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DEFORMATION DUE TO A TORQUE SOURCE IN LAYERED ORTHOTROPIC ELASTIC MEDIUM IN WELDED CONTACT WITH ANOTHER ORTHOTROPIC ELASTIC MEDIUM

Renu Tagra[1*], Jagdish Nandal[2] and Anil Kumar[3]

Author Affiliation:
[1]Assistant Professor, G.V.M. Girls College, Sonepat, Haryana 131001, India
E-mail: renutagra22.rt@gmail.com
[2]Professor, Department of Mathematics, Maharshi Dayanand University, Rohtak, Haryana 124001, India
E-mail: js_nandal_2k5@yahoo.in,
[3]Assistant Professor, Pt. Neki Ram Sharma Government College, Rohtak, Haryana 124001, India
E-mail: dranil120975@gmail.com

***Corresponding Author:**
Renu Tagra, Department of Mathematics, G.V.M. Girls College, Sonepat, Haryana 131001, India
E-mail: renutagra22.rt@gmail.com

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Abstract
Two Orthotropic half-spaces (M-I and M-II) are in welded contact, where M-I has a layer of thickness H. At layer, there is a horizontal discontinuity which intersects with one of the axes. Horizontal discontinuity is due to a torque source and it passes through the point (H, 0, 0). By using Fourier Transform Method, we obtained displacement components for both the half-spaces and observed that the displacement is same throughout M-I. Numerically obtained results are depicted graphically to show that the half-space having discontinuity is more deformed as compared to other half-space. Effect of anisotropy is also studied and observed that orthotropic medium is more deformed as compared to isotropic medium.

Keywords: Horizontal Discontinuity, Welded Contact, Torque Source, Orthotropic, Layered, Boundary Conditions

1. INTRODUCTION

The elastic theory of Dislocation was first introduced by Steketee(1958). Steketee obtained displacement and stress components in an isotropic half-space by using Green function approach. The theory studied by Steketee (1958) and Maruyama (1964) was based on three dimensional elastic models. Further, Maruyama (1966) obtained all the sets of Green functions for semi-infinite isotropic elastic half-space and this study was based on two dimensional elastic sources. Ishii and Takagi (1967) investigated the influence of horizontal discontinuity due to a torque source in two isotropic media in welded contact. Rybicki (1971) studied the elastic field of a long strike-slip in a layered isotropic half-space; Rybicki extended the theory of Maruyama by obtaining the formulas for elastic field of a long strike-slip fault in a multilayered half-space. Singh and Garg (1987) also studied two-dimensional elastic problem in a multilayered isotropic elastic half-space. All these studies are based on isotropic elastic half-spaces.

In this paper, we studied deformations in orthotropic layered elastic half-space which is in welded contact with another orthotropic elastic half-space. We considered two dimensional elastic media and the deformations are resulting from horizontal discontinuity (which coincides with one of the axes) due to a torque source. We used Fourier Transform method and results of Love (1944) due to a torque source to find the displacement components.

2. BASIC EQUATIONS

In the absence of body forces, the stress-strain relation is:

$$\tau_{ij,j} = 0 \ (i = 1, 2, 3) \ , \tag{1}$$

where τ_{ij} is stress tensor of order 2 and comma indicates differentiation with respect to $x_i \ (i = 1, 2, 3)$.

The stress- displacement laws for an anisotropic elastic material an be written as

$$\tau_{ij} = C_{ijks} \ e_{ks} \ , \tag{2}$$

where

$$e_{ks} = \frac{1}{2}(u_{k,s} + u_{s,k}) \tag{3}$$

and C_{ijks} are the elastic stiffness 'coefficients satisfying the relation:

$$C_{ijks} = C_{jiks} = C_{ksij} \ . \tag{4}$$

From (1), (2) and (3), we obtained

$$C_{ijks} \ u_{k,sj} = 0 \ . \tag{5}$$

Here we consider two dimensional elastic deformations; in this type of deformation displacement components $u_i, (i = 1, 2, 3)$ are independent of x_3 and for anti-plane strain deformation:

$$u_1 = u_2 = 0, \quad u_3 = u_3 \ (x_1, x_2) \ . \tag{6}$$

For orthotropic medium:

$$C_{14} = C_{15} = C_{16} = C_{24} = C_{25} = C_{26} = C_{36} = C_{45} = C_{46} = C_{56} = 0 \ . \tag{7}$$

equation (7), we have used the contracted Voigt notation for the stiffness's coefficients according to the scheme: 11→1, 22→2, 33→3, 23→4, 13→5, 16→6

Using (7), the equation (5) becomes

$$C_{55} \ u_{3,11} + C_{44} \ u_{3\,22} = 0 \ . \tag{8}$$

From equation (3), the non-zero stresses are:

$$\tau_{23} = C_{44} \ u_{3,2} , \qquad \tau_{13} = C_{55} \ u_{3,1} \ . \tag{9}$$

3. FORMULATION AND SOLUTION OF THE PROBLEM

We consider two orthotropic elastic half-spaces (M-I and M-II) of different stiffness' coefficients C_{ij} and C_{ij}' which are in welded contact. M-I contains a layer of thickness H which coincides with horizontal discontinuity at (H, 0, 0) as shown in **Figure 1**.

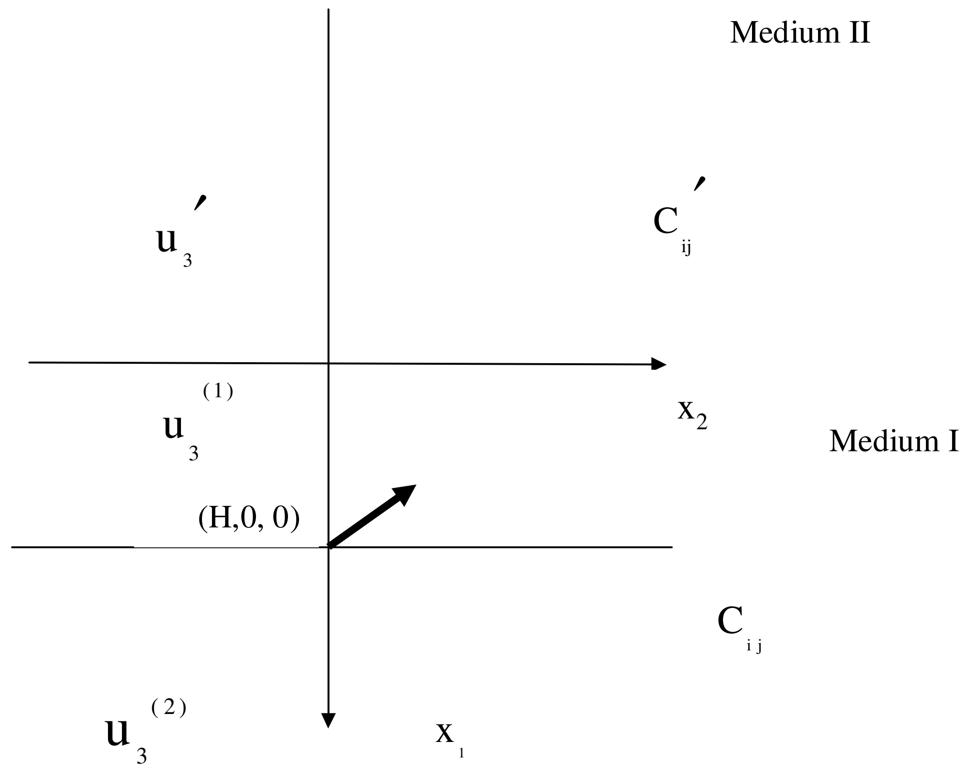

Figure 1:

We define the Fourier Transform by:

$$\overline{u_3}(x_1, k) = \int_{-\infty}^{\infty} u_3(x_1, x_2)\ e^{ikx_2}\ dx_2\,, \tag{10}$$

so that

$$u_3(x_1, x_2) = \int_{-\infty}^{\infty} \overline{u_3}(x_1, k)\ e^{ikx_2}\ dk\,. \tag{11}$$

Performing the Fourier Transform on equation (8), we obtained

$$\left(C_{55}\ \frac{d^2}{dx_1^{\,2}} - C_{44}\ k^2\right) \overline{u_3} = 0\,. \tag{12}$$

The solutions satisfying (12) are taken as

$$\overline{u_3'} = A_2 e^{m_1'|k|x_1}\,, \tag{13}$$

$$\overline{u_3}^{(1)} = A_1 e^{m_1|k|x_1} + B_1 e^{-m_1|k|x_1}, \tag{14}$$

$$\overline{u_3}^{(2)} = C e^{-m_1|k|x_1}. \tag{15}$$

where $m_1 = \frac{C_{44}}{C_{55}}$ and A_1, A_2, B_1 and C be functions of k.

The boundary conditions are:

$$\overline{u_3}' = \overline{u_3}^{(1)} \quad \text{at } x_1 = 0, \tag{16}$$

$$\overline{\tau_{31}}' = \overline{\tau_{31}}^{(1)} \quad \text{at } x_1 = 0 \tag{17}$$

and

$$\overline{u_3}^{(1)} = \overline{u_3}^{(2)} \quad \text{at } x_1 = H, \tag{18}$$

$$\overline{\tau_{31}}^{(1)} - \overline{\tau_{31}}^{(2)} = Z \quad \text{at } x_1 = H. \tag{19}$$

Here Z is the parameter which shows the strength of the source. A_1, A_2, B_1 and C are determined from equations (13)-(19) as below:

$$A_1 = \frac{1}{2}\left(\frac{C_{55}m_1 - C_{55}'m_1'}{C_{55}m_1 + C_{55}'m_1'}\right)\frac{Z}{m_1|k|C_{55}} e^{m_1|k|H}, \tag{20}$$

$$A_2 = \left(\frac{1}{C_{55}m_1 + C_{55}'m_1'}\right)\frac{Z}{|k|} e^{-m_1|k|H}, \tag{21}$$

$$B_1 = \frac{1}{2}\frac{Z}{m_1|k|C_{55}} e^{-m_1|k|H}, \tag{22}$$

$$C = \frac{1}{2}\left(\frac{C_{55}m_1 - C_{55}'m_1'}{C_{55}m_1 + C_{55}'m_1'}\right)\frac{Z}{m_1|k|C_{55}} e^{-m_1|k|H} + \frac{Z}{2m_1|k|C_{55}} e^{m_1|k|H}. \tag{23}$$

Substituting values of A_1, A_2, B_1 and C into equations (13), (14) and (15); we obtain

$$\overline{u_3}' = \left(\frac{1}{C_{55}m_1 + C_{55}'m_1'}\right)\frac{Z}{|k|} e^{-m_1|k|(H-x_1)}, \tag{24}$$

$$\overline{u_3}^{(1)} = \frac{1}{2}\left(\frac{C_{55}m_1 - C_{55}'m_1'}{C_{55}m_1 + C_{55}'m_1'}\right)\frac{Z}{m_1|k|C_{55}} e^{-m_1|k|(H+x_1)} + \frac{Z}{2m_1|k|C_{55}} e^{-m_1|k|(H-x_1)}, \tag{25}$$

$$\overline{u_3}^{(2)} = \frac{1}{2}\left(\frac{C_{55}m_1 - C_{55}'m_1'}{C_{55}m_1 + C_{55}'m_1'}\right)\frac{Z}{m_1|k|C_{55}} e^{-m_1|k|(x_1+H)} + \frac{Z}{2m_1|k|C_{55}} e^{-m_1|k|(x_1-H)}. \tag{26}$$

Performing the Inverse Fourier Transform, we obtain

$$u_3' = \frac{1}{\pi}\left(\frac{Z}{C_{55}m_1 + C_{55}'m_1'}\right)\int_0^\infty \frac{1}{k} e^{-m_1|k|(H-x_1)} \cos kx_2 \, dk, \tag{27}$$

$$u_3 = u_3^{(1)} = u_3^{(2)} = \frac{1}{2\pi}\left(\frac{C_{55}m_1 - C_{55}'m_1'}{C_{55}m_1 + C_{55}'m_1'}\right)\frac{Z}{m_1\ C_{55}}\int_0^{\infty}\frac{1}{k}\, e^{-m_1|k|(H+x_1)}\cos kx_2\, dk +$$

$$\frac{Z}{2\pi\, m_1\ C_{55}}\int_0^{\infty}\frac{1}{k} e^{-m_1|k|(H-x_1)}\cos kx_2\, dk\,. \tag{28}$$

By using the ordinary method(Love, 1944), Displacement components due to torque source is

$$u_3 = \frac{1}{2\pi}\left(\frac{C_{55}m_1 - C_{55}'m_1'}{C_{55}m_1 + C_{55}'m_1'}\right)\frac{Z}{m_1\ C_{55}}\int_0^{\infty} e^{-m_1|k|(H+x_1)}\cos kx_2\, dk$$

$$+\ \frac{Z}{2\pi\, m_1\ C_{55}}\int_0^{\infty} e^{-m_1|k|(H-x_1)}\cos kx_2\, dk\,, \tag{29}$$

$$u_3' = \frac{1}{\pi}\left(\frac{Z}{C_{55}m_1 + C_{55}'m_1'}\right)\int_0^{\infty} e^{-m_1|k|(H-x_1)}\cos kx_2\, dk\,. \tag{30}$$

By solving equations (29) and (30), we obtain

$$u_3 = \frac{1}{2\pi}\left(\frac{C_{55}m_1 - C_{55}'m_1'}{C_{55}m_1 + C_{55}'m_1'}\right)\frac{Z}{C_{55}}\left\{\frac{(H+x_1)}{x_2^{\,2} + m_1^{\,2}\,(H+x_1)^2}\right\}$$

$$+\ \frac{Z}{2\pi\, C_{55}}\left\{\frac{(H-x_1)}{x_2^{\,2} + m_1^{\,2}\,(H-x_1)^2}\right\}, \tag{31}$$

$$u_3' = \frac{1}{\pi}\left(\frac{Z}{C_{55}m_1 + C_{55}'m_1'}\right)\left\{\frac{m_1\,(H-x_1)}{x_2^{\,2} + m_1^{\,2}\,(H-x_1)^2}\right\}. \tag{32}$$

Differentiating (31) and (32) with respect to x_1 and x_2, we obtained

$$u_{3,1} = \frac{Z x_2^{\,2}}{2\pi\, C_{55}}\left[\frac{C_{55}m_1 - C_{55}'m_1'}{C_{55}m_1 + C_{55}'m_1'}\frac{1}{\left\{x_2^{\,2} + m_1^{\,2}\left(H + x_1^{\,2}\right)\right\}^2} + \frac{1}{\left\{x_2^{\,2} + m_1^{\,2}\left(H - x_1^{\,2}\right)\right\}^2}\right], \tag{33}$$

$$u_{3,2} = \frac{Z x_2}{\pi\, C_{55}}\left[\frac{C_{55}m_1 - C_{55}'m_1'}{C_{55}m_1 + C_{55}'m_1'}\frac{-\left(H + x_1\right)}{\left\{x_2^{\,2} + m_1^{\,2}\left(H + x_1^{\,2}\right)\right\}^2} + \frac{\left(H - x_1\right)}{\left\{x_2^{\,2} + m_1^{\,2}\left(H - x_1^{\,2}\right)\right\}^2}\right], \tag{34}$$

$$u'_{3,1}=\frac{m_1 Z x_2^{\,2}}{\pi\, C_{55}}\left[\frac{1}{C_{55}m_1+C'_{55}m'_1}\;\frac{1}{\left\{x_2^{\,2}+m_1^{\,2}\left(H-x_1^{\,2}\right)\right\}^2}\right], \tag{35}$$

$$u'_{3,2}=\frac{2\,m_1 Z x_2}{\pi\, C_{55}}\left[\frac{1}{C_{55}m_1+C'_{55}m'_1}\;\frac{-(H-X_1)}{\left\{x_2^{\,2}+m_1^{\,2}\left(H-x_1^{\,2}\right)\right\}^2}\right]. \tag{36}$$

Using (9), (33), (34), (35) and (36), we obtained stresses for both media.
Hence

$$U_3=\frac{1-\delta}{1+\delta}\left\{\frac{1+X_1}{X_2^{\,2}+m_1^{\,2}\left(1+X_1^{\,2}\right)}\right\}+\left\{\frac{1-X_1}{X_2^{\,2}+m_1^{\,2}\left(1-X_1^{\,2}\right)}\right\}, \tag{37}$$

$$U_3'=\frac{1}{1+\delta}\left\{\frac{1-X_1}{X_2^{\,2}+m_1^{\,2}\left(1-X_1^{\,2}\right)}\right\}, \tag{38}$$

where

$$U_3=\frac{2\,\pi\, H\, C_{55}}{Z},\quad U_3'=\frac{\pi\, H\, C_{55}}{Z},\quad X_1=\frac{x_1}{H},\quad X_2=\frac{x_2}{H},\quad \delta=\frac{C'_{55}\; m'_1}{C_{55}\; m_1}.$$

Equation (34) is solution of displacement component where discontinuity does not exist.

4. PARTICULAR CASE

When value of $m_1=m_1^{'}=1$, then results (37) and (38) are same as for isotropic medium due to a torque source.

$$U_3=\frac{1-\delta_1}{1+\delta_1}\left\{\frac{1+X_1}{X_2^{\,2}+\left(1+X_1^{\,2}\right)}\right\}+\left\{\frac{1-X_1}{X_2^{\,2}+\left(1-X_1^{\,2}\right)}\right\}, \tag{39}$$

$$U_3'=\frac{1}{1+\delta_1}\left\{\frac{1-X_1}{X_2^{\,2}+\left(1-X_1^{\,2}\right)}\right\}, \tag{40}$$

where

$$\delta_1=\frac{C'_{55}}{C_{55}}.$$

5. NUMERICAL DISCUSSION

The graphical representation for variations of displacements (U_3 and U_3') with respect to distance from the fault (X_2) are given within the range $-10\le X_2\le 10$.

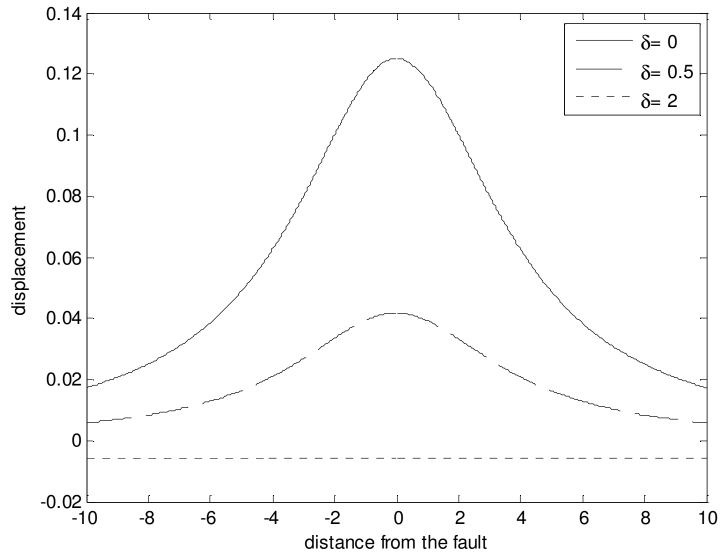

Figure 2:

Figure 2 shows the variations of displacement (U_3) with respect to X_2 for δ = 0, 0.5 and 2 when X_1 =1 and m_1 = 2. It is observed that the displacements corresponding to δ = 2 are smaller than δ = 0.5 which in turn are smaller than δ= 0 within the range $-10 \leq X_2 \leq -10$.

From figure 2, we observed that the displacement is symmetric about $X_2 = 0$. For δ = 0, 0.5, firstly the displacement increases and then decreases afterwards.

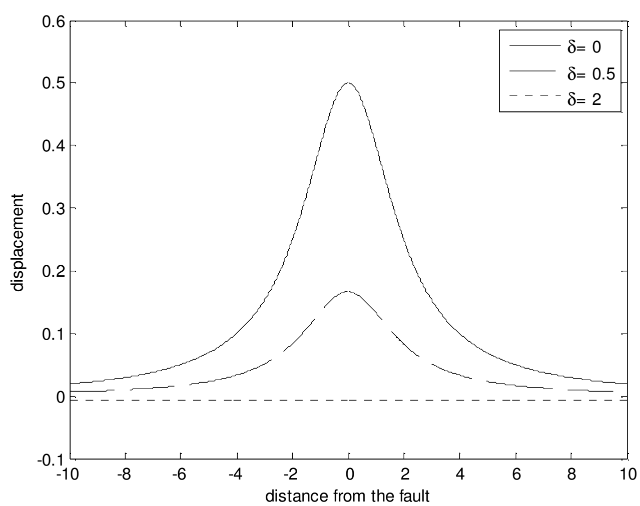

Figure 3:

Figure 3 shows the variations of displacement (U_3) with respect to X_2 for δ = 0, 0.5 and 2 when X_1 =1 and m_1 = 1.

From figure 2 and 3 we observed that displacement for orthotropic medium are larger as compared to isotropic medium.

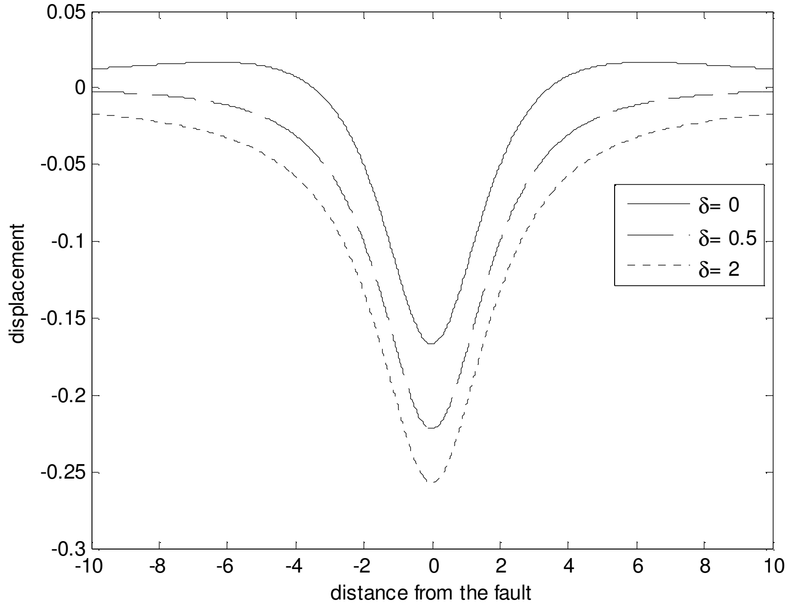

Figure 4:

Figure 4 shows the variations of displacement (U_3) with respect to X_2 for δ = 0, 0.5 and 2 when $X_1 = 2$ and $m_1 = 2$.

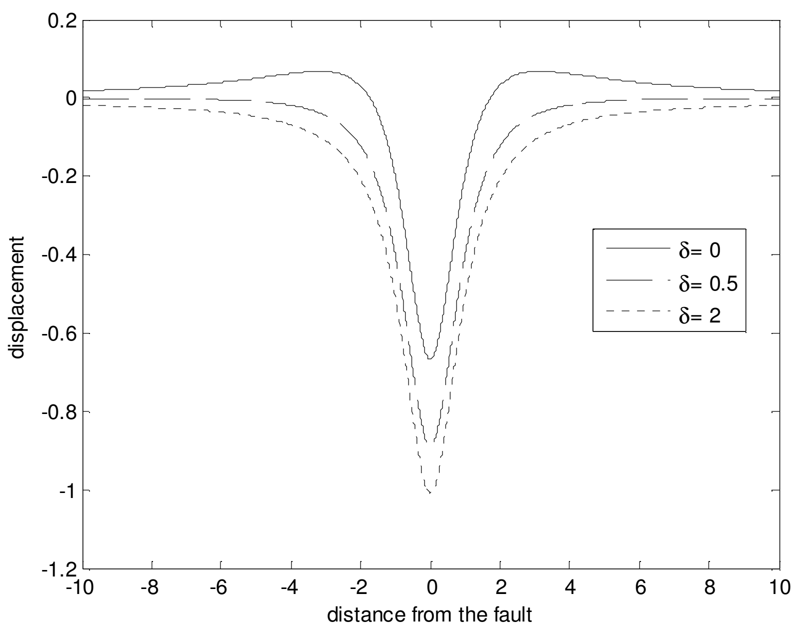

Figure 5:

Figure 5 shows the variations of displacement (U_3) with respect to X_2 for δ = 0, 0.5 and 2 when $X_1 = 2$ and $m_1 = 1$.

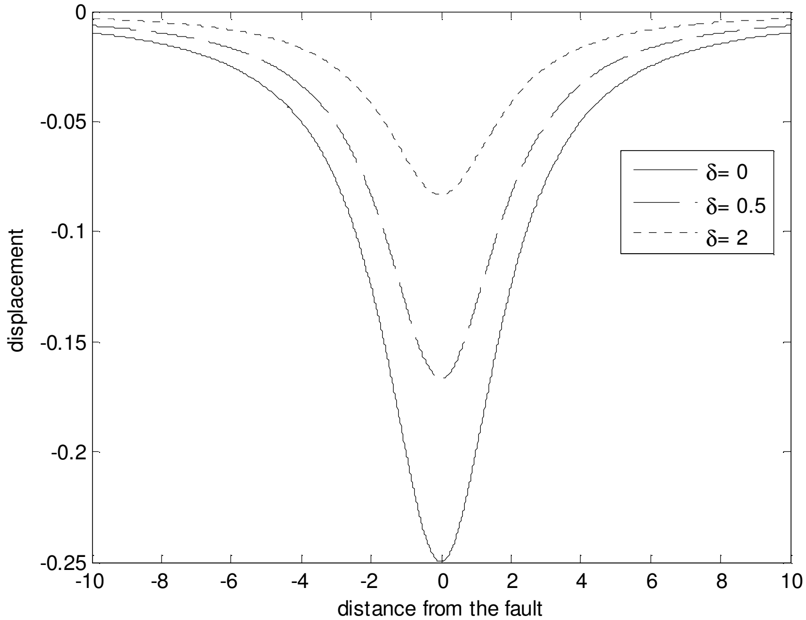

Figure 6:

Figure 6 shows the variations of displacement (U_3') with respect to X_2 for δ = 0, 0.5 and 2 when X_1 =2 and m_1 = 2.

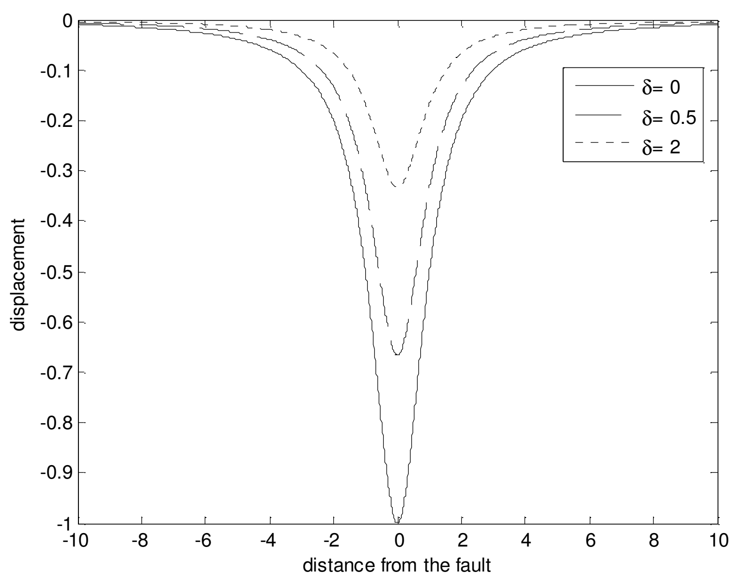

Figure 7:

Figure 7 shows the variations of displacement (U_3') with respect to X_2 for δ = 0, 0.5 and 2 when X_1 =2 and m_1 = 1.

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