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Application of Magneto-Williamson Model in Nanofluid Flow

A. Rameshkumar[1,*], L. Aro Jeba Stanly[2]

Author Affiliation:
[1]Assistant Professor, Dept of Mathematics, Srimad Andavan Arts & Science College, Thiruvanai Kovil, Tiruchirappalli, Tamil Nadu, India-620005.
E-mail: andavanmathsramesh@gmail.com
[2]Research Scholar, Dept of Mathematics, Srimad Andavan Arts & Science College, Thiruvanai Kovil, Tiruchirappalli, Tamil Nadu, India-620005.
E-mail: arojebastanly44@gmail.com

***Corresponding Author:**
A. Rameshkumar, Assistant Professor, Dept. of Mathematics, Srimad Andavan Arts & Science College, Thiruvanai Kovil, Tiruchirappalli, Tamil Nadu, India-620005.

E-mail: andavanmathsramesh@gmail.com

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Abstract

This paper investigates the effect of the nano particle effect on magneto hydrodynamic boundary layer over a stretching in the current of heat generation or absorption with heat and mass fluxes. The governing partial differential equations of the system are transformed into a system of ordinary differential equations and solved numerically with the effect of the non-Newtonian Williamson parameter, heat capacities ratio parameter, heat generation or absorption parameter, Schmidt number on the fluid properties as well as on the skin friction, heat and mass transfer rates are described and explained in detailed through graph.

Keywords: Magneto Hydro Dynamic, nano particle, Williamson fluid model, heat generation. and algorithm.

1. INTRODUCTION

Examinations of Magneto Hydro Dynamic (MHD) limit layer stream and warmth exchange of gooey liquids over a level sheet are essential in many assembling procedures, for example, polymer expulsion, drawing of copper wires, nonstop extending of plastic movies and manufactured filaments, hot moving, wire drawing, glass-fiber, metal expulsion, and metal turning. Among these reviews, Ahmed et al. [1] started the investigation of MHD viscous Casson fluid flow and heat transfer with second-order slip velocity and thermal slip over a permeable stretching sheet in the presence of internal heat generation/absorption and thermal radiation, Ali J. Chamkha [2]concentrated the hydromagnetic stream and warmth and mass exchange over a penetrable chamber moving with a direct speed within the sight of warmth era/ingestion, concoction response, suction/infusion impacts and uniform transverse attractive field.

Bachok and Ishak [3] researched the magnetohydrodynamic (MHD) blended convection stagnation point stream towards a vertical surface inundated in an incompressible micropolar liquid with endorsed divider warm flux unravelled numerically by a limited contrast technique. Basiri Parsa et al. [4] researched the MHD limit layer stream over an extending surface with interior warmth era or absorption. Bhaskar Reddy et al. [5] concentrated the impact of variable warm conductivity and fractional speed slip on hydromagnetic two-dimensional limit layer stream of a nanofluid with Cu nanoparticles over an extending sheet with convective limit condition furthermore inferred that the speed diminishes as the attractive parameter increases. Devi et al. [6] concentrated the dissemination impacts on MHD nonlinear stream and warmth exchange past a permeable surface with endorsed warm flux. Das et al. [7] researched the MHD limit layer slip stream and warmth exchange of nanofluid past a vertical extending sheet with non-uniform warmth era/absorption. Dapra, Scarpi G .[8] deduced a perturbation solution for pulsatile flow of a non-Newtonian Williamson fluid in a rock fracture. El-Amin et al. [9] investigated the free convection with mass exchange stream for a micropolar liquid limited by a vertical unbounded surface with an exponentially rotting warm era, under the activity of a transverse attractive field.

El-Amin,[10] studied the combined effect of internal heat generation and magnetic field on free convection and mass transfer flow in a micropolar fluid with constant suction. Gangadhar [11] explored the impacts of inner warmth era and gooey dispersal on limit layer stream over a vertical plate with a convective surface limit condition and reasoned that neighbourhood skin rubbing coefficient increments and nearby Nusselt number coefficient diminishes with an expansion in both Eckert number and nearby warmth era parameter.

Gangadhar [12] researched the radiation, warm era gooey scattering and magneto hydrodynamic consequences for the laminar limit layer about a level plate in a uniform stream of liquid (Blasius stream), and about a moving plate in a calm surrounding liquid (Sakiadis stream) both under a convective surface limit condition.

Gorla, and Sidawi, [13] explored the free convection on a vertical stretching surface with suction and blowing. Hitesh Kumar [14] concentrated on the impacts of radiation and warmth sink over an extending surface within the sight of a transverse attractive field on two-dimensional limit layer consistent stream and warmth exchange of a gooey incompressible liquid. Hunegnaw Dessie, Naikoti Kishan [15] examined the MHD limit layer stream and warmth exchange of a liquid with variable consistency through a permeable medium towards an extending sheet by taking into consideration the impacts of thick dispersal in nearness of warmth source/sink and presumed that because of the inward warmth sink the warm limit layer increments, though it diminishes with warmth source.

In the present study we study the relation of magneto hydrodynamic limit layer stream on warmth exchange of Williamson nanofluid stream within the sight of warmth era or ingestion and warmth and mass fluxes. The administering limit layer conditions have been changed to a two-point limit esteem issue in likeness factors and the resultant issue is settled numerically. The impacts of different representing parameters on the liquid speed, temperature, nanoparticle volume grinding, diminished Nusselt number and nanoparticle volume erosion slope are represented in figures and investigated in detail.

2. MATHEMATICAL FORMULATION

We here consider a two-dimensional consistent stream of an incompressible nano Williamson liquid over an extending surface. A uniform magnetic field is connected in the y-direction ordinary to the stream direction. The magnetic Reynolds number is thought to be little so that the induced magnetic field is dismissed. The plate is stretched along x-axis with a velocity B_x, where $B>0$ is stretching parameter. The pictorial physical representation of the quandary is given in Fig. 1.

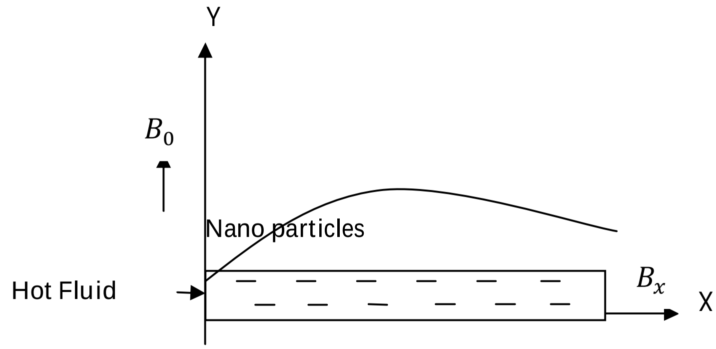

Fig. 1: Schematic representation

The fluid velocity, temperature and nanoparticle concentration near surface are assumed to be U_w, T_w and C_w, respectively. For Williamson fluid model the extra stress tensor τ is defined as

$$\tau = \left[\mu_0 + \frac{(\mu_0 - \mu_\infty)}{1 - \Gamma \dot{\gamma}} \right] \Lambda_1 \qquad (2.1)$$

Where, μ_0 is the limiting viscosity at zero shear rate and μ_∞ is the limiting viscosity at infinite shear rate is a time constant, Λ_1 is the first Rivlin–Erickson tensor and $\dot{\gamma}$ is defined as follows:

$$\dot{\gamma} = \sqrt{\frac{1}{2}\pi} \qquad (2.2)$$

$$\pi = trace(\Lambda_1^2) \qquad (2.3)$$

Here we considered the case for which $\mu_\infty = 0$ and $\Gamma \dot{\gamma} < 1$. Thus Eq. (2.1) can be written as

$$\tau = \left[\frac{\mu_0}{1 - \Gamma \dot{\gamma}} \right] \Lambda_1 \qquad (2.4)$$

Or by using binomial expansion we get

$$\tau = \mu_0 \left[1 + \Gamma \dot{\gamma} \right] \Lambda_1 \qquad (2.5)$$

The above model reduces to Newtonian for $\Gamma = 0$
The equations governing the flow are Continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (2.6)$$

Momentum equation

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\gamma\frac{\partial^2 u}{\partial y^2}+\sqrt{2}\gamma\Gamma\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2}{\rho}u \tag{2.7}$$

Energy equation

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{\rho_P c_P}{\rho c}\left[D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right]-\frac{1}{\rho c_P}q_0\left(T-T_\infty\right) \tag{2.8}$$

Volumetric species equation

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2}+Kr(C-C_\infty) \tag{2.9}$$

The boundary conditions are

$$u=U_w, v=0, \frac{\partial T}{\partial y}=-\frac{q_w}{\alpha}, \frac{\partial C}{\partial y}=-\frac{q_m}{D_B}\ at\ y=0 \tag{2.10}$$

$$u\to 0, T\to T_\infty, C\to C_\infty as\ y\to\infty$$

Since the surface is stretched with velocity B_x, thus $U_w = B_x$ and u and v are horizontal and vertical components of velocity, ν is the kinematic viscosity, q_w , q_m are the heat and mass fluxes per unit area at the surface, respectively.

B_0 is the magnetic field

α isthe nanofluid thermal diffusivity.

ρ is nanofluid density

q_0 is the heat source / sink constant

ρc and $\rho_P c_P$ are heat capacities of nanofluid and nanoparticles

T is temperature

k is nanofluid thermal conductivity

D_B is Brownian diffusion coefficient

C is nanoparticle volumetric fraction

D_T is thermophoretic diffusion coefficient

T_∞ is the ambient fluid temperature.

In order to transform the equations (2.6) to (2.10) into a set of ordinary differential equations, the following similarity transformations and dimensionless variables are introduced

$u = Bxf'(\eta)$, $v = \sqrt{B\nu}f(\eta)$, $\eta = y\sqrt{\frac{B}{\nu}}$

$\theta(\eta) = \frac{T-T_\infty}{T_w-T_\infty}$, $\phi(\eta) = \frac{C-C_\infty}{C_w-C_\infty}$ (2.11)

$Pr = \frac{\nu}{\alpha}$ (Prandtl number = momentum diffusivity / nanofluid thermal diffusivity)

$M = \frac{\sigma B_0^2}{\rho B}$ (Hartmann number)

$Le = \frac{\alpha}{D_B}$ (Lewis number = nanofluid thermal diffusivity / Brownian diffusivity)

$\beta = \Gamma x\sqrt{\frac{2B^3}{\nu}}$ (Non Newtonian Williamson parameter)

$Sc = \frac{\nu}{D_B}$ (Schmidt number = momentum diffusivity / Brownian diffusivity)

$Q = \frac{q_0}{B\rho C_P}$ (Heat source parameter)

$Nc = \frac{\rho_P C_P (C_W - C_\infty)}{\rho c}$ (Heat capacities ratio = nano particle heat capacity / nanofluid heat capacity)
$Nt = \frac{D_B T_\infty (C_W - C_\infty)}{D_T (T_W - T_\infty)}$ (Diffusivity ratio = Brownian diffusivity / thermophoretic diffusivity)
where
$f(\eta)$ is the dimensionless stream function
θ-the dimensionless temperature
ϕ - the dimensionless nanoparticle volume fraction
η-the similarity variable.

In view of the equation (2.11), the equations (2.7) to (2.10) transform into

$$f''' + ff'' - f'^2 + \beta f'' f''' - Mf' = 0 \tag{2.12}$$

$$\theta'' + \Pr f\theta' + \frac{Nc}{Le}\theta'\phi' + \frac{Nc}{LeNt}\theta'^2 + Q\theta = 0 \tag{2.13}$$

$$\phi'' + Scf\phi' + \frac{1}{Nt}\theta'' + ScK\phi = 0 \tag{2.14}$$

where f, θ and ϕ are functions of η and prime denotes derivatives with respect to η.
The transformed boundary conditions can be written as

$$f = 0,\ f' = 1, \theta' = -1, \phi' = -1 \quad \text{at} \quad \eta = 0$$
$$f' \to 0, \theta \to 0, \phi \to 0 \quad \text{as} \quad \eta \to \infty \tag{2.15}$$

where the prime denotes differentiation with respect to η.
If we put $\beta = 0$, our problem reduces to the one for Newtonian Nano (fluid flow) and for $D_B = D_T = 0$ in eq. (2.8) our heat equation reduces to the classical boundary layer heat equation in the absence of heat generation / absorption.

The quantities of physical interest are the values of $f''(0), 1/\theta(0)$ and $1/\phi(0)$ which represent the skin friction, heat and mass transfer rates of the surface, respectively.
where $\mathrm{Re} = \frac{Bx^2}{\nu}$ is the local Reynolds number.

3. RESULT ANALYSIS

In order to provide a clear insight of the physical problem, the velocity, temperature and nanoparticle volume friction have been analyzed by assigning numerical values to the governing parameters Magnetic parameter (M), Non-Newtonian Williamson parameter (β), Heat capacities ratio (Nc), Diffusivity ratio (Nt), Heat generation / absorption (Q), Prandtl number (Pr), Lewis number (Le), Schmidt number (Sc), local Nusselt number (Nu) and local Sherwood number (Sc) respectively. Numerical computations are shown graphically in figures.1-7.

Figs. 2(a)-(c) depict the effect of the magnetic parameter (M) on the velocity, temperature and mass volume fraction profiles. It illustrates that the velocity of the fluid decreases with increase in the values of M and temperature, mass volume friction of the fluid increases with increasing values of M.

Figs. 3(a)-(c) show the effect of the non-Newtonian Williamson parameter (β) on the velocity, temperature and mass volume fraction profiles. It illustrates that the velocity of the fluid decreases with an increase in β and temperature, mass volume friction of the fluid increases with increasing values of β.

Figs. 4(a) & (b) exhibit the effect of Heat capacities ratio Nc on temperature and mass volume friction profiles. It illustrates that on increasing the values of Nc the temperature of the fluid increases and mass volume friction of the fluid is decreases.

Figs. 5(a) & (b) display the effect of Diffusivity ratio Nt on temperature and mass volume friction profiles. These figures illustrate that with an increase in the values of Nt, the temperature and mass volume friction of the fluid decreases.

Figs. 6(a) & (b) represent the effect of heat generation/absorption (Q) on temperature and mass volume friction profiles. It illustrates that on increasing values of (Q) the temperature and mass volume friction of the fluid is decreases.

Fig. 7 depicts the effect of Prandtl number (Pr) on temperature profile. It illustrates that temperature of the fluid decreases with increasing values of the Prandtl number.

Fig. 8 shows the effect of Lewis number (Le) on temperature profile. It illustrates that temperature of the fluid decreases with increasing values of the Lewis number.

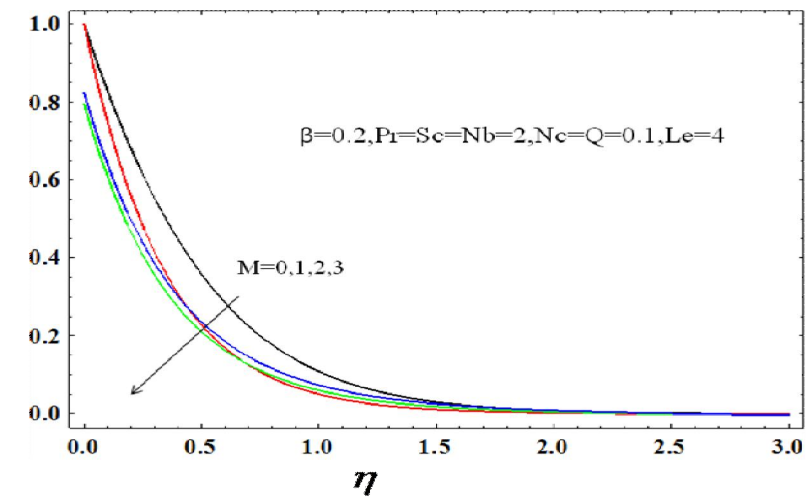

Fig. 2(a): Velocity for different values of M

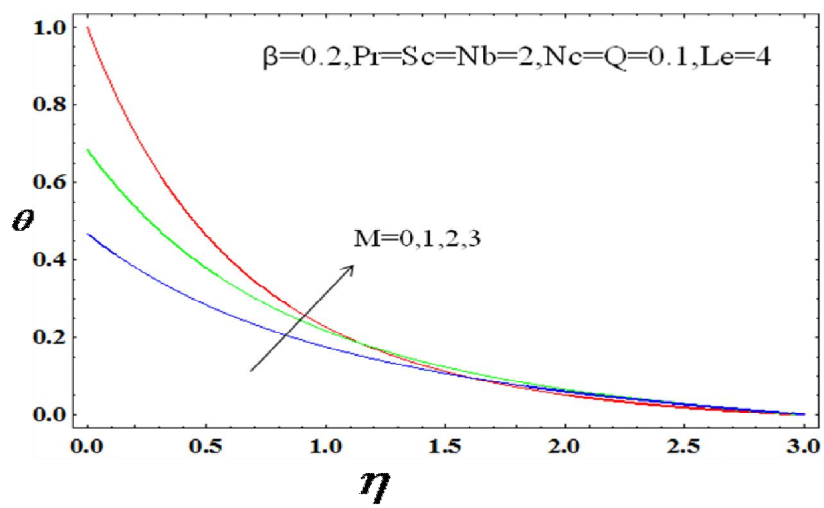

Fig. 1(b): Temperature for different values of M

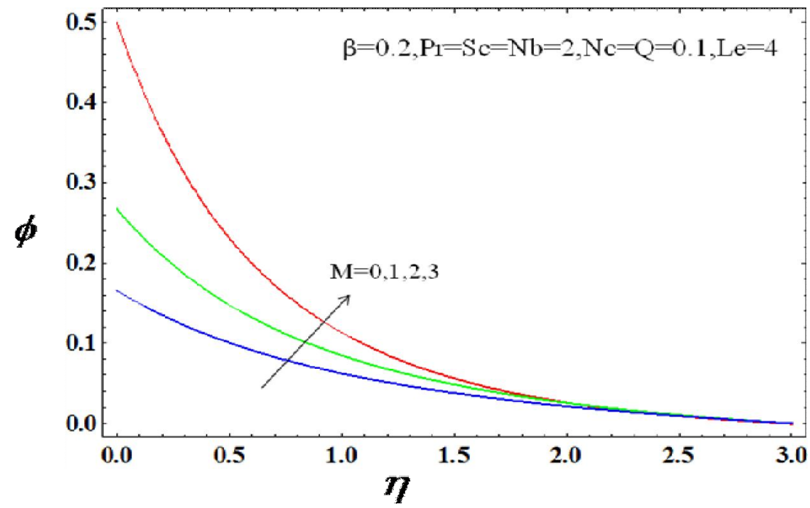

Fig. 2(C): Nano particle volume fraction for different values of M

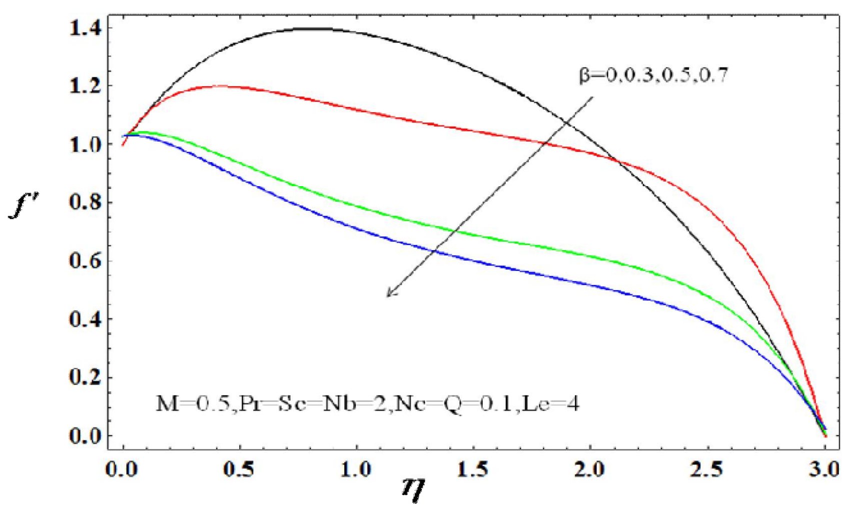

Fig. 3(a): Velocity for different values of β

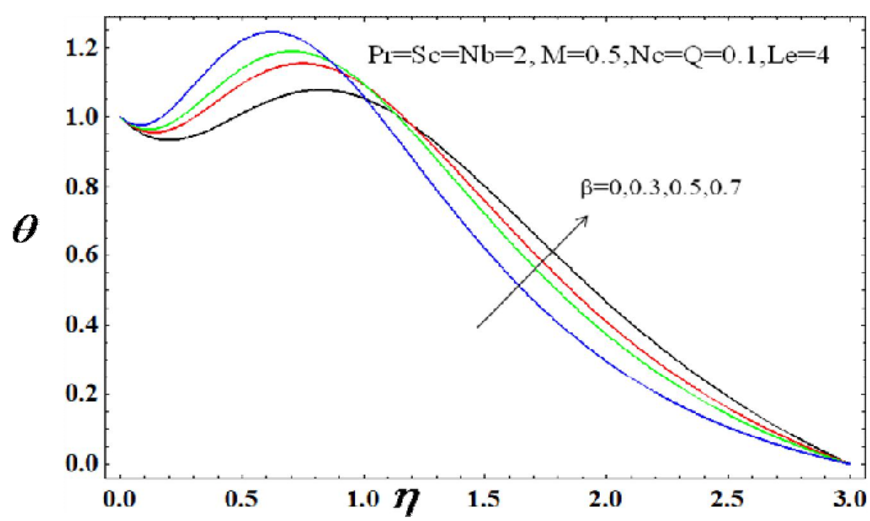

Fig. 3(b): Temperature for different values of β

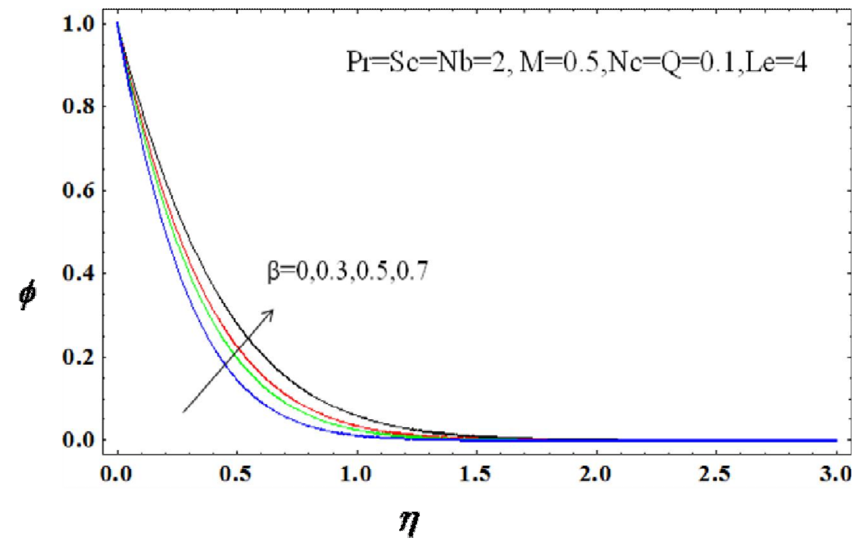

Fig. 3(C): Nanoparticle volume fraction for different values of β

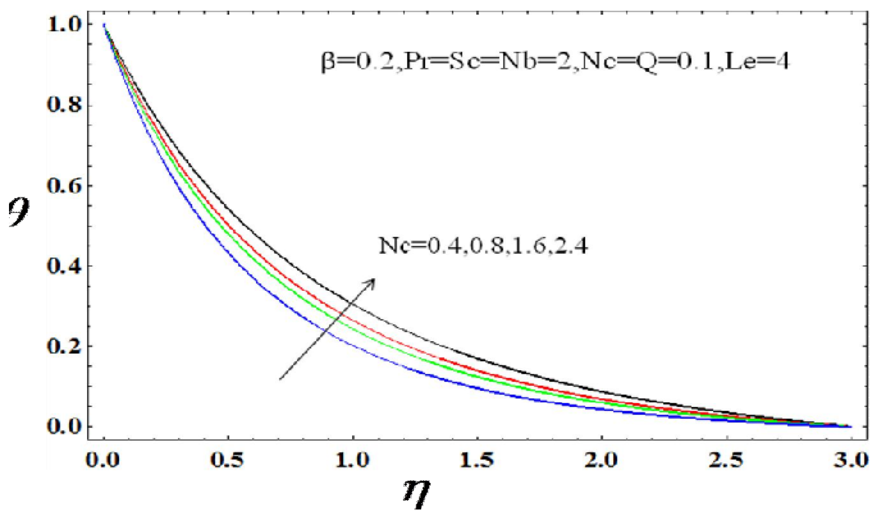

Fig 4(a): Temperature for different values of Nc

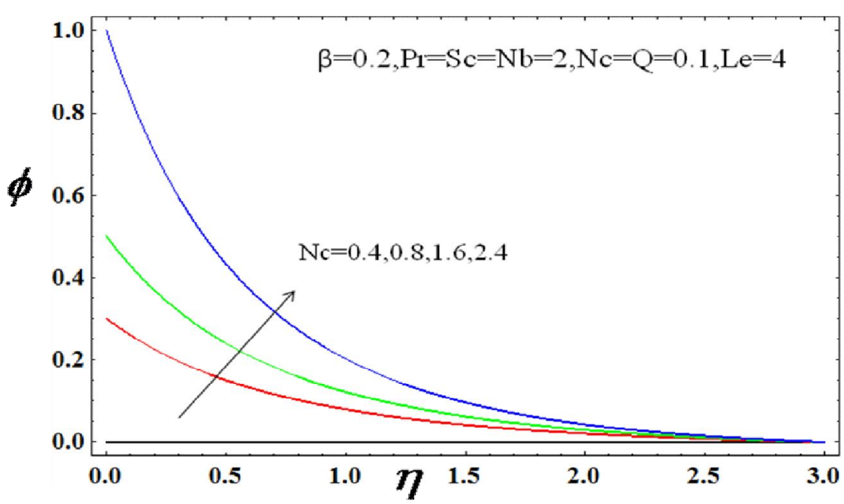

Fig 4(b): Nano particles volumes fraction for different values of Nc

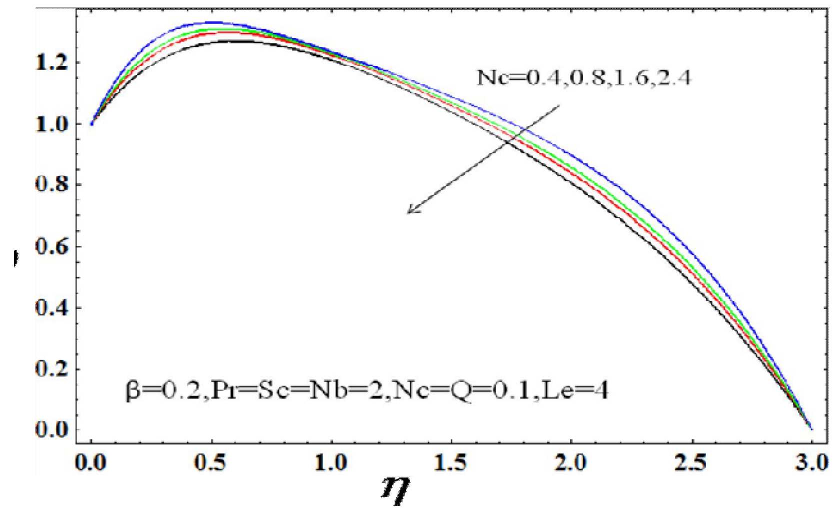

Fig. 5(a): Temperature for different values of Nb

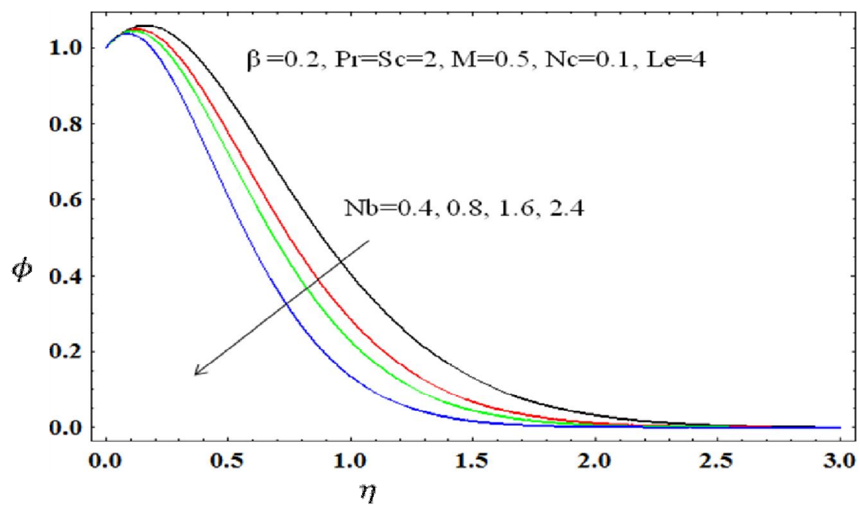

Fig. 5(b): Nanoparticle volume fraction for different values of Nb

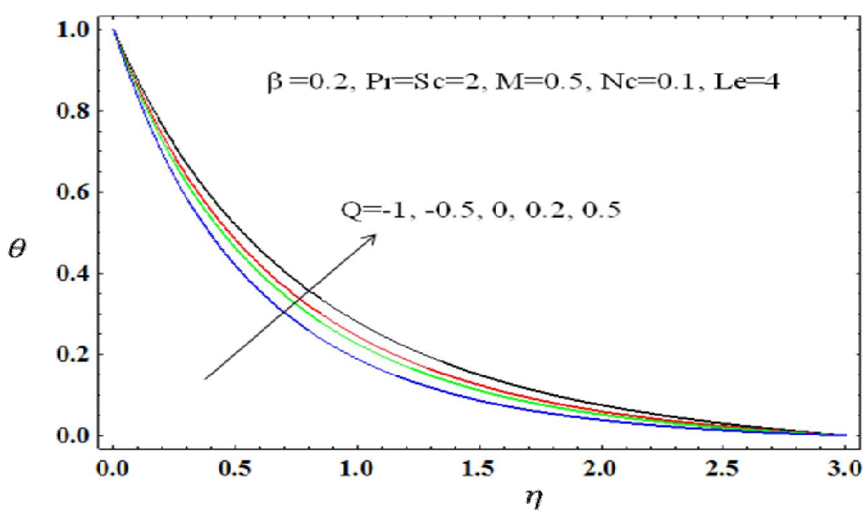

Fig. 6 (a): Temperature for different values of Q

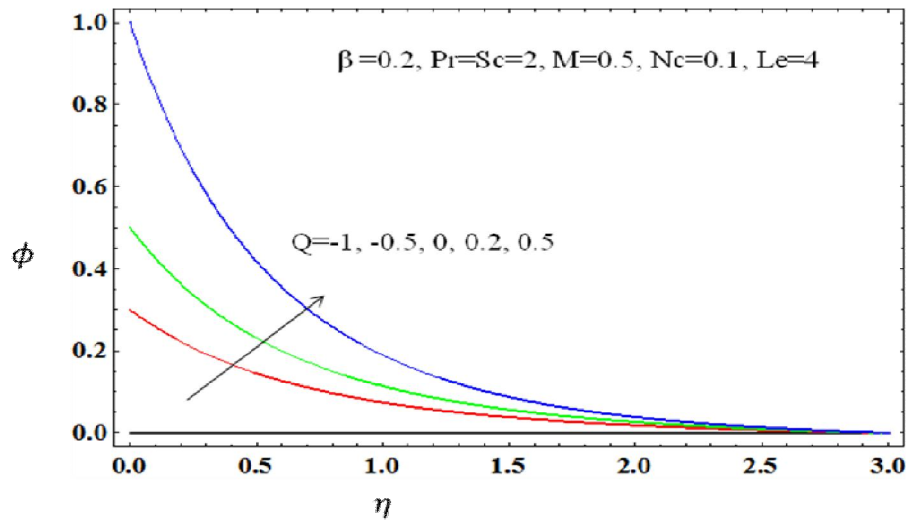

Fig. 6(b): Nanoparticle volume fraction for different values of Q

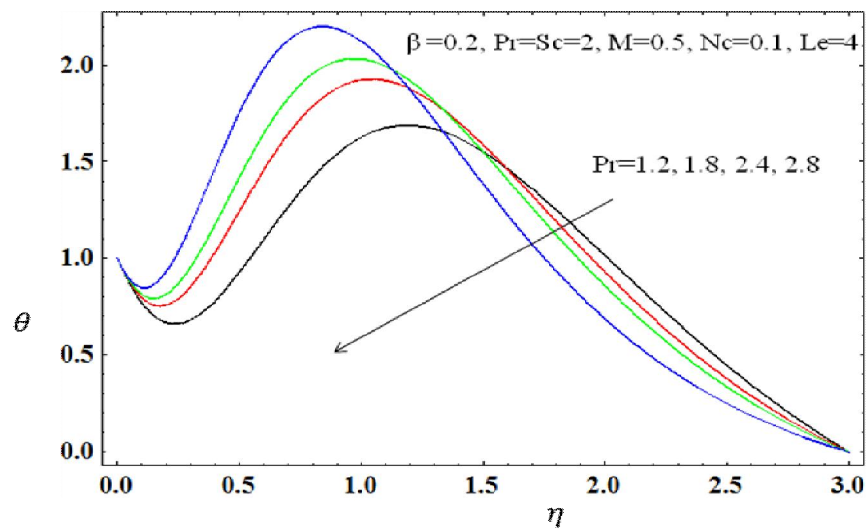

Fig. 7: Temperature for different values of Pr

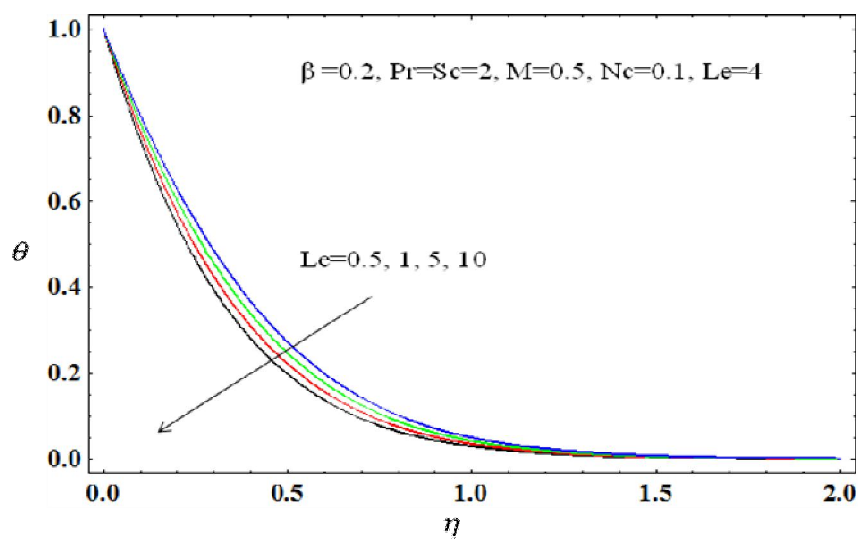

Fig. 8: Nanoparticle volume fraction for different values of Le

4. CONCLUSION

The velocity of the fluid decreases with increase in the values of Magnetic parameter M and Non-Newtonian Williamson parameter β. Temperature increases with increasing values of the Non Newtonian Williamson parameter, Magnetic parameter, Heat capacity ratio and heat source whereas the temperature decreases for increasing values of Diffusivity ratio and Prandtl number. Nano particle volume friction of the fluid increases with increasing values of Magnetic parameter, Non Newtonian Williamson parameter, Heat capacity ratio and heat source. When the values of Diffusivity ratio and Lewis number are increased then the nano particle volume friction of fluid decreases.

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