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Effect of Radiation and Oblateness on the Equilibrium Points in the Elliptic Restricted Three Body Problem

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Abstract

In this paper we have obtained the location of equilibrium points in the elliptic restricted three body problem. We suppose that the bigger primary is oblate and the smaller primary as radiating. We find the positions of triangular equilibrium points in our problem and note that the positions of triangular points are shifted away from the line joining the primaries than in the classical case. Thus the positions are affected due to introduction of eccentricity, semi-major axis, radiation and oblateness factors of both primaries.

Keywords: Equilibrium points, Radiation, Oblateness, Elliptic, RTBP.

1. INTRODUCTION

The elliptic restricted three-body problem (ER3BP) describes the three–dimensional motion of a small particle, called the third body (infinitesimal mass) under the gravitational attraction force of two finite bodies, called the primaries, which revolve in elliptic orbits in a plane around their common centre of mass. The major drawback of the circular restricted three body problem (CR3BP) in celestial mechanics is its inability to treat the long time behavior of practically important dynamical systems. The principal reason is that significant effects might be expected because of the eccentricity of the orbits of the primaries. The orbits of most celestial bodies are elliptic rather than circular, as such the ER3BP analyses the dynamical systems more accurately.

The restricted three-body problem is unable to discuss the motion of the infinitesimal body when at least one of the participating bodies is an intense emitter of radiation. When a star acts upon a particle in a cloud of gas and dust, the dominant factor is by no means gravity, but the repulsive force of the radiation pressures. The motion of a particle in the double stellar system may be of particular interest, because this system forms considerable part of all stellar systems. The photogravitational CR3BP formulated by Radzievsky (1950) [25] may be the simplest model for this problem. Among the various possible motions of the particle, the equilibrium positions around the equilibrium points of a rotating (together with the stars) system of coordinates have practical applications. The photogravitational restricted three-body problem models describe adequately the motion of a particle of a gas-dust cloud which is in the field of two gravitating and rotating stars. The summary action of gravitational and light repulsive forces may be characterized by the mass reduction factor q. The existence and stability of equilibrium points were studied by Chernokov (1970) [9] and Kuniftsyn [12,13] .

The rotation of a star produces an equatorial bulge due to centrifugal forces; as a result, the stars are often oblate in shape. The bodies in the classical restricted three-body problem are strictly spherical, but some planets (Earth, Jupiter and Saturn) and stars (Archernet, Antares and Altair) are sufficiently oblate to make the departure form sphericity very significant in the study of celestial and stellar systems. Taking one or both primaries as sources of radiation or oblate spheroids or both, the effects of oblateness and radiation pressure of the primaries on the existence and stability of equilibrium points in the CR3BP were analyzed by Sharma (1987) [20], Singh and Ishwar (1999)[22], Ishwar and Kushvah (2006)[2].

The ER3BP generalizes the original CR3BP, and improves its applicability, while some outstanding and useful properties of the circular model still hold true or can be adapted to the elliptic case. In particular, possible positions of equilibrium occur when the three bodies form an equilateral triangle. An application of this model can be seen in the motion of the Trojan asteroids around the triangular points L4. The asteroids in this case are only influenced by the gravitational forces of the Sun and the Jupiter, and the orbit of Jupiter around the Sun is assumed to be a fixed ellipse. The influence of the eccentricities of the orbits of the primary bodies with or without radiation pressure(s) on the existence of the equilibrium points and their stability was touched upon to some extent by Kumar and Choudhary (1990)[4], Markelloss et al. (1992)[17]. Singh and Umar (2012) [23] investigated the motion of dust particle in orbit with a dark oblate, degenerate primary and a stellar secondary companion moving in elliptic orbits around their common centre of mass.

We attempt here to examine the motion of infinitesimal body in the ER3BP when bigger primary is oblate and smaller primary as radiating. We find the position of triangular equilibrium points in our problem. The first section describes the introduction and second section provides equations of motion and the positions of triangular equilibrium points. The conclusions are drawn in the third section of the paper.

2. EQUATIONS OF MOTION

Since the orbits of the primaries are elliptic, in order to maintain the primaries in fixed positions–the rotating reference frame-must be a system which rotates uniformly with axes which expand and shrink, the equations of motion of the infinitesimal mass are presented here in dimensionless units in such a rotating pulsating coordinate systems.

$$x'' - 2y' = \Omega_x^*, \qquad y'' + 2x' = \Omega_y^*, \qquad z'' = \Omega_z^* \tag{1}$$

With the force function

$$\Omega = \left(1-e^2\right)^{-1/2}\left[\frac{x^2+y^2}{2} + \frac{1}{n^2}\left(\frac{(1-\mu)}{r_1} + \frac{\mu q_2}{r_2} + \frac{(1-\mu)A_1}{2r_1^3}\right)\right] \tag{2}$$

The mean motion, *n*, is given by

$$n^2 = \frac{\left(1+e^2\right)^{1/2}\left(1+\frac{3}{2}A_1\right)}{a\left(1-e^2\right)} \tag{3}$$

$$r_i^2 = \left(x+x_i\right)^2 + y^2 + z^2, \quad x_1 = -\mu, \; x_2 = 1-\mu, \; \mu = \frac{m_2}{m_1+m_2} \tag{4}$$

Here, m_1, m_2 are the masses of the bigger and smaller primaries positioned at the points $\left(x_i, 0, 0\right), i = 1,2,;$ q_1, q_2 are their mass reduction factors; A_1, A_2 are their oblateness coefficients; $r_i, \left(i = 1,2\right)$ are the distances of the infinitesimal mass from the bigger and smaller primaries, respectively; while *a* and *e* are respectively the semi-major axis and eccentricity of the orbits.

The equilibrium points are the solutions of the equations
$\Omega_x = \Omega_y = \Omega_z = 0$.

that is,

$$\begin{aligned} & x - \frac{1}{n^2}\left(\frac{(1-\mu)(x+\mu)}{r_1^3} + \frac{\mu q_2(x+\mu-1)}{r_2^3} + \frac{3(1-\mu)(x+\mu)A_1}{2r_1^5}\right) = 0 \\ & y\left[1 - \frac{1}{n^2}\left(\frac{(1-\mu)}{r_1^3} + \frac{\mu q_2}{r_2^3} + \frac{3(1-\mu)A_1}{2r_1^5}\right)\right] = 0 \\ & z\left(\frac{(1-\mu)}{r_1^3} + \frac{\mu q_2}{r_2^3} + \frac{3(1-\mu)A_1}{2r_1^5}\right) = 0 \end{aligned} \tag{5}$$

The solutions of the first two equations of system (5) with $y \neq 0, z = 0$ give the positions of the triangular points. Form which we obtain

$$n^2 = \frac{1}{r_1^3} + \frac{3A_1}{2r_1^3}, \qquad n^2 = \frac{q_2}{r_2^3} \tag{6}$$

with $r_2^3 = \frac{q_2}{n^2}$ when oblateness of the bigger primary is absent, and $r_1^3 = \frac{1}{n^2}$ in the absence of oblateness of the smaller one; these values change slightly by ε_1 (say), with the introduction of oblateness, so that

$$r_1 = \frac{1}{n^{2/3}} + \varepsilon_1, \qquad r_2 = \frac{q_2^{1/3}}{n^{2/3}} + \varepsilon_2 \qquad \varepsilon_1, \varepsilon_2 << 1 \tag{7}$$

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onsidering only terms in A_1 and e^2 and neglecting their products, (3) gives

$$n^2 = \frac{1}{a}\left(1 + \frac{3}{2}A_1 + \frac{3}{2}e^2\right) \tag{8}$$

Equation (8) together with A_1 = 0 and (7) give us

$$r_1 = (a)^{1/3}\left(1 - \frac{e^2}{2}\right) + \varepsilon_1$$

$$r_2 = (aq_2)^{1/3}\left(1 - \frac{e^2}{2}\right) + \varepsilon_2 \quad (9)$$

From (6), (7) and (8) and neglecting higher order terms in ε_1, A_1, e^2, we get

$$\varepsilon_1 = -\frac{(a)^{1/3}}{2}\left(A_1 - A_1(a)^{-2/3}\right)$$

$$\varepsilon_2 = -\frac{(a)^{1/3}}{2}(A_1) \quad (10)$$

Substituting for ε_1 in (9), we obtain

$$r_1^2 = (a)^{2/3}\left(1 - e^2 - A_1 + A_1(a)^{-2/3}\right)$$

$$r_2 = (aq_2)^{2/3}\left(1 - e^2 - A_1\right) \quad (11)$$

Using (4) and (11), we get

$$x = \frac{1}{2} - \mu + \frac{1}{2}\left[(a)^{2/3}\left(1 - e^2 - A_1 + A_1(a)^{-2/3}\right) - (aq_2)^{2/3}\left(1 - e^2 - A_1\right)\right]$$

$$y = \pm\left[(a)^{2/3}\left(1 - e^2 - A_1 + A_1(a)^{-2/3}\right) - \frac{1}{4}\left(1 + 2(a)^{2/3}\left(1 - e^2 - A_1 + A_1(a)^{-2/3}\right)\right) - 2(aq_2)^{2/3}\left(1 - e^2 - A_1\right)\right]^{1/2} \quad (12)$$

The triangular Lagrangian points denoted $L_{4,5}(x_1 \pm y)$ are given by (12).

3. CONCLUSIONS

We find that the existences of the triangular points are affected by oblateness coefficient and radiation factor. We conclude that there is a shift in both co-ordinates towards the x-axis and y-axis respectively.

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