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Neighborhood Cordial (Nhd-C) Labeling - A New Method of Graph Labeling

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Abstract

We introduce a new technique of graph labeling named as Neighborhood Cordial (Nhd-C) Labeling of Graphs and show that paths, cycles, FL (Cn), S (C_3,n), pathunion Pm(C_3), Pm (C_4),$K_{1,m}$: $K_{1,m}$, *TreeYn* are nhd-c graphs.

Keywords: Nhd, neighborhood, cordial, pathunion, snakes, flag graph etc.

Subject Classification: 05C78

1. INTRODUCTION

The graphs considered here are finite, simple and connected. For general definitions and terminology we refer to Harary [4], and Gallian [4]. The concept of cordial labeling was introduced by I. Cahit. [4].Then up till now a number of cordial labelings have been proposed by different authors [3],[5],[9]. The concept of nhd-p labeling was introduced by Patel and Shrimali [8] in 2015. Making use of neighborhood concept of vertex we combine it with cordiality to obtain Neighborhood Cordial (Nhd-C) Labeling Of Graph.

Let G be a (p,q) graph .Define a function f:V(G)→{1,2,…p} such that for every vertex v ϵ V(G), deg(v)>1 the gcd {f(u)/uϵN(v)} is considered to label of the vertex v. This produces the function f*: V (G) →{0,1} and f*(u) =1 when gcd {f(u)/uϵN(v)} is 1 and 0 otherwise. Further for degree one vertex (pendent vertex) the f*label is taken as 0 if it is adjacent to vertex x with f(x) as a even number and f*(x) is 1 otherwise. Further number of vertices with label 0 and that with label 1 differ at most by

1.We use the convention $v_f(0,1) = (x,y)$ to indicate the number of vertices with label 0 are x and that with label 1 are y in number. The graphs that admit nhd-cordial labeling are called as nhd-c graphs.
In this paper we have defined and shown that paths, cycles, Flag of cycles, S(C3,n) ,y-tree y_n, pathunion $Pm(C_3)$,$Pm(C_4)$$K_{1,m}: K_{1,m}$etc.areNhd-c graphs.

2. DEFINITIONS

2.1 Path Pn is a sequence of vertices and edges given by path (uv) =($v_1,e_1,v_2,e_2,v_3,e_3\ldots v_n$.) It has n-1 edges and n vertices.

2.2 Cycle Cn is a closed path (uu).It is generally given by (v_1=u,e_1,v_2,e_2,…, v_n,e_n,u). We also can write a (uu) cycle as (u=v_1,v_2, v_3..,v_n,u) .

2.3 S (C_3,n) is a snake on n-1 blocks of C_3.We start with path (v_1,v_2,..,v_n) .Between every two consecutive vertices of Pn, namely v_i and v_{i+1} take new vertex u_{i1} and join it by new edges (v_iu_{i1}),($u_{i1}v_{i+1}$).Thus S(c3,n) has 3(n-1) edges and 2n-1 vertices.

2.4 FL (G) is obtained by attaching an edge to a suitable vertex of G.It has |V(G)| +1 vertices and |E(G)|+1edges.

2.5 Pm (G) is pathunion of G. We start with a path p_m and m copies of a (p,q) graph G are taken. At each vertex of path Pm a copy of G is attached. The point of attachment is same for all copies of G. It has mp vertices and q+m-1 edges.

2.6 A *y*-tree is a graph obtained from a path by appending an edge to a vertex of a path adjacent to an end point and it is denoted by *Yn* where *n* is the number of vertices in the tree.

2.7 The graph $K_{1,m}: K_{1,n}$ is obtained by joining the center *u* of the star $K_{1,a}$ and the center *v* of another star $K_{1,m}$ to a new vertex *w*. The number of vertices is $n + m + 3$ and the number of edges is $n + m + 2$.

3. MAIN RESULTS

Theorem 3.1: Path Pn is nhd-c graph.

Proof: The ordinary labels of path are defined using vertices as be (v_1,v_2,v_3,…,v_n). Define f:V(Pn)→ {1,2,3,…n} given by $f(v_i) = i$, i = 1,2,..n. If n = 2x this produces $v_f(0,1) = (x,x)$ and for n = 2x+1 we have $v_f(0,1) = (x+1,x)$.

Theorem 3.2: Cn is nhd-c graph for an even number n, where, n>3.

Proof: Define f: V (Cn)→{1,2,..n} as $f(v_i)$ =i, where, i=1,2,..,n. Taking n =2x, we have $v_f(0,1) = (x,x)$ from where the result follows.

Theorem 3.3: Fl (Cn) is nhd-c graph.

Proof: A Flag of Cn is defined using vertices as v1,v2,v3…,v_n,v_{n+1}. Define f: V(G)→{1,2,3,…,2x+1} given by $f(v_i)$= i, where, i=1,2,..,n.

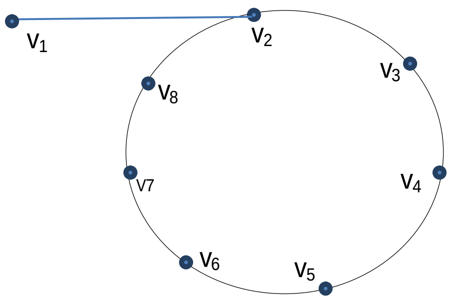


Fig 3.1: Ordinary labeling of

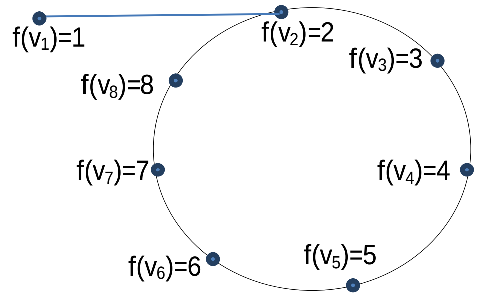


Fig 3.2: Nhd-c labeling of $FL(C_7)$

The numbers observed are for n = 2x ,$v_f(0,1)$ =(x+1,x) and for n = 2x+1 ,$v_f(0,1)$ =(x,x).

Theorem 3.4: S (C_3,n), a snake on n-1 blocks is nhd-c graph for n>3.

Proof: A C_3 snake S (C_3,n) is obtained by taking a path defined by using the vertices as P_n given by $v_1,v_2,v_3,\ldots v_n$. A new vertex u_i is taken between vertices v_i ,v_{i+1} (not on edge v_iv_{i+1}) and new edges (v_iw_i) and (w_iv_{i+1}).A function f :V(G)→{1,2,3..,2(n-1)+1} given by $f(v_i)$ = 2i for i = 1,2,4,5,..,n-2; $f(v_3)$ = 2(n-1), $f(v_{n-1})$=6, $f(v_n)$=3, $f(u_1)$=2n-1, $f(u_2)$=1, $f(u_i)$=2i-1 for i=3,4,… .

We get number distribution as $v_f(0,1)$ =(n-1,n)

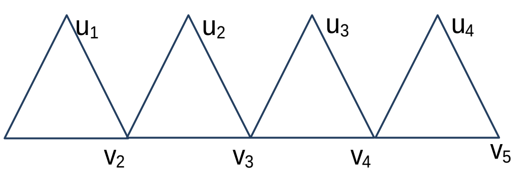


Fig 3.3: S(C_3,4) with ordinary labeling

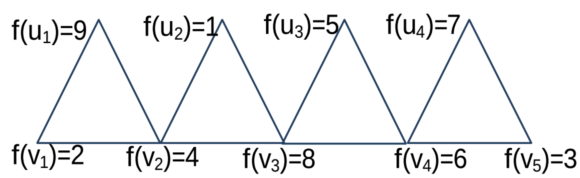


Fig3.4: S(C_3,4) with nhd-c labeling

Theorem 3.5: Pm (C3) is nhd-c graph iff m is an even number.

Proof: Take a path defined by using vertices as p_m=$v_1,v_2,v_3,\ldots,v_n$. The vertices of C_3 at i^{th} vertex of path Pn are given by $w_{i,1}$=v_i,$w_{i,2}$,$w_{i,3}$. It has p= 3n vertices and q=4n-1 edges.

A function f :V(G)→{ 1,2, 3..,p} given by: $f(v_i)$=2+(i-1)6; $f(w_{i1})$= $f(v_i)$+2; $f(w_{i2})$= $f(v_i)$+4;$f(v_{x+j})$=6(j-1)+1;$f(w_{x+j,1})$=$f(v_{x+j})$+2; $f(w_{x+j,2})$=$f(v_{x+j})$+4 for i = 1,2,..x and j=1,2,..x where, x=$\frac{p}{2}$, p being even number.

The proof then follows.

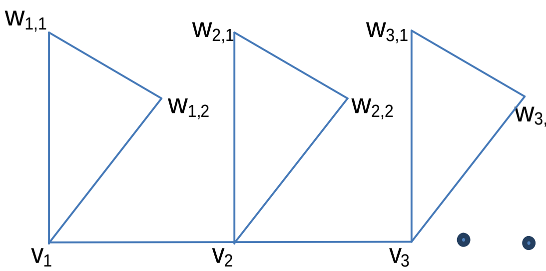


Fig 3.5: $P_m(C_3)$ ordinary labeling

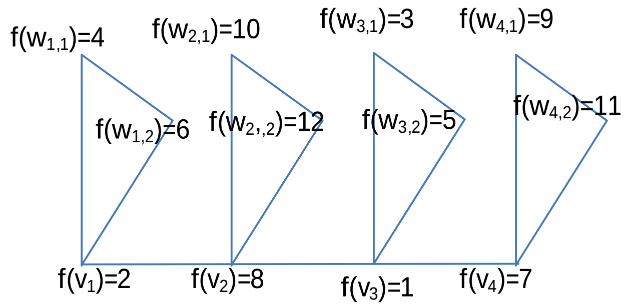


Fig 3.6: $P_m(C_3)$ nhd-c labeling

Theorem 3.6: Pm (C_4) is nhd-c graph.

Proof: Take a path defined using vertices as $p_m=v_1,v_2,v_3,\ldots,v_n$. The vertices of C_4 at i^{th} vertex of path Pn are given by $w_{i,1}=v_i,w_{i,2},w_{i,3}$,$w_{i,4}$. It has p= 4n vertices and q=5n-1 edges. A function f :V(G)→{ 1,2, 3..,p} given by

$f(v_i)=4(i-1)+1$,i =1,2,..n
$f(w_{ij})=f(v_i)+j$,j=1,2,3. This gives label numbers distribution as $v_f(0,1)=(2n,2n)$

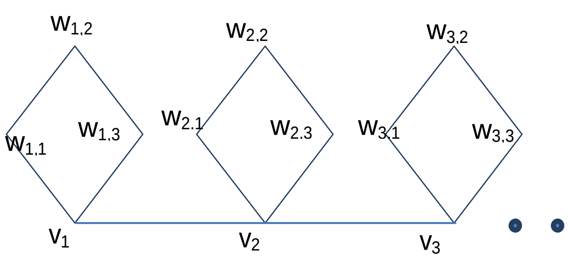


Fig 3.7: ordinary labeling of $P_m(C_4)$

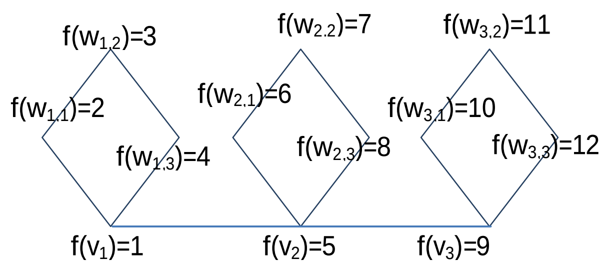


Fig 3.8: nhd-c labeling of $P_m(C_4)$

Theorem 3.7: A y-tree y_n is nhd-c graph for n>5.

Proof: A y-tree is defined by vertices as $y_1,y_2,y_3,y_4,\ldots,y_{n-1},y_n$.This is shown in figure 3.9 below.

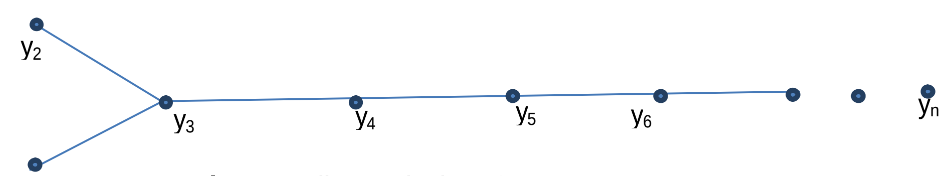


Fig 3.9: ordinary labeling of Y_n

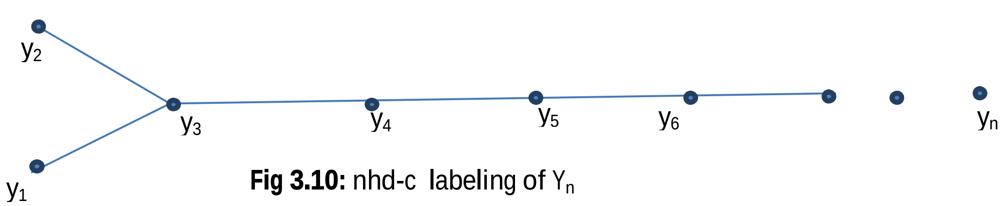


Fig 3.10: nhd-c labeling of Y_n

Define a function f:V(G)→{1,2,3..n}

Case I. When n =6. $f(y_1)=1$, $f(y_2)=5$, $f(y_3)=3$, $f(y_4)=2$, $f(y_5)=6$, $f(y_6)=4$. Note that vf (0,1)=(3,2)

Case II. When n_i i s odd number >6 , say n = 2x+1, then

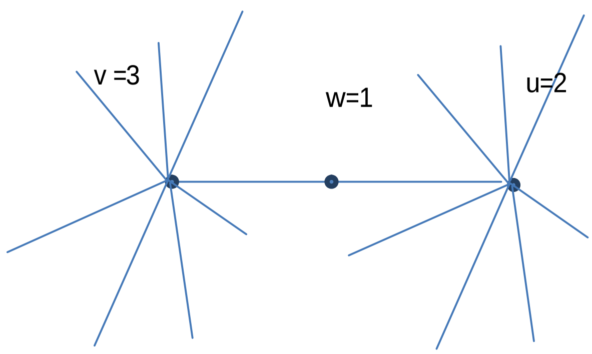
$f(y_1)=5$, $f(y_2)=n$, $f(y_3)=1$,$f(y_i)=2i-1$,i=4,5,…x
$f(y_{x+1})=3$, $f(y_{x+2})=2$, $f(y_{x+3})=6$, $f(y_{x+4})=4$, $f(y_{x+i})=2(i-1)$, i= 5,6,..,x+1. The number distribution is $v_f(0,1)=(x+1,x)$

Case III. When n is even number, say 2x, then
$f(y_1)=5$, $f(y_2)=7$, $f(y_3)=1$, $f(y_4)=9$, $f(y_j)=2j-1$, j=5,..,x-1, $f(y_x)=3$, $f(y_{x+1})=2$, $f(y_{x+2})=6$, $f(y_{x+3})=4$, $f(y_{x+j})=2j$, j= 4,5,…x.

The number distribution is $v_f(0,1)=(x,x)$.

Theorem 3.8: The graph $K_{1,m}$: $K_{1,m}$is a nhd-c graph.

Proof: The vertices adjacent to u are $u_1,u_2,\ldots,u_m$ and that adjacent to v be $v_1,v_2,v_3,\ldots,v_m$. The degree 2 vertex be labeled as w.



Once f*(v) =3 and f*(u) =2 is fixed all the pendent vertices adjacent to v will receive f* image as 1 and that all pendent vertices adjacent to u will receive f* label as 0, this is independent of what actual f-labels are received by u_i and v_i, i = 1.2.,,m. As f*(w)=1, thus the label number distribution is $v_f(0,1)=(m,m+1)$.

4. CONCLUSIONS

We have defined a new type of graph labeling called as nhd-c labeling. We have discussed different families and have shown them to be nhd-c graphs. Further attention on C4-snakes, path unions of even cycles (n>4) as well path unions on odd cycles (n>3) is required.

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