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ANALYSIS OF PRIORITY QUEUEING SYSTEM WITH WORKING BREAKDOWN, REPAIR, IMMEDIATE FEEDBACK AND BERNOULLI VACATION

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Abstract: We analyze an $M^{[X_1]}, M^{[X_2]}/G_1, G_2/1$ queue with two classes of non-preemptive priority service based on working breakdown, repair, immediate feedback and Bernoulli vacation. The server may subject to random breakdown with parameter α, during high priority service (type I), then the server will complete the service for current customer at a slower service rate compared to the regular service rate. On the other hand, during low priority service (type II), it should go for repair immediately. After the completion of each high priority service, there are two choices. First, the server can go for vacation with probability θ, secondly, its serves the next customer which has the probability $(1-\theta)$. In case of customer dissatisfaction after completion of high priority service, immediately they receive service again without joining queue with probability r, or else the customer gets an option to discard, which has a probability of $(1-r)$. We use the established norm, which is the corresponding steady state results for the time dependent probability generating functions. Along with that, the expected time of wait for the expected number of customers in the high and low priority queues are computed. Numerical results along with the graphical representations are shown elaborately.

Keyword: Batch arrival; Priority queue;Working breakdown; Immediate feedback; Bernoulli vacation.

2010AMS Mathematics Subject Classification: $60K25,\ 68M30,\ 90B22$

1.INTRODUCTION

Queueing models have attracted attention of a lot of researchers because of their applications in day-to-day systems. For instance, a common application which is being mentioned in most of the papers is, when a computer system gets an unpredictable number of failures. Now, we can model it as a queueing model in which the servers are designed for unpredictable breakdowns even when a customer is being served.

Takacs [15] introduced the model Bernoulli feedback queue. An immediate feedback is alternative approach to the concept of feedback developed by Kalidass and Kasturi [5]. Choudhury and Kalita [3] studied $M/G/1$ queue with heterogeneous service, optional repeated service subject to server breakdown and delayed repair. Ayyappan and Shyamala [1] investigated an $M^X/G/1$ queue with feedback, breakdowns, vacation under Bernoulli schedule and setup time. Saravanarajan and Chandrasekaran [13] extensively discussed $M/G/1$feedback queue with two types of services, Bernoulli vacation and random breakdown. The updated reference on queue interruptions can be found in Krishnamoorty et al. [7].

Queueing system with working breakdown was introduced by Kalidass and Kasturi [4]. This concept is not the same as working vacations. Kim and Lee [6] studied $M/G/1$ queue with disasters and working breakdowns and an $M/G/1$ retrial queue with disaster under working vacations and breakdowns are extensively discussed by Rajadurai [12].

Madan [9] studied a priority queueing system with service interruptions and an $M/G/1$ queue with preemptive priority service with optional vacation was investigated by Baskar and Palaniammal [2]. Senthil Kumar et al. [14] developed a priority retrial queue with two classes of MAP arrivals using matrix analytic method. Recently, Krishnamoorty and Manjunath [8] studied about priority queueing system where the feedback customers are considered as the low priority customers.

We study single server queueing system subject to working breakdown, repair, immediate feedback and Bernoulli vacation. We assume that after the completion of each high priority service the server can take a vacation and also assume that the high priority customer has an option of getting immediate feedback, if dissatisfied with his service. Further, we also consider the working breakdown during the high priority service, which means that the server may breakdown, but the service does not stop. Here the server will complete his service at a slower rate and after completing the service of the current customer the server will go for repair.

The paper is arranged as follows. The mathematical notations and the definitions are mentioned in section 2, the equations defining the model and the time dependent solutions are mentioned in section 3. The steady state results are obtained in section 4. The expected queue length and expected waiting time are derived in sections 5 and 6, respectively.Few particularcases are mentioned in section 7 and in section 8, some numerical results and their graphical representations are also presented.

2.MODEL DESCRIPTION

2.1 Mathematical Description

1) High priority and low priority customers arrive at the system in batches of variable size in a compound Poisson process. Let $\lambda_h c_i dt (i = 1,2,3,\dots)$ and $\lambda_l c_j dt (j = 1,2,3,\dots)$ be the first order probability that a batch of i and j customers arrives at the system during a short interval of time$(t, t + dt)$, where $0 \le c_i \le 1, \sum_{i=1}^{\infty} c_i = 1, 0 \le c_j \le 1, \sum_{j=1}^{\infty} c_j = 1$, and$\lambda_h > 0, \lambda_l > 0$ are the mean arrival rate for high priority and low priority customers entering into the system. Note that the low priority customers will be served only when there are no high priority customers in the queue. Consequently, high priority customers have a non-preemptive priority over the low priority customers.
2) The customers under the high and the low priority service are looked by a single server on a first in - first out basis.
3) Our next assumption is that as soon as the completion of each high priority service takes palce, the server can take a vacation of random length with probability $\theta$for which case the vacation starts immediately, or else, with a probability of $(1-\theta)$ it may serve the next customer, if any.
4) With the completion of a service for the high priority customers, if they are not satisfied with his service, they get the service immediately without joining the queue with probability r or can exit the system with probability $(1-r)$.
5) After return from the vacation, the server starts serving the customer accordingly, if any. The server waits for new arrivals in case, there are no customers.
6) The system can breakdown at any point of time during the service and the breakdowns can occur according to a Poisson stream which has a mean breakdown rate of $\alpha > 0$. During the high priority service, the server will complete his service at a slower rate compared to regular service rate $(\mu_3(\kappa) < \mu_1(\kappa))$, after that it will go for repair. On the other hand, during low priority service, the server stop his service immediately and goes for repair.
7) The stochastic processes are considered to be independent of each other.

For a steady state, we assume that $B_i(0) = 0, B_i(\infty) = 1, V(0) = 0, V(\infty) = 1, R(0) = 0, R(\infty) = 1$ are continuous at $\kappa = 0$ $(i = 1,2,3)$.

2.2 Definitions

Let

- $N_1(t)$ represent the high priority queue size at time t.
- $N_2(t)$ represent the low priority queue size at time t.
- $B_i^0(t), i = 1,2,3$ be the elapsed service times of the high, low priority and working breakdown services respectively.
- $V^0(t)$ be the elapsed vacation time at time t.
- $R^0(t)$ be the elapsed repair time at time t.
- $Y(t)$ denotes the server state at time t, given by

$$Y(t)= \begin{cases} 0, if the server is idle; \\ 1, if the server is busy with high priority customers \\ during regular service; \\ 2, if the server is busy with low priority customers \\ during regular service period; \\ 3, if the server is busy with high priority customers \\ during working breakdown period; \\ 4, if the server is on vacation; \\ 5, if the server is under repair; \end{cases}$$

The high and low priority service time, working breakdown service time, vacation time and repair time all follows the general (arbitrary) distribution and the notions used for their Cumulative Distribution Function (CDF), the probability density function (pdf) and the Laplace transform (LT) are given in table 1.

Table 1: Notations

Server state	CDF	pdf	LT	Hazard rate
High priority service	$B_1(t)$	$b_1(t)$	$\overline{B}_1(s)$	$\mu_1(\kappa)$
Low priority service	$B_2(t)$	$b_2(t)$	$\overline{B}_2(s)$	$\mu_2(\kappa)$
Working breakdown service	$B_3(t)$	$b_3(t)$	$\overline{B}_3(s)$	$\mu_3(\kappa)$
Vacation	$V(t)$	$v(t)$	$\overline{V}(s)$	$\gamma(\kappa)$
Repair	$R(t)$	$r(t)$	$\overline{R}(s)$	$\eta(\kappa)$

Next, we define the probability $I_0(t) = Pr\{N_1(t) = 0, N_2(t) = 0, Y(t) = 0\}, t > 0$
and probability densities,

$$P_{m,n}^{(1)}(\kappa,t)d\kappa = Pr\{N_1(t) = m, N_2(t) = n, Y(t) = 1; \kappa \le B_1^0(t) < \kappa + d\kappa\},$$
$$P_{m,n}^{(2)}(\kappa,t)d\kappa = Pr\{N_1(t) = m, N_2(t) = n, Y(t) = 2; \kappa \le B_2^0(t) < \kappa + d\kappa\},$$
$$Q_{m,n}^{(1)}(\kappa,t)d\kappa = Pr\{N_1(t) = m, N_2(t) = n, Y(t) = 3; \kappa \le B_3^0(t) < \kappa + d\kappa\},$$
$$V_{m,n}(\kappa,t)d\kappa = Pr\{N_1(t) = m, N_2(t) = n, Y(t) = 4; \kappa \le V^0(t) < \kappa + d\kappa\},$$
$$R_{m,n}(\kappa,t)d\kappa = Pr\{N_1(t) = m, N_2(t) = n, Y(t) = 5; \kappa \le R^0(t) < \kappa + d\kappa\}$$

for $\kappa > 0, t > 0,\ m,n \ge 0$.

3.EQUATIONS GOVERNING THE SYSTEM

Here, we construct a set of Kolomogorov forward equations using the supplementary variables technique as follows:

$$\frac{\partial}{\partial t}P_{m,n}^{(1)}(\kappa,t) + \frac{\partial}{\partial \kappa}P_{m,n}^{(1)}(\kappa,t) = -\big(\lambda_h + \lambda_l + \alpha + \mu_1(\kappa)\big)P_{m,n}^{(1)}(\kappa,t)$$
$$+(1-\delta_{m0})\lambda_h \textstyle\sum_{i=1}^{m} c_i P_{m-i,n}^{(1)}(\kappa,t) + (1-\delta_{n0})\lambda_l \sum_{j=1}^{n} c_j P_{m,n-j}^{(1)}(\kappa,t) \quad (1)$$

$$\frac{\partial}{\partial t}P_{m,n}^{(2)}(\kappa,t) + \frac{\partial}{\partial \kappa}P_{m,n}^{(2)}(\kappa,t) = -(\lambda_h + \lambda_l + \alpha + \mu_2(\kappa))P_{m,n}^{(2)}(\kappa,t)$$

$$+(1-\delta_{m0})\lambda_h \sum_{i=1}^{m} c_i P^{(2)}_{m-i,n}(\kappa,t) + (1-\delta_{n0})\lambda_l \sum_{j=1}^{n} c_j P^{(2)}_{m,n-j}(\kappa,t) \quad (2)$$

$$\frac{\partial}{\partial t}V_{m,n}(\kappa,t) + \frac{\partial}{\partial \kappa}V_{m,n}(\kappa,t) = -(\lambda_h+\lambda_l+\gamma(\kappa))V_{m,n}(\kappa,t)$$
$$+(1-\delta_{m0})\lambda_h \sum_{i=1}^{m} c_i V_{m-i,n}(\kappa,t) + (1-\delta_{n0})\lambda_l \sum_{j=1}^{n} c_j V_{m,n-j}(\kappa,t) \quad (3)$$

$$\frac{\partial}{\partial t}Q^{(1)}_{m,n}(\kappa,t) + \frac{\partial}{\partial \kappa}Q^{(1)}_{m,n}(\kappa,t) = -(\lambda_h+\lambda_l+\mu_3(\kappa))Q^{(1)}_{m,n}(\kappa,t)$$
$$+(1-\delta_{m0})\lambda_h \sum_{i=1}^{m} c_i Q^{(1)}_{m-i,n}(\kappa,t) + (1-\delta_{n0})\lambda_l \sum_{j=1}^{n} c_j Q^{(1)}_{m,n-j}(\kappa,t) \quad (4)$$

$$\frac{\partial}{\partial t}R_{m,n}(\kappa,t) + \frac{\partial}{\partial \kappa}R_{m,n}(\kappa,t) = -(\lambda_h+\lambda_l+\eta(\kappa))R_{m,n}(\kappa,t)$$
$$+(1-\delta_{m0})\lambda_h \sum_{i=1}^{m} c_i R_{m-i,n}(\kappa,t) + (1-\delta_{n0})\lambda_l \sum_{j=1}^{n} c_j R_{m,n-j}(\kappa,t) \quad (5)$$

$$\frac{d}{dt}I_0(t) = -(\lambda_h+\lambda_l)I_0(t) + (1-\theta)(1-r)\int_0^\infty P^{(1)}_{0,0}(\kappa,t)\mu_1(\kappa)d\kappa$$
$$+\int_0^\infty P^{(2)}_{0,0}(\kappa,t)\mu_2(\kappa)d\kappa + \int_0^\infty V_{0,0}(\kappa,t)\gamma(\kappa)d\kappa + \int_0^\infty R_{0,0}(\kappa,t)\eta(\kappa)d\kappa \quad (6)$$

Boundary conditions at $\kappa = 0$,

$$P^{(1)}_{m,n}(0,t) = \delta_{n0}\lambda_h c_{m+1} I_0(t) + r\int_0^\infty P^{(1)}_{m,n}(\kappa,t)\mu_1(\kappa)d\kappa + \int_0^\infty P^{(2)}_{m+1,n}(\kappa,t)\mu_2(\kappa)d\kappa$$
$$+(1-\theta)(1-r)\int_0^\infty P^{(1)}_{m+1,n}(\kappa,t)\mu_1(\kappa)d\kappa + \int_0^\infty V_{m+1,n}(\kappa,t)\gamma(\kappa)d\kappa$$
$$+\int_0^\infty R_{m+1,n}(\kappa,t)\eta(\kappa)d\kappa \quad (7)$$

$$P^{(2)}_{0,n}(0,t) = \lambda_l c_{n+1} I_0(t) + (1-\theta)(1-r)\int_0^\infty P^{(1)}_{0,n+1}(\kappa,t)\mu_1(\kappa)d\kappa$$
$$+\int_0^\infty P^{(2)}_{0,n+1}(\kappa,t)\mu_2(\kappa)d\kappa + \int_0^\infty V_{0,n+1}(\kappa,t)\gamma(\kappa)d\kappa + \int_0^\infty R_{0,n+1}(\kappa,t)\eta(\kappa)d\kappa \quad (8)$$

$$V_{m,n}(0,t) = \theta(1-r)\int_0^\infty P^{(1)}_{m,n}(\kappa,t)\mu_1(\kappa)d\kappa \quad (9)$$

$$Q^{(1)}_{m,n}(0,t) = \alpha\int_0^\infty P^{(1)}_{m,n}(\kappa,t)d\kappa \quad (10)$$

$$R_{m,n}(0,t) = \int_0^\infty Q^{(1)}_{m,n}(\kappa,t)\mu_3(\kappa)d\kappa + \alpha\int_0^\infty P^{(2)}_{m,n-1}(\kappa,t)d\kappa \quad (11)$$

The initial conditions are,

$$P^{(1)}_{m,n}(0) = P^{(2)}_{m,n}(0) = Q^{(1)}_{m,n}(0) = V_{m,n}(0) = R_{m,n}(0) = 0 \text{ and } I_0(0) = 1;\, m,n \geq 0. \quad (12)$$

The Probability Generating Function(PGF) of this model is

$$A(\kappa,t,z_p) = \sum_{m=0}^{\infty} z_p^m A_m(\kappa,t); \quad A(\kappa,t,z_q) = \sum_{n=0}^{\infty} z_q^n A_n(\kappa,t);$$
$$A(\kappa,t,z_p,z_q) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} z_p^m z_q^n A_{m,n}(\kappa,t)$$

where $A = P^{(1)}, P^{(2)}, Q^{(1)}, V, R$. Which are convergent inside the circle given by $|z_p| \leq 1, |z_q| \leq 1$. By taking the Laplace transforms of the equations (1) to (11) and solving these equations, we get,

$$\overline{P}^{(1)}(\kappa,s,z_p,z_q) = \overline{P}^{(1)}(0,s,z_p,z_q)\left[1-\overline{B}_1\left(\phi_1(s,z_p,z_q)\right)\right]e^{-\phi_1(s,z_p,z_q)\kappa}, \quad (13)$$
$$\overline{P}^{(2)}(\kappa,s,z_p,z_q) = \overline{P}^{(2)}(0,s,z_p,z_q)[1-\overline{B}_2(\phi_1(s,z_p,z_q))]e^{-\phi_1(s,z_p,z_q)\kappa}, \quad (14)$$
$$\overline{Q}^{(1)}(\kappa,s,z_p,z_q) = \overline{Q}^{(1)}(0,s,z_p,z_q)\left[1-\overline{B}_3\left(\phi_2(s,z_p,z_q)\right)\right]e^{-\phi_2(s,z_p,z_q)\kappa}, \quad (15)$$

$$\overline{V}(\kappa,s,z_p,z_q)=\overline{V}(0,s,z_p,z_q)\left[1-\overline{V}\left(\phi_2(s,z_p,z_q)\right)\right]e^{-\phi_2(s,z_p,z_q)\kappa}, \tag{16}$$

$$\overline{R}(\kappa,s,z_p,z_q)=\overline{R}(0,s,z_p,z_q)\left[1-\overline{R}\left(\phi_2(s,z_p,z_q)\right)\right]e^{-\phi_2(s,z_p,z_q)\kappa}, \tag{17}$$

$$\overline{Q}^{(1)}(0,s,z_p,z_q)=\alpha\overline{P}^{(1)}(0,s,z_p,z_q)\left[\frac{1-\overline{B}_1\left(\phi_1(s,z_p,z_q)\right)}{\phi_1(s,z_p,z_q)}\right], \tag{18}$$

$$\overline{R}(0,s,z_p,z_q)=\alpha\overline{P}^{(1)}(0,s,z_p,z_q)\overline{B}_3(\phi_2(s,z_p,z_q))[\frac{1-\overline{B}_1(\phi_1(s,z_p,z_q))}{\phi_1(s,z_p,z_q)}]$$
$$+\alpha\overline{P}_0^{(2)}(0,s,z_p,z_q)\left[\frac{1-\overline{B}_2\left(\phi_1(s,z_p,z_q)\right)}{\phi_1(s,z_p,z_q)}\right], \tag{19}$$

$$\overline{V}(0,s,z_p,z_q)=\theta(1-r)\overline{P}^{(1)}(0,s,z_p,z_q)\overline{B}_1\left(\phi_1(s,z_p,z_q)\right) \tag{20}$$

$$\overline{Q}_0^{(1)}(0,s,z_q)=\alpha\overline{P}_0^{(1)}(0,s,z_q)\left[\frac{1-\overline{B}_1\left(\psi_1(s,z_q)\right)}{\psi_1(s,z_q)}\right], \tag{21}$$

$$\overline{R}_0(0,s,z_q)=\alpha\overline{P}_0^{(1)}(0,s,z_q)\overline{B}_3(\psi_2(s,z_q))[\frac{1-\overline{B}_1(\psi_1(s,z_q))}{\psi_1(s,z_q)}]$$
$$+\alpha\overline{P}_0^{(2)}(0,s,z_q)\left[\frac{1-\overline{B}_2\left(\psi_1(s,z_q)\right)}{\psi_1(s,z_q)}\right], \tag{22}$$

$$\overline{V}_0(0,s,z_q)=\theta(1-r)\overline{P}_0^{(1)}(0,s,z_q)\overline{B}_1\left(\psi_1(s,z_q)\right). \tag{23}$$

However, by definition $\overline{P}^{(2)}(0,s,z_p,z_q)=\overline{P}_0^{(2)}(0,s,z_q)$,where
$\psi_1(s,z_q)=\lambda_h+\lambda_l[1-C(z_q)]+\alpha$
$\psi_2(s,z_q)=\lambda_h+\lambda_l[1-C(z_q)]$
$\phi_1(s,z_p,z_q)=\lambda_h[1-c(z_p)]+\lambda_l[1-C(z_q)]+\alpha$
$\phi_2(s,z_p,z_q)=\lambda_h[1-c(z_p)]+\lambda_l[1-C(z_q)]$

using equations (13) to (23) in (7) and (8), we get

$$\overline{P}^{(1)}(0,s,z_p,z_q)\{z_p-[z_pr+(1-\theta)(1-r)]\overline{B}_1(\phi_1(s,z_p,z_q))-\alpha[\frac{1-\overline{B}_1(\phi_1(s,z_p,z_q))}{\phi_1(s,z_p,z_q)}]$$
$$\times\overline{B}_3(\phi_2(s,z_p,z_q))\overline{R}(\phi_2(s,z_p,z_q))-\theta(1-r)\overline{B}_1(\phi_1(s,z_p,z_q))\overline{V}(\phi_2(s,z_p,z_q))\}$$
$$=\lambda_hC(z_p)\overline{I}_0(s)\ +\overline{P}_0^{(2)}(0,s,z_q)\{\overline{B}_2\left(\phi_1(s,z_p,z_q)\right)-\overline{B}_2\left(\psi_1(s,z_q)\right)$$
$$+\alpha[\frac{1-\overline{B}_2\left(\phi_1(s,z_p,z_q)\right)}{\phi_1(s,z_p,z_q)}]\overline{R}(\phi_2(s,z_p,z_q))-\alpha[\frac{1-\overline{B}_2\left(\psi_1(s,z_q)\right)}{\psi_1(s,z_q)}]\overline{R}(\psi_2(s,z_q))\}$$
$$-\overline{P}_0^{(1)}(0,s,z_q)\ \{(1-r)\overline{B}_1(\psi_1(s,z_q))+\alpha\overline{B}_3(\psi_2(s,z_q))[\frac{1-\overline{B}_1\left(\psi_1(s,z_q)\right)}{\psi_1(s,z_q)}]\overline{R}_1(\psi_1(s,z_q))$$
$$-\theta(1-r)\overline{B}_1(\psi_1(s,z_q))[1-\overline{V}(\psi_2(s,z_q))]\} \tag{24}$$

$$\overline{P}_0^{(2)}(0,s,z_q)\left\{z_q-\overline{B}_2\left(\psi_1(s,z_q)\right)-\alpha\left[\frac{1-\overline{B}_2\left(\psi_1(s,z_q)\right)}{\psi_1(s,z_q)}\right]\overline{R}\left(\psi_2(s,z_q)\right)\right\}$$
$$=1-\psi_2(s,z_q)\overline{I}_0(s)+\overline{P}_0^{(1)}(0,s,z_q)\{(1-r)\overline{B}_1(\psi_1(s,z_q))+\alpha[\frac{1-\overline{B}_1\left(\psi_1(s,z_q)\right)}{\psi_1(s,z_q)}]$$
$$\times\overline{B}_3(\psi_2(s,z_q))\overline{R}(\psi_2(s,z_q))-\theta(1-r)\overline{B}_1(\psi_1(s,z_q))[1-\overline{V}(\psi_2(s,z_q))]\} \tag{25}$$

We have to solve the equations (24) and (25). Letting $z_p=g(z_q)$ in (24) we get,

$$\overline{P}_0^{(1)}(0,s,z_q)=\frac{\left\{\begin{array}{c}\lambda_hC(g(z_q))\overline{I}_0(s)+\overline{P}_0^{(2)}(0,s,z_q)\{\overline{B}_2(\sigma_1(s,z_q))-\overline{B}_2(\psi_1(s,z_q))\\+\ \ \alpha[\frac{1-\overline{B}_2(\sigma_1(s,z_q))}{\sigma_1(s,z_q)}]\overline{R}(\sigma_2(s,z_q))-\alpha[\frac{1-\overline{B}_2(\psi_1(s,z_q))}{\psi_1(s,z_q)}]\overline{R}(\psi_2(s,z_q))\}\end{array}\right\}}{\left\{\begin{array}{c}(1-r)\overline{B}_1(\psi_1(s,z_q))\ \ +\alpha\overline{B}_3(\psi_2(s,z_q))[\frac{1-\overline{B}_1(\psi_1(s,z_q))}{\psi_1(s,z_q)}]\overline{R}_1(\psi_2(s,z_q))\\-\theta(1-r)\overline{B}_1(\psi_1(s,z_q))[1-\overline{V}(\psi_2(s,z_q))]\end{array}\right\}} \tag{26}$$

Substituting (26) into (25) and (24)

$$\overline{P}_0^{(2)}(0,s,z_q)=\frac{\{1-\sigma_2(s,z_q)\overline{I}_0(s)\}}{\left\{z_q-\overline{B}_2(\sigma_1(s,z_q))-\alpha\left[\frac{1-\overline{B}_2(\sigma_1(s,z_q))}{\sigma_1(s,z_q)}\right]\overline{R}(\sigma_2(s,z_q))\right\}} \tag{27}$$

$$\overline{P}^{(1)}(0,s,z_p,z_q)=\frac{\left\{\begin{array}{l}1-\phi_2(s,z_p,z_q)\overline{I}_0(s)+\overline{P}_0^{(2)}(0,s,z_q)\{\overline{B}_2(\phi_1(s,z_p,z_q))-\overline{B}_2(\sigma_1(s,z_q))\\ +\quad \alpha[\frac{1-\overline{B}_2(\phi_1(s,z_p,z_q))}{\phi_1(s,z_p,z_q)}]\overline{R}(\phi_2(s,z_p,z_q))-\alpha[\frac{1-\overline{B}_2(\sigma_1(s,z_q))}{\sigma_1(s,z_q)}]\overline{R}(\sigma_2(s,z_q))\}\end{array}\right\}}{\left\{\begin{array}{l}z_p-[z_pr+(1-\theta)(1-r)]\overline{B}_1(\phi_1(s,z_p,z_q))-\alpha[\frac{1-\overline{B}_1(\phi_1(s,z_p,z_q))}{\phi_1(s,z_p,z_q)}]\\ \times\overline{B}_3(\phi_2(s,z_p,z_q))\overline{R}(\phi_2(s,z_p,z_q))-\theta(1-r)\overline{B}_1(\phi_1(s,z_p,z_q))\overline{V}(\phi_2(s,z_p,z_q))\end{array}\right\}} \tag{28}$$

Theorem 1 *The probability generating function of the Laplace transforms of thenumber of customers in the high and the low priority queue while the system was in regular service, working breakdown service, repair and vacation is given by:*

$$\overline{P}^{(1)}(s,z_p,z_q)=\overline{P}^{(1)}(0,s,z_p,z_q)\left[\frac{1-\overline{B}_1(\phi_1(s,z_p,z_q))}{\phi_1(s,z_p,z_q)}\right], \tag{29}$$

$$\overline{P}^{(2)}(s,z_p,z_q)=\overline{P}_0^{(2)}(0,s,z_q)\left[\frac{1-\overline{B}_2(\phi_1(s,z_p,z_q))}{\phi_1(s,z_p,z_q)}\right], \tag{30}$$

$$\overline{Q}^{(1)}(s,z_p,z_q)=\alpha\overline{P}^{(1)}(0,s,z_p,z_q)\left[\frac{1-\overline{B}_1(\phi_1(s,z_p,z_q))}{\phi_1(s,z_p,z_q)}\right]\left[\frac{1-\overline{B}_3(\phi_2(s,z_p,z_q))}{\phi_2(s,z_p,z_q)}\right], \tag{31}$$

$$\begin{aligned}\overline{R}^{(1)}(s,z_p,z_q)&=\alpha\overline{P}^{(1)}(0,s,z_p,z_q)\overline{B}_3\left(\phi_2(s,z_p,z_q)\right)\left[\frac{1-\overline{B}_1\left(\phi_1(s,z_p,z_q)\right)}{\phi_1(s,z_p,z_q)}\right]\left[\frac{1-\overline{R}\left(\phi_2(s,z_p,z_q)\right)}{\phi_2(s,z_p,z_q)}\right]\\ &\quad+\alpha\overline{P}_0^{(2)}(0,s,z_q)[\frac{1-\overline{B}_2(\phi_1(s,z_p,z_q))}{\phi_1(s,z_p,z_q)}][\frac{1-\overline{R}(\phi_2(s,z_p,z_q))}{\phi_2(s,z_p,z_q)}],\end{aligned} \tag{32}$$

$$\overline{V}(s,z_p,z_q)=\theta(1-r)\overline{P}^{(1)}(0,s,z_p,z_q)\overline{B}_1\left(\phi_1(s,z_p,z_q)\right)\left[\frac{1-\overline{V}(\phi_2(s,z_p,z_q))}{\phi_2(s,z_p,z_q)}\right]. \tag{33}$$

4.STEADY STATE ANALYSIS: LIMITING BEHAVIOUR

Now, we study the steady state probability distribution for our queueing model. By applying the well-known Tauberian property,

$$\lim_{s\to 0} s\overline{f}(s)=\lim_{t\to\infty} f(t),$$

The normalizing condition

$$P^{(1)}(1,1)+P^{(2)}(1,1)+V(1,1)+Q^{(1)}(1,1)+R(1,1)+I_0=1$$

We get the probability generating function of the queue size irrespective of the state of the system.

$$W_q(z_p,z_q)=\frac{Nr(z_p,z_q)}{Dr(z_p,z_q)} \tag{34}$$

where,

$$Nr(z_p,z_q)=-I_0\phi_1(z_p,z_q)\sigma_1(z_q)\phi_2(z_p,z_q)f(z_p,z_q)+P_0^{(2)}(0,z_q)\{f(z_p,z_q)j(z_p,z_q)+g(z_p,z_q)k(z_p,z_q)\},$$

$$Dr(z_p,z_q)=\phi_1(z_p,z_q)\phi_2(z_p,z_q)k(z_p,z_q),$$

$$\begin{aligned}f(z_p,z_q)&=\left[1-\overline{B}_1\left(\phi_1(z_p,z_q)\right)\right]\phi_2(z_p,z_q)+\alpha\left[1-\overline{B}_3\left(\phi_2(z_p,z_q)\right)\overline{R}\left(\phi_2(z_p,z_q)\right)\right]\\ &\quad+\theta(1-r)\phi_1(z_p,z_q)\overline{B}_1\left(\phi_1(z_p,z_q)\right)\left[1-\overline{V}\left(\phi_2(z_p,z_q)\right)\right],\end{aligned}$$

$$g(z_p,z_q)=\left[1-\overline{B}_2\left(\phi_1(z_p,z_q)\right)\right]\phi_2(z_p,z_q)+\alpha\left[1-\overline{R}\left(\phi_2(z_p,z_q)\right)\right],$$

$$\begin{aligned}j(z_p,z_q)&=\phi_1(z_p,z_q)\sigma_1(z_q)\left[\overline{B}_2\left(\phi_1(z_p,z_q)\right)-\overline{B}_2\left(\sigma_1(z_q)\right)\right]+\alpha\sigma_1(z_q)\\ &\quad\times\left[1-\overline{B}_2\left(\phi_1(z_p,z_q)\right)\right]\overline{R}(\phi_2(z_p,z_q))-\alpha\phi_1(z_p,z_q)[1-\overline{B}_2(\sigma_1(z_q))]\overline{R}(\sigma_2(z_q)),\end{aligned}$$

$$\begin{aligned}k(z_p,z_q)&=z_p\phi_1(z_p,z_q)\sigma_1(z_q)-[z_pr+(1-\theta)(1-r)]\phi_1(z_p,z_q)\sigma_1(z_q)\overline{B}_1(\phi_1(z_p,z_q))\\ &\quad-\alpha\sigma_1(z_q)\overline{B}_3(\phi_2(z_p,z_q))[1-\overline{B}_1(\phi_1(z_p,z_q))]\overline{R}(\phi_2(z_p,z_q))\end{aligned}$$

$$-\theta(1-r)\phi_1(z_p,z_q)\sigma_1(z_q)\overline{B}_1(\phi_1(z_p,z_q))\overline{V}(\phi_2(z_p,z_q)).$$

To find the unknown probability I_0 by using the normalizing condition $W_q(1,1)+I_0=1$, we get

$$I_0=\frac{\alpha(\lambda_h+\lambda_l)E(X)k'(1,1)}{\left\{\alpha(\lambda_h+\lambda_l)E(X)\{k'(1,1)-\alpha f'(1,1)\}-P_0^{(2)}(0,1)\{f'(1,1)j'(1,1)+g'(1,1)k'(1,1)\}\right\}} \tag{35}$$

The utilization factor is given by $\rho=1-I_0$, that is

$$\rho=\frac{\left\{\alpha^2(\lambda_h+\lambda_l)E(X)f'(1,1)+P_0^{(2)}(0,1)\{f'(1,1)j'(1,1)+g'(1,1)k'(1,1)\}\right\}}{\left\{-\alpha(\lambda_h+\lambda_l)E(X)\{k'(1,1)-\alpha f'(1,1)\}+P_0^{(2)}(0,1)\{f'(1,1)j'(1,1)+g'(1,1)k'(1,1)\}\right\}} \tag{36}$$

where,

$$f'(1,1)=-(\lambda_h+\lambda_l)E(X)[1-\overline{B}_1(\alpha)]\{1+\alpha[E(B_3)+E(R)]\}-\theta(1-r)\alpha(\lambda_h+\lambda_l)E(X)\overline{B}_1(\alpha)E(V),$$
$$g'(1,1)=-(\lambda_h+\lambda_l)E(X)[1-\overline{B}_2(\alpha)]\{1+\alpha E(R)\},$$
$$j'(1,1)=\alpha\lambda_h(1-E(X_1))E(X)[1-\overline{B}_2(\alpha)]$$
$$k'(1,1)=\alpha^2[1-r\overline{B}_1(\alpha)]-(\lambda_h+\lambda_l)E(X)\alpha[1-\overline{B}_1(\alpha)]\{1+\alpha[E(B_3)+E(R)]\}$$
$$-\theta(1-r)\alpha^2(\lambda_h+\lambda_l)E(X)\overline{B}_1(\alpha)E(V),$$

$$\overline{P}_0^{(2)}(0,1)=\frac{I_0\alpha(\lambda_hE(X_1)+\lambda_l)E(X)}{\left\{\alpha-(\lambda_hE(X_1)+\lambda_l)E(X)[1-\overline{B}_2(\alpha)]\{1+\alpha E(R)\}\right\}}$$

5.THE EXPECTED QUEUE LENGTH

The expected length of the high priority queue is

$$L_{q_h}=\frac{d}{dz_p}W_{q_h}(z_p,1)|_{z_p=1} \tag{37}$$

and the expected length of the low priority queue is

$$L_{q_l}=\frac{d}{dz_q}W_{q_l}(1,z_q)|_{z_q=1} \tag{38}$$

then

$$L_{q_h}=\frac{DR''(1)NR'''(1)-DR'''(1)NR''(1)}{3(DR''(1))^2} \tag{39}$$

$$L_{q_l}=\frac{dr''(1)nr'''(1)-dr'''(1)nr''(1)}{3(dr''(1))^2} \tag{40}$$

where

$$NR''(1)=2I_0\alpha^2\lambda_hE(X)f_h'(1)+2P_0^{(2)}(0,1)\{f_h'(1)j_h'(1)+g_h'(1)k_h'(1)\}$$
$$NR'''(1)=-I_0\alpha\{6(\lambda_hE(X))^2f_h'(1)-3\alpha\lambda_hE(X^2)f_h'(1)-3\alpha\lambda_hE(X)f_h''(1)\}$$
$$+3P_0^{(2)}(0,1)\{f_h''(1)j_h'(1)+f_h'(1)j_h''(1)+g_h''(1)k_h'(1)+g_h'(1)k_h'(1)\}$$
$$DR''(1)=-2\alpha\lambda_hE(X)k_h'(1)$$
$$DR'''(1)=6(\lambda_hE(X))^2k_h'(1)-3\alpha\lambda_hE(X^2)k_h'(1)-3\alpha\lambda_hE(X)k_h''(1)$$
$$f_h'(1)=-\lambda_hE(X)[1-\overline{B}_1(\alpha)]\{1+\alpha[E(B_3)+E(R)]\}-\theta(1-r)\alpha\lambda_hE(X)\overline{B}_1(\alpha)E(V)$$
$$f_h''(1)=-2(\lambda_hE(X))^2\overline{B}_1'(\alpha)\{1+\alpha[E(B_3)+E(R)]-\theta(1-r)\alpha E(V)\}$$
$$-\lambda_hE(X^2)[1-B_1(\alpha)]\{1+\alpha[E(B_3)+E(R)]\}-\alpha(\lambda_hE(X))^2[1-\overline{B}_1(\alpha)]$$
$$\times\{E(B_3^2)+E(R^2)+2E(B_3)E(R)\}+2\theta(1-r)(\lambda_hE(X))^2\overline{B}_1(\alpha)E(V)$$
$$-\alpha\theta(1-r)\lambda_hE(X^2)\overline{B}_1(\alpha)E(V)-\alpha\theta(1-r)(\lambda_hE(X))^2\overline{B}_1E(V^2)$$
$$g_h'(1)=-\lambda_hE(X)[1-\overline{B}_2(\alpha)]\{1+\alpha E(R)\}$$
$$g_h''(1)=-2(\lambda_hE(X))^2\overline{B}_2'(\alpha)\{1+\alpha E(R)\}-\lambda_hE(X^2)[1-B_2(\alpha)]\{1+\alpha E(R)\}$$
$$-\alpha(\lambda_hE(X))^2[1-\overline{B}_2(\alpha)]E(R^2)$$
$$j_h'(1)=\alpha\lambda_hE(X)[1-\overline{B}_2(\alpha)]\{1+\alpha E(R)\}$$
$$j_h''(1)=\alpha\lambda_hE(X^2)[1-\overline{B}_2(\alpha)]\{1+\alpha E(R)\}+2\alpha(\lambda_hE(X))^2\overline{B}_2'(\alpha)\{1+\alpha E(R)\}$$
$$+\alpha^2(\lambda_hE(X))^2[1-\overline{B}_2(\alpha)]E(R^2)$$
$$k_h'(1)=\alpha^2[1-r\overline{B}_1(\alpha)]-\alpha\lambda_hE(X)[1-\overline{B}_1(\alpha)]\{1+\alpha[E(B_3)+E(R)]\}-\alpha^2\theta(1-r)\lambda_hE(X)E(V)$$
$$k_h''(1)=-\alpha\lambda_hE(X^2)\{[1-\overline{B}_1(\alpha)]\{1+\alpha[E(B_3)+E(R)]\}+\alpha\theta(1-r)\overline{B}_1(\alpha)E(V)\}$$

$$-2\alpha(\lambda_h E(X))^2\overline{B}_1{}'(\alpha)\{1+\alpha[E(B_3)+E(R)]-\alpha\theta(1-r)E(V)\}$$
$$-2\alpha\lambda_h E(X)\{1-r\overline{B}_1(\alpha)-\alpha r\overline{B}_1{}'(\alpha)\}+2\alpha\theta(1-r)(\lambda_h E(X))^2\overline{B}_1(\alpha)E(V)$$
$$-\alpha^2(\lambda_h E(X))^2[1-\overline{B}_1(\alpha)]\{E(B_3^2)+E(R^2)+2E(B_3)E(R)\}$$
$$-\alpha^2\theta(1-r)(\lambda_h E(X))^2\overline{B}_1(\alpha)E(V^2)\}$$
$$nr''(1)=2I_0\alpha^2\lambda_l E(X)f_l{}'(1)+2P_0^{(2)}(0,1)\{f_l{}'(1)j_l{}'(1)+g_l{}'(1)k_l{}'(1)\}$$
$$nr'''(1)=-I_0\{6\alpha(\lambda_l E(X))^2 f_l{}'(1)+6\alpha(\lambda_l E(X))(\lambda_h E(X_1)+\lambda_l)E(X)f_l{}'(1)$$
$$-3\alpha^2\lambda_l E(X^2)f_l{}'(1)-3\alpha^2\lambda_l E(X)f_l{}''(1)\}+6P_0^{(2)'}(0,1)\{f_l{}'(1)j_l{}'(1)+g_l{}'(1)k_l{}'(1)\}$$
$$+3P_0^{(2)}(0,1)\{f_l{}''(1)j_l{}'(1)+f_l{}'(1)j_l{}''(1)+g_l{}''(1)k_l{}'(1)+g_l{}'(1)k_l{}''(1)\}$$
$$dr''(1)=-2\alpha\lambda_l E(X)k_l{}'(1)$$
$$dr'''(1)=6(\lambda_l E(X))^2k_l{}'(1)-3\alpha\lambda_l E(X^2)k_l{}'(1)-3\alpha\lambda_l E(X)k_l{}''(1)$$
$$f_l{}'(1)=-\lambda_l E(X)[1-\overline{B}_1(\alpha)]\{1+\alpha[E(B_3)+E(R)]\}-\theta(1-r)\alpha\lambda_l E(X)\overline{B}_1(\alpha)E(V)$$
$$f_l{}''(1)=-2(\lambda_l E(X))^2\overline{B}_1{}'(\alpha)\{1+\alpha[E(B_3)+E(R)]-\theta(1-r)\alpha E(V)\}$$
$$-\lambda_l E(X^2)[1-B_1(\alpha)]\{1+\alpha[E(B_3)+E(R)]\}-\alpha(\lambda_l E(X))^2[1-\overline{B}_1(\alpha)]$$
$$\times\{E(B_3^2)+E(R^2)+2E(B_3)E(R)\}+2\theta(1-r)(\lambda_l E(X))^2\overline{B}_1(\alpha)E(V)$$
$$-\alpha\theta(1-r)\lambda_l E(X^2)\overline{B}_1(\alpha)E(V)-\alpha\theta(1-r)(\lambda_l E(X))^2\overline{B}_1(\alpha)E(V^2)$$
$$g_l{}'(1)=-\lambda_l E(X)[1-\overline{B}_2(\alpha)]\{1+\alpha E(R)\}$$
$$g_l{}''(1)=-2(\lambda_l E(X))^2\overline{B}_2{}'(\alpha)\{1+\alpha E(R)\}-\lambda_l E(X^2)[1-B_2(\alpha)]\{1+\alpha E(R)\}$$
$$-\alpha(\lambda_l E(X))^2[1-\overline{B}_2(\alpha)]E(R^2)$$
$$j_l{}'(1)=\alpha[1-\overline{B}_2(\alpha)]\{(\lambda_l E(X)-(\lambda_h E(X_1)+\lambda_l)E(X)\}$$
$$j_l{}''(1)=2\alpha\overline{B}_2{}'(\alpha)[1+\alpha E(R)]\{(\lambda_l E(X))^2-((\lambda_h E(X_1)+\lambda_l)E(X))^2\}$$
$$+\alpha^2[1-\overline{B}_2(\alpha)]E(R^2)\{(\lambda_l E(X))^2-((\lambda_h E(X_1)+\lambda_l)E(X))^2\}$$
$$-\alpha[1-\overline{B}_2(\alpha)][1+\alpha E(R)]\{\lambda_h E(X_1)^2E(X^2)+\lambda_h E(X_1^2)E(X)\}$$
$$k_l{}'(1)=-\alpha\lambda_l E(X)\{[1-\overline{B}_1(\alpha)]\{1+\alpha[E(B_3)+E(R)\}+\alpha\theta(1-r)\overline{B}_1E(V)\}$$
$$k_l{}''(1)=-\alpha^2(\lambda_l E(X))^2[1-\overline{B}_1(\alpha)]\{E(B_3^2)+E(R^2)+2E(B_3)E(R)\}$$
$$+2\alpha\theta(1-r)(\lambda_l E(X))^2\overline{B}_1(\alpha)E(V)-\alpha^2\theta(1-r)(\lambda_l E(X))^2\overline{B}_1(\alpha)E(V^2)$$
$$+2(\lambda_l E(X))(\lambda_h E(X_1)+\lambda_l)E(X)\{[1-\overline{B}_1(\alpha)]\{1+\alpha[E(B_3)+E(R)]\}$$
$$+\alpha\theta(1-r)\overline{B}_1(\alpha)E(V)\}-\alpha\lambda_l E(X^2)\{[1-\overline{B}_1(\alpha)]\{1+\alpha[E(B_3)+E(R)]\}$$
$$+\alpha\theta(1-r)\overline{B}_1E(V)\}-2\alpha(\lambda_l E(X))^2\overline{B}_1{}'(\alpha)\{1+\alpha[E(B_3)+E(R)]-\alpha\theta(1-r)E(V)\}$$

6. THE EXPECTED WAITING TIME

Expected waiting time of a customer in the high priority queue is

$$W_{q_h}=\frac{L_{q_h}}{\lambda_h} \tag{41}$$

Expected waiting time of a customer in the low priority queue is

$$W_{q_l}=\frac{L_{q_l}}{\lambda_l} \tag{42}$$

7. PARTICULAR CASES

Case 1: If there is no low priority queue, no breakdown, no immediate feedback. i.e $\lambda_l=0,\ \alpha=0,\ r=0$. Then,

$$W(z)=\frac{-I_0\{[1-\overline{B}_1(\lambda_h(1-C(z_p)))]+\theta B_1(\lambda_h(1-C(z_p)))[1-V(\lambda_h(1-C(z_p)))]\}}{z_p-(1-\theta)\overline{B}_1(\lambda_h(1-C(z_p)))-\theta\overline{B}_1(\lambda_h(1-C(z_p)))\overline{V}(\lambda_h(1-C(z_p)))}$$

The above result coincides with the result of Maraghi et al.[10].

Case 2: If there is no low priority queue, no breakdown, no immediate feedback, no bernoulli vacation. i.e $\lambda_l=0$, $\alpha=0,\ r=0,\ \theta=0$. Then,

$$W(z)=\frac{-I_0[1-\overline{B}_1(\lambda_h(1-C(z)))]}{z-\overline{B}_1(\lambda_h(1-C(z)))}$$

The above result coincides with the result of Medhi. J [11].

8. NUMERICAL RESULT

The queueing model which we have mentioned is studied numerically with the following assumption. We consider that the time of service in regular service and working breakdown service, time of repair and time of vacation follows the exponential and the Erlang-2 distributions.

Table 2, shows that an increase in the high priority arrival rate leads to a decrease in the idle time and increases the expected queue length and waiting time of high and low priority queues for the arbitrary values, we choose $\lambda_l = 4, \mu = 8, \mu_w = 7, \alpha = 0.5, \eta = 0.65, \gamma = 12, \theta = 0.8, r = 0.4, E(X) = 1, E(X(X-1)) = 0$, while λ_h varies from 0.3 to 0.7 so as to satisfy the stability condition. In figure 1, we compare the result for the exponential and the Erlang-2 distributions for idle time, expected length of queue and waiting time of the high and the low priority under the values of Table 2.

Table 3, shows that an increase in the regular service rate increases the idle time and decreases the expected queue length and the waiting time of the high priority and the low priority queues for the arbitrary values, we choose $\lambda_h = 2.6$, $\lambda_l = 5, \mu_w = 15, \alpha = 4, \eta = 8, \gamma = 16, \theta = 0.1, r = 0.1, E(X) = 1, E(X(X-1)) = 0$, while,$\mu$ varies from 18.5 to 20 so as to satisfy the stability condition. In figure 2, we compare the result for the exponential and the Erlang-2 distributions for the idle time, the expected length of queue and the waiting time of high and low priority under the values of Table 3.

Table 2: **Impact of λ_h on various queue characteristics**

λ_h	Exponential					Erlang-2				
	I_0	L_{q_h}	L_{q_l}	W_{q_h}	W_{q_l}	I_0	L_{q_h}	L_{q_l}	W_{q_h}	W_{q_l}
0.3	0.1929	0.4018	0.5130	1.3394	0.1283	0.1679	0.3021	0.1352	1.0070	0.0338
0.4	0.1714	0.5891	0.6621	1.4727	0.1655	0.1480	0.4472	0.2774	1.1181	0.0694
0.5	0.1509	0.8136	0.8506	1.6273	0.2127	0.1287	0.6235	0.4614	1.2470	0.1154
0.6	0.1311	1.0846	1.0945	1.8076	0.2736	0.1101	0.8386	0.7071	1.3976	0.1768
0.7	0.1120	1.4137	1.4198	2.0195	0.3550	0.0919	1.1025	1.0486	1.5750	0.2622

Table 3: **Impact of μ_1 on various queue characteristics**

μ_1	Exponential					Erlang-2				
	I_0	L_{q_h}	L_{q_l}	W_{q_h}	W_{q_l}	I_0	L_{q_h}	L_{q_l}	W_{q_h}	W_{q_l}
18.0	0.2423	0.0972	0.1356	0.0374	0.0271	0.2182	0.0776	0.1045	0.0298	0.0209
18.5	0.2546	0.0961	0.1268	0.0369	0.0254	0.2313	0.0770	0.0943	0.0296	0.0189
19.0	0.2665	0.0948	0.1200	0.0365	0.0240	0.2439	0.0762	0.0862	0.0293	0.0172
19.5	0.2780	0.0935	0.1150	0.0360	0.0230	0.2561	0.0754	0.0801	0.0290	0.0160
20.0	0.2891	0.0921	0.1117	0.0354	0.0223	0.2680	0.0744	0.0759	0.0286	0.0152

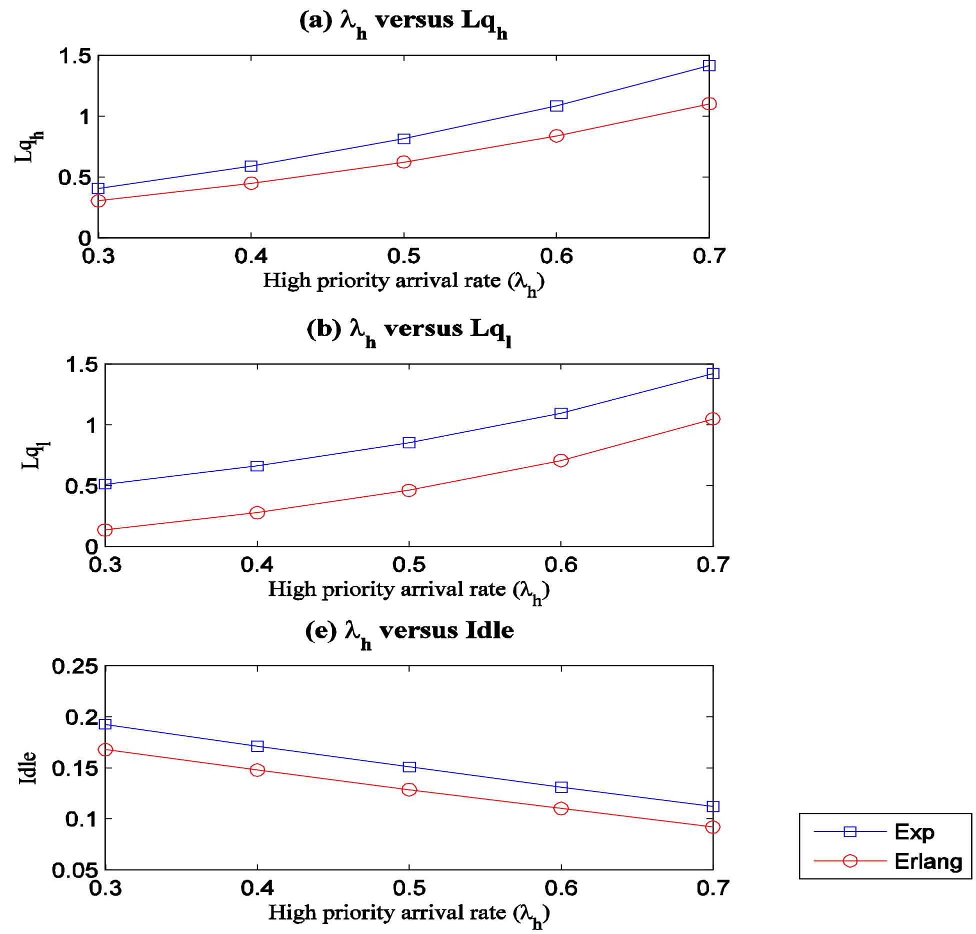

Figure 1:λ_h *versus(a)* L_{q_h}, *(b)* L_{q_l}, *(c) Idle*

In Figures 3 and 4 3-D graphs are illustrated. In Figure 3, the surface displays downward trend as expected for an increase in the value of high priority arrival rate (λ_h) and breakdown rate (α) against the server idle probability. Figure 4 shows that the surface displays an upward trend as expected for an increase in the value of high priority arrival rate (λ_h) and breakdown rate (α) against the expected queue length for high priority respectively.

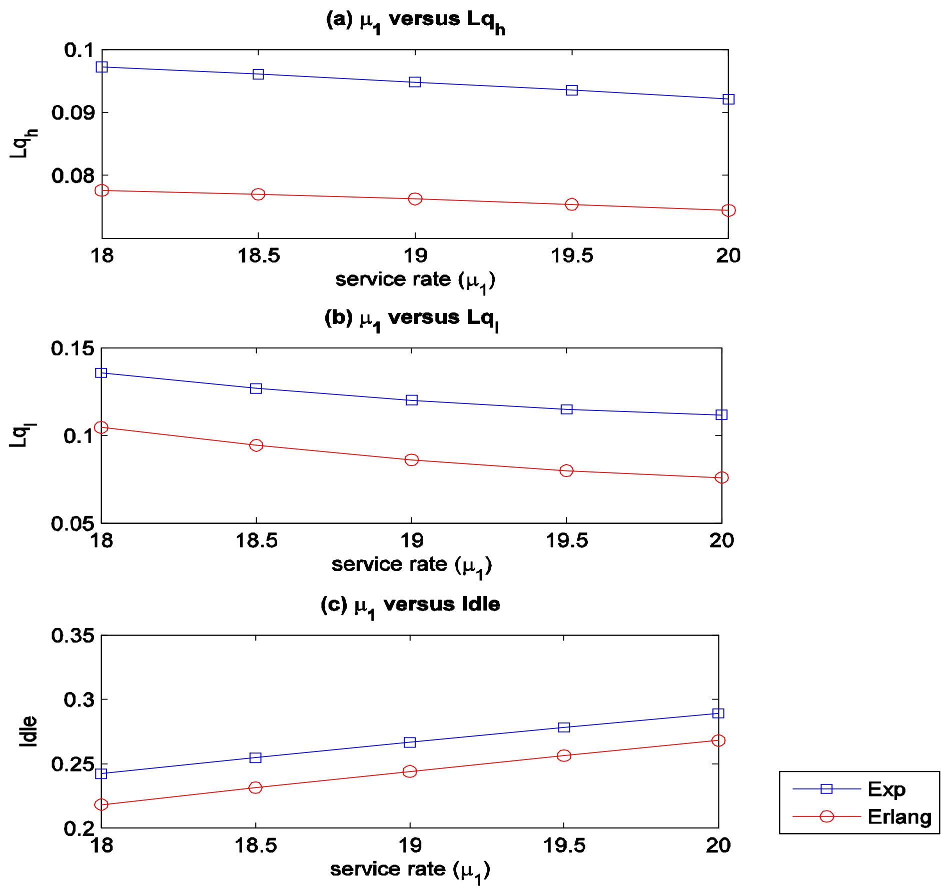

Figure 2:μ_1 *and versus (a)* L_{q_h}*, (b)* L_{q_l}*, (c) Idle*

Table 4:Effect of μ_3 with varying value of μ_1 on various queue characteristics

λ_h	α	I_0	L_{qh}	L_{ql}	W_{qh}	W_{ql}
0.6	2.3	0.5296	0.2229	0.0007	0.3716	0.0014
	2.4	0.5202	0.2297	0.0020	0.3829	0.0041
	2.5	0.5109	0.2363	0.0104	0.3939	0.0207
	2.6	0.5019	0.2428	0.0272	0.4046	0.0544
0.7	2.3	0.4968	0.2980	0.0047	0.4257	0.0094
	2.4	0.4867	0.3072	0.0061	0.4388	0.0122
	2.5	0.4768	0.3161	0.0139	0.4515	0.0277
	2.6	0.4672	0.3248	0.0294	0.4640	0.0587
0.8	2.3	0.4636	0.3875	0.0082	0.4844	0.0165
	2.4	0.4528	0.3996	0.0097	0.4995	0.0194
	2.5	0.4423	0.4113	0.0169	0.5141	0.0337
	2.6	0.4320	0.4227	0.0310	0.5284	0.0620
0.9	2.3	0.4300	0.4933	0.0113	0.5481	0.0226
	2.4	0.4185	0.5087	0.0127	0.5652	0.0255
	2.5	0.4072	0.5237	0.0193	0.5819	0.0387
	2.6	0.3962	0.5382	0.0321	0.5980	0.0642

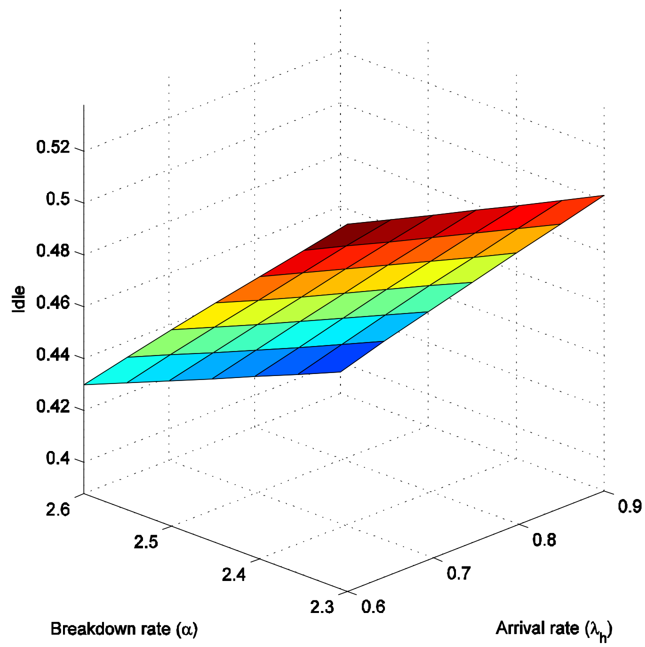

Figure 3: λ_h *and* α *versus Idle*

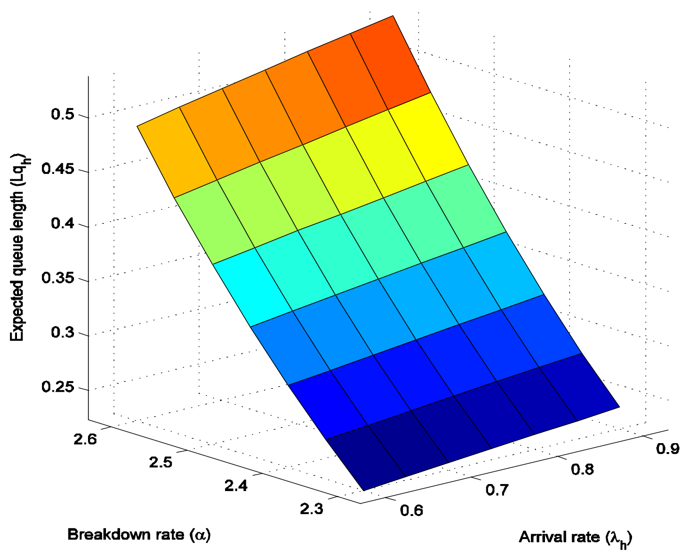

Figure 4: λ_h *and* α *versus* L_{q_h}

9. CONCLUSION

In this paper we study a priority queue based on working breakdown, repair, immediate feedback and Bernoulli vacation. The queue providesservice under non-preemptive priority rule. We derive the probability generating functions of the number of customers in the high and low priority queues using supplementary variable technique, expected queue size, expected waiting time for the high and low priority customers and numerical results are also obtained.

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