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Spatial Reasoning of Field-Dependent Cognitive Students: Analysis of Hyperbola Problem Solving Based on Polya's Stages

[1,2]Rizki Kurniawan Rangkuti*, [1]Siti Khabibah, [1]Rooselyna Ekawati, [3]Irvan, [2]Raden Sri Ayu Ramadhana, [4]Rahmat Taufik Rangkuti

[1] Universitas Negeri Surabaya, Ketintang Surabaya, Indonesia
[2] Universitas Al Washliyah Labuhanbatu, Rantauprapat, Indonesia
[3] Universitas Muhammadiyah Sumatera Utara, Medan, Indonesia
[4] Sekolah Tinggi Agama Islam Negeri Mandailing Natal, Panyabungan, Indonesia

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Abstract: This research is important because of the urgency of information describing that *field-dependent subjects* do not have good learning outcomes, due to their characteristics in gaining knowledge that tends to require assistance from other objects. Therefore, a study is needed on spatial reasoning in *field-dependent subjects* through solving hyperbola problems. The purpose of this study is to analytically identify the steps of solving hyperbola problems based on Polya's four stages to see spatial reasoning. The research method used is a qualitative method. This research was conducted on undergraduate mathematics education students. The research subjects were determined by *purposive sampling.* The research subjects were selected from two *field-dependent students* with moderate abilities as measured using an analytical geometry test. The research data were obtained using hyperbola problem tasks to see spatial reasoning, and supported by interview transcripts. The findings of this study are in the form of aspects of the novelty of spatial reasoning theory in students with a *field-dependent* cognitive style , namely the existence of an imperfect problem-solving process in solving hyperbola problems, resulting in poor hyperbola problem-solving achievement. In the step of understanding hyperbola problems, students write down what is known and what is asked in the problem. In the planning step of the solution, students did not provide sufficient information. In the problem solving step, students solved the hyperbola problem well. In the step of checking the solution results, students did not write a review of the results of the problem solving. The conclusion of this study is that the spatial reasoning of *field-dependent subjects with moderate abilities has poor hyperbola problem solving. The implications of the results of this study provide benefits for the contribution of science to the theory of spatial reasoning which is reviewed based on the field-dependent* cognitive style .

KEYWORDS
analysis, *field-dependent* , students, non-routine problems, hyperbolic problem solving, spatial reasoning

Introduction

UNESCO considers reasoning as a critical ability that is essential for education because it enables people to understand, evaluate, and use information reflectively and critically [1]. In addition, reasoning plays a vital role in the development of critical thinking, problem-solving abilities, and other skills that are essential for success in learning, working, and participating effectively. Reasoning needs to be improved so that the quality of educational value continues to grow. In addition, related to geometry learning, the correlation between scores on geometry achievement tests and spatial ability tests tends to be only moderate [1], [2]. On the other hand, students' reasoning ability to apply their knowledge, skills, and understanding in real situations has been tested by the Organization for Economic Co-operation and Development to assess their ability to identify problems, formulate questions, gather and evaluate information, and draw conclusions or solutions supported by evidence or arguments [2]. PISA reasoning results are represented by the average score and distribution of students from each country that took the assessment. These scores indicate the extent to which students can apply their knowledge, skills, and understanding to solve real-world problems. Countries with lower average scores indicate that their students generally have good reasoning abilities.

Spatial reasoning has received significant attention in mathematics education since the 1970s. The growing interest from cognitive neuroscience, mathematics, psychology, and philosophy across all age groups, genders, and demographics illustrates the transdisciplinary and universal nature of this skill set [3], [4], [5]. Spatial reasoning is an essential component of mental abilities [6]in mathematical thinking processes [7], [8], [9], [10], [11]. Spatial reasoning refers to the ability to continuously generate, maintain, retrieve, and transform visual images so that they are well structured [6], [12].

The three components of this framework are mental rotation, spatial visualization and spatial orientation and serve as the spatial reasoning abilities to be explored [3], [7]. Spatial visualization refers to the ability to imagine complex, multi-step spatial transformations within objects (Frick, 2019; Harris et al., 2021; Linn & Petersen, 1985; Sorby, 2009). This is measured by tasks that involve imagining object-based transformations such as paper folding, or a net conversion to a solid [13], [16], [17]. Spatial visualization as a step-by-step manipulation of objects. That is, performing many rotations and transformations of objects mentally can occur as participants keep a mental record of each application and its impact on the original image at hand [3], [7].

As the name suggests, mental rotation is a cognitive process that involves imagining what a 2D or 3D object would look like after being rotated by a certain angle. [3], [18]. Perspective-taking tasks have been shown to differ from mental rotation at both behavioral and neural levels. Problem-solving [13], [16], [19], [20], [21]ability is at the heart of mathematics; it also includes solving exercises that are not based on algorithms.

Spatial orientation indicates the perceptual perspective of the participant who sees the object being changed or moved [3], [22], [23]. Spatial orientation is the ability to imagine different perspectives, either when navigating through space or imagining alternative views of a stationary object, i.e., an egocentric task (Harris et al., 2021; Hegarty & Waller, 2004).

The development of spatial reasoning ability is stated as one of the goals of mathematics education from Kindergarten to College level [6], [11], [24], [25], [25], [26]so it is very important to prepare students to excel in various fields, students must be trained to develop and cultivate spatial reasoning ability.

There is strong evidence linking spatial reasoning and success in mathematics; however, the underlying processes linking spatial reasoning and mathematical problem solving are not well understood [13], [14], [27], [28], [29], [30], [31], [32], [33]. In this study, we examined a range of spatial skills, namely mental rotation, spatial visualization, and spatial orientation (Harris et al., 2021).

Developmental studies highlight how spatial reasoning supports the development of numerical cognition, but to date there have been few studies examining the role of spatial reasoning in real-time mathematical problem solving [13], [35], [36]. The most fundamental of geometric thinking is spatial reasoning (Harris et al., 2021). In the psychology literature, mental rotation refers to the ability to imagine the rotation of a 2D shape or a whole 3D object [7], [13]. It is often measured with a speeded task that asks problem solvers to decide whether two objects are the same or different [13], [18], [37].

Problem solving is a cognitive processing directed at achieving a goal when there is no obvious solution method for the problem solver [38]. As an essential component of mathematical problem solving, problem solving involves knowledge of semantic constructions and mathematical relationships as well as knowledge of basic numerical skills and strategies [38]. Polya's strategy is an attempt to combine analytical and synthetic approaches. The author considers that the first step, "understanding the problem", is actually the main analytical breakdown of the problem, while the steps "making a plan" and "executing the plan" are related to the synthetic breakdown. The last stage, "feedback", is analytically rooted, so the analytic-synthetic method is closer to Polya's strategy.(Szabo et al., 2020).

Every idea or solution that students obtain through independent internalization will remain in the long term as an active part of their mathematical knowledge and argumentation [39]. Non-routine problem-solving skills and strategies acquired through continuing mathematics education can be used to solve real-life problems in the workplace [39]. Non-routine problem solving is characterized as a higher-level skill that must be acquired after routine problem-solving skills which in turn must be acquired after students have learned basic mathematical concepts and skills [40]. Mathematics education is essential for students' overall cognitive development and problem-solving abilities (Ahmad et al., 2024).

Non-routine problems are problems encountered mostly in real life, which do not have a specific formula that can be solved using various strategies and categorizations that require creative thinking skills [42]. There can be a certain tension between this approach and the non-routine, creative character of problem solving because "problem-solving heuristics based only on logical and deductive reasoning processes distort the true nature of problem solving (van Zanten & van den Heuvel-Panhuizen, 2018). "

Problem solving can be seen from the cognitive style of the subjects studied, because based on the cognitive style the achievement of a test can have different results. There are two types of cognitive styles, namely field-dependent cognitive style and field-independent cognitive style [44]. Field-dependent cognitive style is defined as the tendency to see problems globally [44]. The results of the study found that field-dependent students tend to be passive and dependent (Karamaerouz et al., 2013). This is also reinforced by the results of the study which showed that individuals with a field-dependent cognitive style cannot handle objects that are perceived separately from the elements around them [46].

This study focuses on the field-dependent cognitive style caused by the characteristics possessed by field-dependent subjects who view complex situations globally without identifying the key elements of these complex situations [44]. Subjects with a field-dependent cognitive style are more successful in learning and remembering social material (Karamaerouz et al., 2013). Various studies have found a relationship between a person's personality traits and their dependent or independent nature in a particular field. Some characteristics of subjects with a field-dependent cognitive style are passivity, dependence and acquiescence. Research has shown that subjects with a field-dependent cognitive style are more successful in learning material that is already organized than in learning material that is not sufficiently organized. Subjects with a field-dependent cognitive style tend to experience events globally [48]. Field-dependent cognitive style is a cognitive style with a tendency to rely on references outside of context.

Based on the relevant research results, several gaps were obtained in the research phenomenon, namely previous research was still limited to spatial reasoning studies only in certain fields. It has not been discussed comprehensively in all fields of science, so that spatial reasoning studies have not been found in all fields of scientific application due to the absence of research related to spatial reasoning in many fields of science. In general, previous research is still limited to certain fields of science, even though there is another essence that is very important to see students' spatial reasoning in solving hyperbola problems reviewed based on field-dependent cognitive styles, therefore to answer the phenomena of several previous studies, it is necessary to conduct a study to find a theory about spatial reasoning in solving hyperbola problems on field-dependent subjects. The hope of this research is that the theme of spatial reasoning can be applied to a broader field of science with a review that has never been discussed.

The purpose of this study is to analyze the steps of solving hyperbola problems on field-dependent subjects to explore the spatial reasoning of undergraduate mathematics education students. Significant contributions to this study provide benefits to academics to provide the latest references from the results of spatial reasoning research on undergraduate mathematics education students reviewed based on field-dependent cognitive styles.

Research Methodology

2.1 Research Model

This study uses a descriptive qualitative approach, this is because this study aims to identify analytically the steps of solving hyperbola problems based on Polya's four stages. This study uses a qualitative case study type model, namely to obtain more in-depth information about solving hyperbola problems.

2.2 Participants and Data Collection

Participants in this study were obtained from analytical geometry ability tests and cognitive style tests. From these results, the characteristics of students with moderate abilities in analytical geometry courses and field-dependent cognitive styles were selected. This study was conducted on undergraduate students of Mathematics Education at the South Tapanuli Institute of Education. The research subjects were determined by purposive sampling with the provision that they had taken analytical geometry courses. This is because hyperbola is included in the analytical geometry section.

2.3 Research Tools

The research data were obtained by using hyperbola problem tasks to see the spatial reasoning of the subjects, because hyperbola problems contain spatial reasoning indicators. The criteria for the analytical geometry ability category in the subjects in this study are presented in Table 1.

Table 1. Analytical Geometry Ability Level Criteria

Test Scores	Criteria
$90 \leq AGALC \leq 100$	High
$80 \leq AGALC < 90$	Medium
$70 \leq AGALC < 80$	Low

Based on Table 1, the average value of students in the medium group with a field-dependent cognitive style is taken to then analyze the problem-solving steps based on Polya's theory. The following is a relationship that describes Polya's problem-solving steps in analyzing spatial visualization abilities, mental rotation and spatial orientation which are indicators of spatial reasoning.

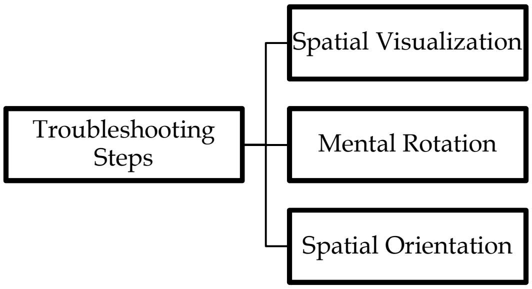

Figure 1. Problem Solving Analysis Flow in Spatial Reasoning Indicators

Based on Figure 1, it is described that the analysis of the steps for solving hyperbolic problems was carried out on three indicators of spatial reasoning, this leads to being able to answer the objectives of this study.

2.4 Data Analysis

The analysis of hyperbolic problem-solving data based on spatial reasoning indicators in this study refers to the data presented in Figure 1. After the spatial reasoning data is categorized, the next step is to analyze the hyperbolic problem-solving steps based on the field-dependent cognitive style with the stages of data collection, data reduction, data presentation, and drawing conclusions. The data analysis chart in this study can be described in the following chart.

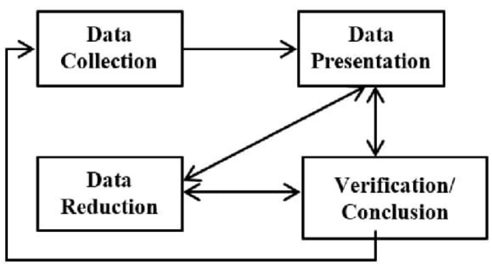

Figure 2. Qualitative Research Data Analysis Chart

Research Result

Initially, the analytical geometry ability test was given to 22 students, then continued with a cognitive style test. From the results of the analysis of the two tests given, several students were selected who had moderate abilities and field-dependent cognitive styles so that they met the criteria and became research subjects. Details of the results of the analytical geometry ability test with the cognitive style test on the research subjects are presented in Table 2 below.

Table 2. Results of Analytical Geometry Ability and Cognitive Style Tests

Subject Code	Student Initials	Test Scores	Cognitive Style
High Group	S-14	96	Non Field Dependent
	S-5	93	Non Field Dependent
	S-9	92	Non Field Dependent
	S-17	90	Non Field Dependent
	S-2	91	Non Field Dependent
Medium Group	S-17	89	Non Field Dependent

Subject Code	Student Initials	Test Scores	Cognitive Style
	S-7	88	Non Field Dependent
	S-15	88	Non Field Dependent
	S-12	87	Non Field Dependent
	S-3	86	Non Field Dependent
	S-16	85	Non Field Dependent
	S-8	85	Field Dependent
	S-1	84	Field Dependent
	S-19	83	Field Dependent
	S-13	82	Field Dependent
	S-21	81	Field Dependent
	S-10	80	Field Dependent
Low Group	S-6	78	Field Dependent
	S-18	74	Field Dependent
	S-20	73	Field Dependent
	S-22	71	Field Dependent
	S-11	70	Field Dependent

Based on the table above, it can be described that there are five high-ability students with two students with a field-dependent cognitive style, twelve medium-ability students with four students with a field-dependent cognitive style, and five low-ability students with all of them having a field-dependent cognitive style. Therefore, two research subjects were taken from the medium group with a field-dependent cognitive style using purposive sampling technique, namely S-8, S-1, S-19, S-13, S-21 and S-10. The selection of the two research subjects was based on the consideration of the average score of each medium group with a field-dependent cognitive style and considerations from research colleagues. The two selected subjects can be seen in Table 3 below.

Table 3. Research Subjects

Subject Code	Student Initials	Test Scores	Cognitive Style
Medium Group	S-8	85	Field Dependent
	S-1	84	Field Dependent
	S-21	81	Field Dependent

Subject Code	Student Initials	Test Scores	Cognitive Style
	S-13	82	Field Dependent
	S-10	80	Field Dependent
	S-21	81	Field Dependent

Based on the review of the results of solving hyperbola problems as a whole, it has good achievements with incomplete problem solving based on the problem solving steps according to Polya's theory, there are two complete steps and two incomplete steps from the four problem solving steps. A summary of the results of solving hyperbola problems worked on by the subjects based on three spatial reasoning indicators is presented in Table 4 below.

Table 4. Results of Hyperbola Problem Assignments Based on Spatial Reasoning Indicators

Troubleshooting Steps	Spatial Reasoning Indicators		
	Spatial Visualization	Mental Rotation	Spatial Orientation
1. Understand the problem	Wrote down the information that was known and asked through examples.	Wrote down the informatio n that is known and asked in the problem.	Wrote down the information that is known and asked in the problem.
2. Plan a solution	Does not categorize the location of objects.	Does not write the peak point on the horizontal axis.	Does not make a plan using a formula.
3. Implement the solution	Described other objects while still paying attention to patterns.	Rotated the hyperbolic object while still paying attention to the rotation angle.	Calculated the distance between two points.
4. Checking the results	Does not check the correctness of the results based on previous knowledge, and the final answer was correct.	Does not check the correctness of the results by reviewing them clockwise, and the final answer was incorrect.	Does not check the correctness of the results through the form of mathematical inequality. The final answer was incorrect.

Based on Table 4 above, it is known that the steps for solving hyperbola problems at the stage of planning solutions and checking the results of the solution did not find a clear description to direct to solving the problem. However, at the stage of solving the problem, it is known that students are able to describe the answer as a form of solution to each spatial visualization reasoning. It was found that the answers they gave turned out to be correct, although no stage was found in re-checking the solution to the hyperbola problem.

Research Discussion

The selected research subjects were subjects who had moderate abilities in analytical geometry courses and field-dependent cognitive styles were given questions in the form of hyperbola problem assignments that aimed to be able to explore spatial reasoning information. This was intended to obtain a characteristic picture of how to solve hyperbola problems in student subjects with field-dependent cognitive styles that had never been known based on research results published in journals. The following is a discussion related to the analysis of hyperbola problem solving steps based on

Polya's four stages to see the spatial reasoning of undergraduate students in mathematics education at the Tapanuli Selatan Institute of Education which was reviewed based on the field-dependent cognitive style .

4.1 Solving Hyperbola Problems in Spatial Visualization Ability

In the step of understanding the mental rotation problem, it is known that the subject wrote down the information that was known and asked from the problem. The known things are three pieces of horizontal hyperbola objects, the subject considers the known things to be initial information in identifying something important to formulate the solution steps. Meanwhile, the thing that was asked was determining the steps in constructing a complete hyperbola object from the pieces of the horizontal hyperbola object. In this case, the subject provides direction in solving the spatial visualization problem. The subject's steps in understanding the spatial visualization problem are given in the image below.

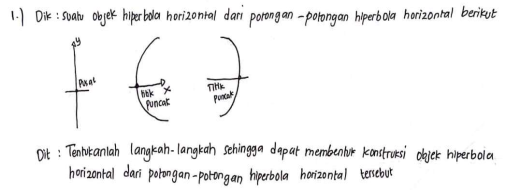

Figure 3. Steps to Understand Spatial Visualization Problems

Based on Figure 3 above, in the step of understanding the spatial visualization problem, it is known that the subject did not write down sufficient and necessary conditions for information based on the given problem, so that the subject does not categorize the location of objects from the three horizontal hyperbola sections. In this stage, there was also no important information found to support problem solving so that no new knowledge was found outside the given problem. In the step of implementing spatial visualization problem solving, the subject is able to solve it by describing other objects while still paying attention to the pattern. The pattern referred to in this case is a form of analogy of the three pieces of the horizontal hyperbola object with the alphabetical letter attribute. The use of the analogy is a mental form in the cognitive process as a result of overall analytical thinking. After the analogy, the subject combines the three pieces of the object to form a complete horizontal hyperbola. The process of combining the three pieces of the object is part of the unconscious reasoning ability which could be a result of previous knowledge related to horizontal hyperbola. The subject combines them following the memory that is still stored so that the effort to construct the hyperbola is successful. The subject's steps in solving the spatial visualization problem are given in the image below.

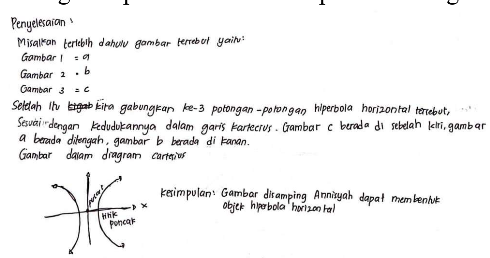

Figure 4. Steps to Solve Spatial Visualization Problems

Based on Figure 4 above, in the steps of solving the spatial visualization problem, it is known that the subject did not check the correctness of the results based on previous knowledge, but the answer given is correct. This can be seen from the results of the problem solving sheet where no checking stages were found after the answer stage was given, so that there is a possibility that the solution given is wrong. However, based on observations from researchers, the solution given by the subject is correct.

Based on Polya's four stages in solving hyperbola problems in spatial visualization abilities in subjects with a *field-dependent cognitive style* , there is a fairly good solution by providing the first and third steps in solving Polya's problems and the answers given by the subjects are correct.

The results of the interviews with field-dependent subjects during the problem-solving process in spatial visualization are given in the form of the following transcript:

Researcher : After reading and observing the question number, what can you understand based on the spatial visualization problem?

Subject : What I understand based on observations of the spatial visualization problem is that there are three hyperbola pieces that will be constructed so that they can form a horizontal hyperbola object construction.
Researcher : How do you construct the three pieces of the hyperbola object?
Subject : My way of constructing it is by making an example of the three pieces of the hyperbola, then combining the three pieces with the pattern of the first image example on the left, the second image example in the middle and the third image example on the right.
Researcher : What can you conclude from solving this problem?
Subject : What I can conclude is that the result of constructing the three hyperbola pieces will form a complete horizontal hyperbola.

4.2 Solving Hyperbola Problems on Mental Rotation Ability

In the step of understanding the mental rotation problem, it is known that the subject writes down the information that is known and asked from the question. The thing that is known is a vertical hyperbola, the subject writes down the known is the stage of remembering how a vertical hyperbola is formed which will make it easier to formulate a solution. Meanwhile, the thing that is asked is determining the steps to be able to form the rotation result of a vertical hyperbola object that is rotated by an angle of 120. In this case, the subject focuses on solving the mental rotation problem. The subject's steps in understanding the mental rotation problem are given in the image below.

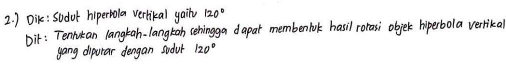

Figure 5. Steps to Understand Mental Rotation Problems

Based on Figure 5 above, in the step of understanding the mental rotation problem, it is known that the subject did not write down the sufficient and necessary conditions for information based on the problem given, so that the subject did not write the peak point on the horizontal axis. In this stage, there was also no important information found to support problem solving so that no new knowledge was found outside the problem given.

In the step of implementing the mental rotation problem solving, the subject is able to mentally rotate the hyperbola object while still paying attention to the given rotation angle. The method used by still paying attention to the angle is intended so that in estimating the angle there is no mistake so that the rotation process to be carried out is right on target. The rotation process of the vertical hyperbola object begins by shifting the *y- axis* , while the *x- axis* remains in position. This is intended so that the subject by shifting the *y- axis* can estimate the requested rotation angle correctly, so that the results obtained do not show a complete Cartesian coordinate system even though according to the subject the vertical hyperbola object after being rotated by 120 has been done as well as possible. The subject's steps in solving the mental rotation problem can be given in the image below.

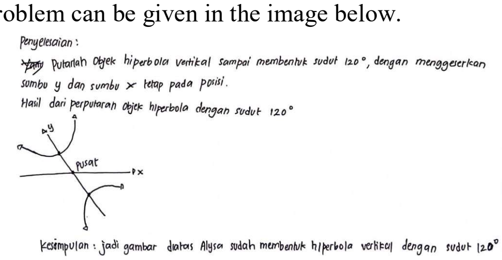

Figure 6. Steps to Solve Mental Rotation Problems

Based on Figure 6 above, in the steps to solve the mental rotation problem, it is known that the subject did not check the correctness of the results by reviewing whether it was clockwise, until finally the final answer was wrong. This can be seen from the results of the problem solving sheet where no checking process was found after the answer stage was given, so that there is a possibility that the solution given is wrong. Based on observations from researchers, the solution given by the subject is wrong.

Based on Polya's four stages in solving hyperbola problems in mental rotation ability in subjects with a *field-dependent cognitive style* , the solution is less good by providing the first and third steps in solving Polya's problems and the answers given by the subjects are wrong.

The results of the interview with the field-dependent subject during the problem-solving process in mental rotation are given in the following transcript form:

Researcher : After reading and observing the question number, what can you understand based on the mental rotation problem?
Subject : What I understand based on observations of the mental rotation problem is that a vertical hyperbolic object will be rotated by an angle of 120
Researcher : How do you rotate the vertical hyperbola object?
Subject : My method of rotating the vertical hyperbola object is to mentally shift the y-axis while the x-axis remains in position, resulting in a rotation of the vertical hyperbola object at an angle of 120
Researcher : What can you conclude from solving this problem?
Subject : What I can conclude is that the result of the vertical hyperbolic rotation is the result of a rotation of 120

4.3 Solving Hyperbola Problems in Spatial Orientation Ability

In the step of understanding the spatial orientation problem, it is known that the subject wrote down the information that is known and asked from the question. The thing that is known is a hyperbola with a center point H (p , q) and the distance between the right center point and the focal point of the hyperbola is 3 units, namely d (A1, f1) = 3. The information on the center point and focal point written by the subject as something that is known is intended to make it easier to identify the thing that is the question of the spatial orientation problem of the hyperbola. Meanwhile, the thing that is asked is to determine the steps so that the distance from the left peak point to the focal point can be determined. After obtaining the distance, then draw the hyperbola object along with information on the distance from the focal point to the center point of the hyperbola. In this case, the subject focuses comprehensively on solving the spatial orientation problem. The subject's steps in understanding the spatial orientation problem are given in the image below.

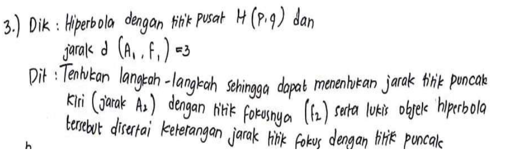

Figure 7. Steps to Understand Spatial Orientation Problems

Based on Figure 7 above, in the step of understanding the spatial orientation problem in the step of planning to solve the spatial orientation problem, it is known that the subject did not write down the sufficient and necessary conditions for information based on the problem given, so that the subject did not make a solution plan using a certain approach. In this stage, there was also no additional important information found to support problem solving so that no new knowledge was found outside the problem given.

In the step of implementing spatial orientation problem solving, the subject is able to calculate the distance between two points given through estimation. The method used in estimating the distance between the left peak point and the horizontal hyperbola focal point is by using information from known things and the images given in it. The estimation process begins with the subject's observation that the shape of the two hyperbola objects is not congruent (does not have the same shape and does not have the same angle size), so it can be concluded that the distance of the right peak point of the horizontal hyperbola to its focal point, which is 3 units, is not the same as the distance of the left peak point of the horizontal hyperbola to its focal point, which is minus 3 units. This is because the subject is unable to interpret the existence of a dotted line that shows the correspondence between one point and another and the distance between the four lines is the same. The subject's steps in solving the spatial orientation problem can be given in the image below.

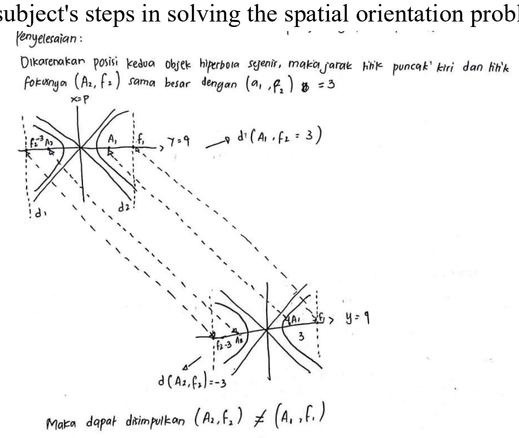

Figure 8. Steps to Solve Spatial Orientation Problems

Based on Figure 8 above in the steps to solve the spatial orientation problem in the steps to check the results of solving the spatial orientation problem, it is known that the field-dependent subject did not check the solution of the results through the form of mathematical inequality, until finally the final answer was wrong. This can be seen from the results of the problem solving sheet where no checking process was found after the answer stage was given, so that there is a possibility that the solution given is wrong. Based on observations from researchers, the solution given by the subject is wrong.

Based on Polya's four stages in solving horizontal hyperbola problems in spatial orientation abilities in subjects with a *field-dependent cognitive style* , the solution is less good by providing the first and third steps in solving Polya's problems and the answers given by the subjects are wrong.

The results of the interview with the field-dependent subject during the problem-solving process on spatial orientation ability are given in the following transcript form:

Researcher : After reading and observing the question number, what can you understand based on the spatial orientation problem?
Subject : What I understand based on observations of the spatial orientation problem is determining the distance of the left peak point to its focal point and drawing a hyperbola object with information on the distance of the focal point to its peak point (if the distance of the right peak point to its focal point is known).
Researcher : How do you determine the distance from the left vertex to the focal point of the hyperbola?
Subject : My method of determining the distance between the left vertex and the focal point is by observing that the two hyperbolic objects are similar, so that the distance between the left vertex and the focal point is the same as the distance between the right vertex and the focal point.
Researcher : What can you conclude from solving this problem?
Subject : What I can conclude is that the distance from the left vertex to the focal point is 3 units.

Conclusion

Based on the results of the analysis of the research data obtained, a conclusion can be drawn based on the research objective, namely that the spatial reasoning of *field-dependent subjects* with moderate abilities has poor hyperbolic problem solving based on the analysis of problem-solving steps based on Polya's theory.

In the step of understanding hyperbola problems based on problem solving analysis on spatial visualization ability, mental rotation and spatial orientation, it is known that the subject wrote down what is known and what is asked according to the given problem. Furthermore, in the step of planning problem solving, it is known that the subject did not write down sufficient and necessary conditions for information on the three indicators of spatial reasoning. In the third step, namely solving hyperbola problems. It is known that the subject solved the problem on spatial visualization ability, mental rotation and spatial orientation. The last step is checking the results of the solution. It is known that the subject did not show steps in reviewing the results of the solution that had been done, this shows that in spatial visualization ability the results of the solution are correct while in mental rotation ability and spatial orientation the results of the solution are wrong. Thus it can be concluded that the subject's problem solving of hyperbola with a field-dependent cognitive style has poor problem solving.

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