(\frac{Y^2+(2n\gamma+Z-R)^2{\alpha_1}^2}{Y^2+(2n\gamma+Z)^2{\alpha_1}^2}\right)$$
$$-\alpha_1(1-2n\gamma+Z)\left(\frac{(2n\gamma-Z)\alpha_1}{Y^2+(2n\gamma-Z)^2{\alpha_1}^2} - \frac{(2n\gamma-Z-w)\alpha_1}{Y^2+(2n\gamma-Z-R)^2{\alpha_1}^2}\right)$$
$$+Y^2R\frac{(4n\gamma-2Z-R)}{(Y^2+(2n\gamma-Z)^2{\alpha_1}^2)(Y^2+(2n\gamma-Z-R)^2{\alpha_1}^2)}$$
$$+\frac{1}{2}\log\left(\frac{Y^2+(2n\gamma-Z-R)^2{\alpha_1}^2}{Y^2+(2n\gamma-Z)^2{\alpha_1}^2}\right)\Bigg\}\Bigg] \quad [20]$$

$$\tau_{13} = \frac{b_0 c_1}{2\pi\alpha_1}\Bigg[\sum_{n=0}^{\infty} S_1^n \Bigg\{\alpha_1^2 Y(1+2n\gamma+Z)\left(\frac{1}{Y^2+(2n\gamma+Z+R)^2\alpha_1^2}-\frac{1}{Y^2+(2n\gamma+Z)^2\alpha_1^2}\right)$$
$$+\alpha_1\left(tan^{-1}\left(\frac{(2n\gamma+Z+R)\alpha_1}{Y}\right)-tan^{-1}\left(\frac{(2n\gamma+Z)\alpha_1}{Y}\right)\right)$$
$$+\frac{\alpha_1^2 YR\left(Y^2-\alpha_1^2(2n\gamma+Z)(2n\gamma+Z+R)\right)}{(Y^2+\alpha_1^2(2n\gamma+Z+R)^2)(Y^2+\alpha_1^2(2n\gamma+Z)^2)}$$
$$-\alpha_1^2 Y(1+2n\gamma-Z)\left(\frac{1}{Y^2+(2n\gamma-Z+R)^2\alpha_1^2}-\frac{1}{Y^2+(2n\gamma-Z)^2\alpha_1^2}\right)$$
$$+\alpha_1\left(tan^{-1}\left(\frac{(2n\gamma-Z+R)\alpha_1}{Y}\right)-tan^{-1}\left(\frac{(2n\gamma-Z)\alpha_1}{Y}\right)\right)$$
$$+\frac{\alpha_1^2 YR\left(Y^2-\alpha_1^2(2n\gamma-Z)(2n\gamma-Z+R)\right)}{(Y^2+\alpha_1^2(2n\gamma-Z+R)^2)(Y^2+\alpha_1^2(2n\gamma-Z)^2)}\Bigg\}$$
$$+\sum_{n=0}^{\infty} S_1^n \Bigg\{\alpha_1^2 Y(1-2n\gamma-Z)\left(\frac{1}{Y^2+(2n\gamma+Z-R)^2\alpha_1^2}-\frac{1}{Y^2+(2n\gamma+Z)^2\alpha_1^2}\right)$$
$$-\alpha_1\left(tan^{-1}\left(\frac{(2n\gamma+Z-R)\alpha_1}{Y}\right)-tan^{-1}\left(\frac{(2n\gamma+Z)\alpha_1}{Y}\right)\right)$$
$$-\frac{\alpha_1^2 YR\left(Y^2-\alpha_1^2(2n\gamma+Z)(2n\gamma+Z-R)\right)}{(Y^2+\alpha_1^2(2n\gamma+Z-R)^2)(Y^2+\alpha_1^2(2n\gamma+Z)^2)}$$
$$+\alpha_1^2 Y(1-2n\gamma+Z)\left(\frac{1}{Y^2+(2n\gamma-Z-R)^2\alpha_1^2}-\frac{1}{Y^2+(2n\gamma-Z)^2\alpha_1^2}\right)$$
$$-\alpha_1\left(tan^{-1}\left(\frac{(2n\gamma-Z-R)\alpha_1}{Y}\right)-tan^{-1}\left(\frac{(2n\gamma-Z)\alpha_1}{Y}\right)\right)$$
$$-\frac{\alpha_1^2 YR\left(Y^2-\alpha_1^2(2n\gamma-Z)(2n\gamma-Z-R)\right)}{(Y^2+\alpha_1^2(2n\gamma-Z-R)^2)(Y^2+\alpha_1^2(2n\gamma-Z)^2)}\Bigg\}\Bigg] \qquad [21]$$

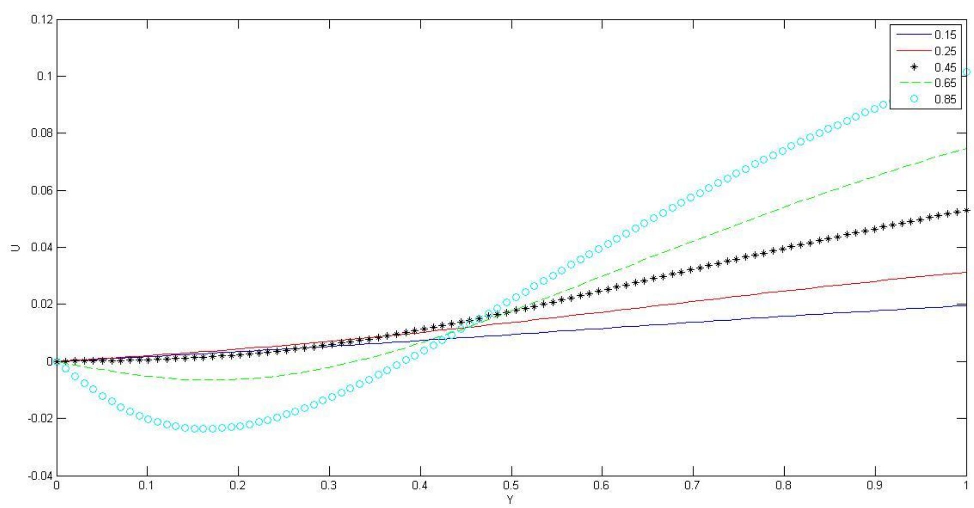
3. NUMERICAL RESULTS AND DISCUSSIONS

In this section, we study the effect of parabolic and linear slips. We consider Topaz to represent the layer (medium I) which is rare magnesia silicate material which usually forms in fractures and cavities of igneous rocks and Olivine to represent half-space (medium II) which is believed to be the most common mineral in the earth's subsurface. For medium I $\alpha_1 = 0.99$, $c_1 = 13.24$ X 10^4 MPa as given by Love (1944)[6]. For medium II as given by Verma (1960) [12] $\alpha_1 = 0.9894$, $c_1 = 8.10$ X 10^4 MPa.

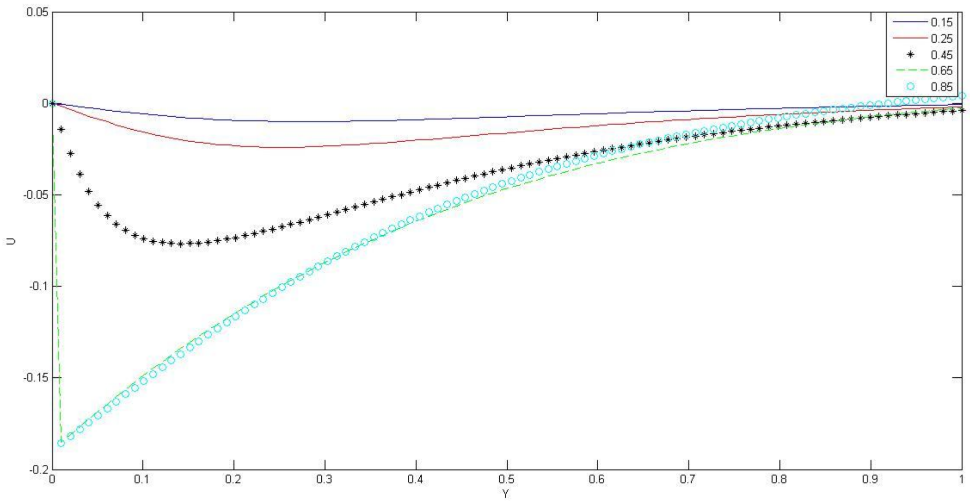
The comparison of the horizontal dimensionless displacements in an orthotropic elastic layered half-space due to parabolic and linear slip profiles for Z=0.5,1 and $\gamma = 0.5$ has been made at different values of R (w / d ratio) in figures 2(a)- (b). From these figures it has been observed that in the vicinity of the fault, as the ratio increases, there is a significant difference occuring in the displacements. The boundary conditions [4c] have been verified from the figures that at surface the displacements are zero for each slip profile. It is further observed that for sufficiently large depth the slip becomes uniform and the corresponding results for displacements and stresses can be obtained as a particular case obtained from our results.



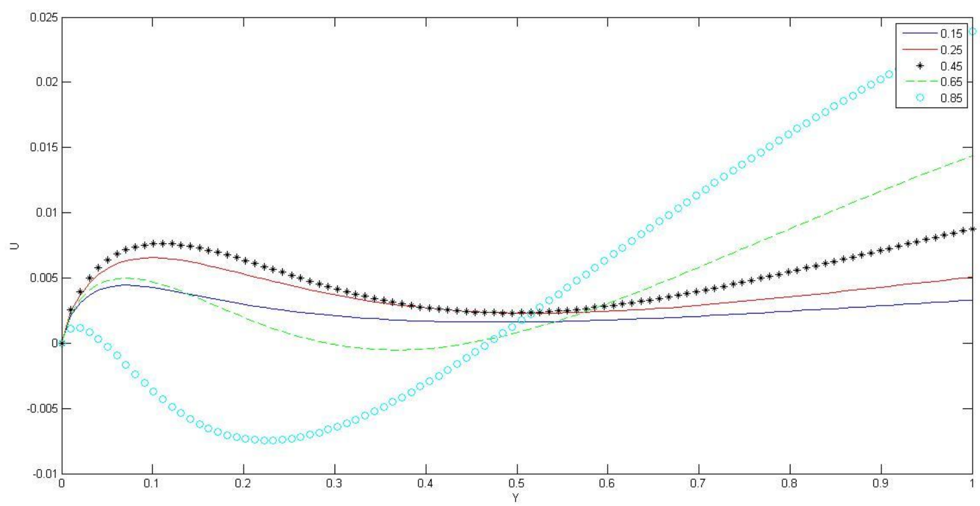
(a)



(b)



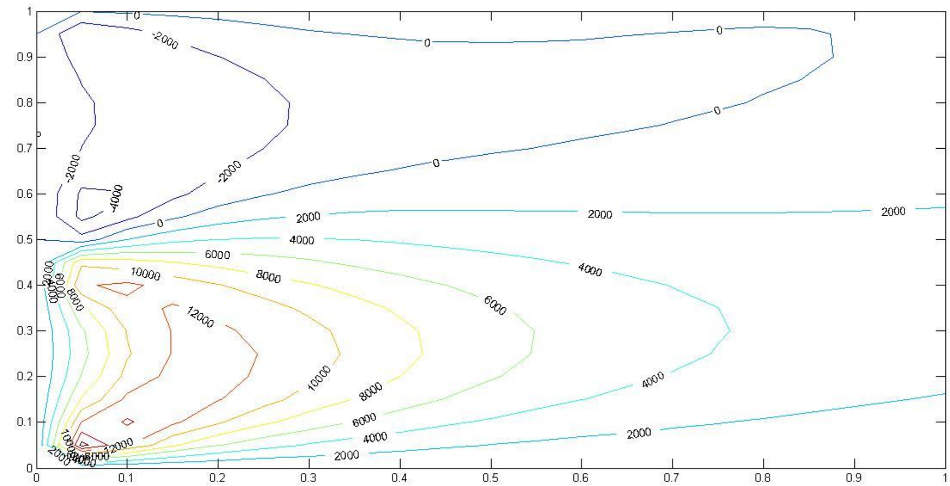
(c)



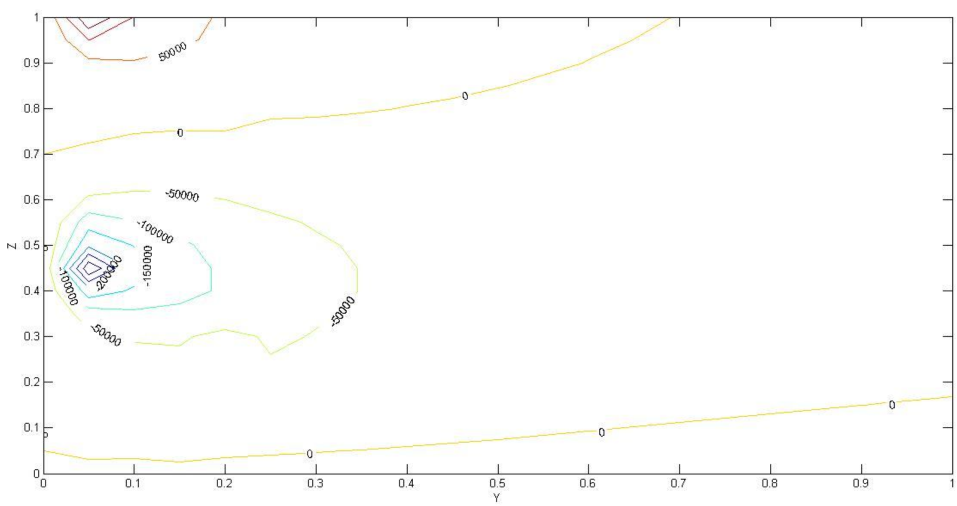
(d)

Figure 2: Variation of the horizontal dimensionless displacement in an orthotropic elastic layered half-space at different width-depth ratio (R=0.15,0.25,0.45,0.65,0.85) form the upper edge of the fault Y for (a) $Z = 0.5, \gamma = 0.5$ for parabolic slip, (b) $Z = 1, \gamma = 0.5$ for parabolic slip, (c) $Z = 0.5, \gamma = 0.5$ for linear slip , (d) $Z = 1, \gamma = 0.5$ for linear slip.

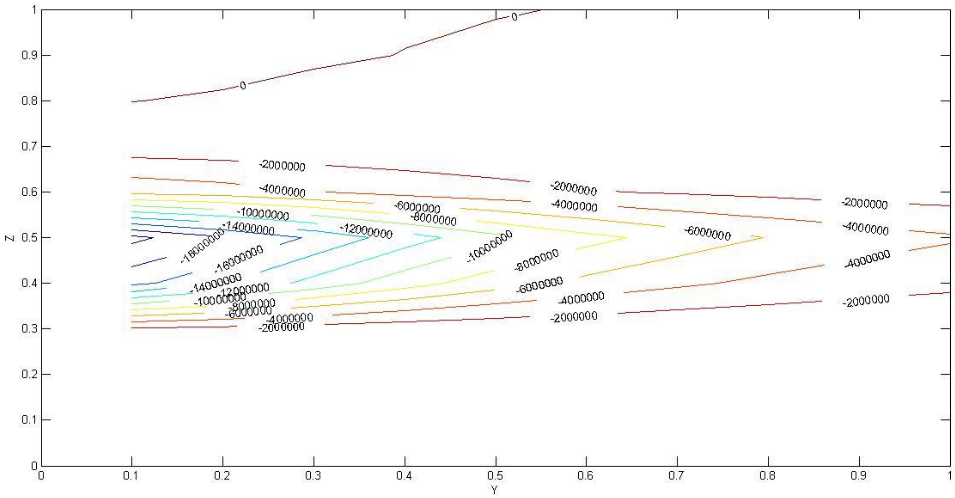
Contour maps of shear-stress τ_{12} *and* τ_{13} for different slip profiles (parabolic and linear) have been depicted in figures (e) – (h) at Width-Depth ratio R=0.45 and γ=0.5.


0
-2000
-4000
2000
4000
6000
8000
10000
12000
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

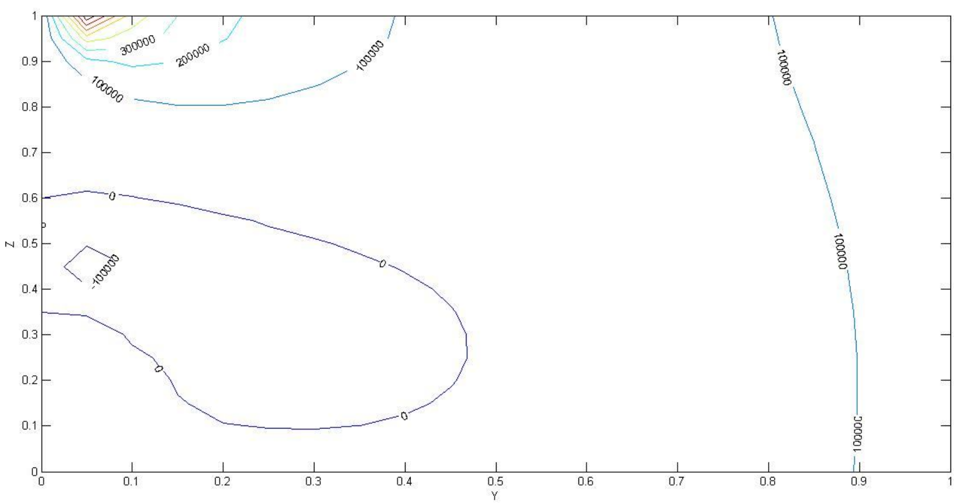
(e)


Z
Y
0
50000
-50000
-100000
-150000
-200000
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

(f)



(g)



(h)

Figure 2: (e) Contour map for Shear-Stress τ_{12} due to parabolic slip with the horizontal distance Y at R=0.45 and $\gamma = 0.5$, (f) Contour map for Shear-Stress τ_{13} due to parabolic slip with the horizontal distance Y at R=0.45 and $\gamma = 0.5$., (g) Contour map for Shear-Stress τ_{12} due to linear slip with the horizontal distance Y at R=0.45 and $\gamma = 0.5$., (h) Contour map for Shear-Stress τ_{13} due to linear slip with the horizontal distance Y at R=0.45 and $\gamma = 0.5$.

4. ACKNOWLEDGEMENT

One of the authors (DKM) is thankful to the University Grants Commission, New Delhi for Major Research Project vide F.No.43-437/2014 (SR).

REFERENCES

[1]. Amano, H.(1981), Stress field in a semi-infinite elastic medium due to internal stress sources, and an interpretation of tectonics in and around Japan. J. Phys. Earth, 29:539-573.

[2]. Chugh, S., Madan, D.K., Singh, K.(2011), Static deformation of an orthotropic elastic layered medium due to a non-uniform discontinuity along a very long strike-slip fault. International Journal of Engineering, Science and Technology, 3(1):69-86.

[3]. Crampin, S. (1994), The Fracture Critically of Cristal Rocks. Geophysical Journal International, 118(2): 428-438.

[4]. Dziewonski, A.M., Anderson, D.L., (1981), Preliminary reference Earth Model. Physics of the Earth and Planetary Interiors, 25(2): 297-356.

[5]. Garg, N.R., Madan, D.K, Sharma, R.K. (1996). Two-dimensional deformation of an orthotropic elastic medium due to the seismic sources. Physics of the Earth and Planetary Interiors, 94(1): 43-62.

[6]. Love, A.E.H. (1944)., A Treatise on the Mathematical Theory of Elasticity. Dover Publications: New York.

[7]. Madan, D.K., Singh, K., Aggarwal, R., Gupta, A., (2005), Displacement and stresses in anisotropic medium due to non –uniform slip along a very long strike-slip fault. ISET Journal of Earthquake Technology, 42(1);1-11.

[8]. Maruyama, T. (1966), On two dimensional elastic dislocations in an infinite and semi-infinite medium. Bulletin of the Earthquake Research Institute, 44(3): 811-871.

[9]. Okada, Y., (1992), Internal deformation due to shear and tensile faults in a half-space. Bulletin of the Seismological Society of America, 82(2): 1018-1040

[10]. Ruina, A., Katzman, Y., Conrad, G.,Horowitz, F.G., Some theory and experiments related to frictional behavior of rocks at low normal stress. Pure and Applied Geophysics, 1-82.

[11]. Singh, S.J., Punia, M. and Rani, S., (1994), Crustal deformation due to non-uniform slip along a long fault. Geophysical Journal International, 118(2):411-427.

[12]. Verma, R.K. (1990), Elasticity of some high-density crystals. Journal of Geophysical Research, 65:757-766.